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Three flavour oscillation interpretation of neutrino data

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Abstract. I consider the mixing of the three active neutrino flavours and obtain the constraints on the parameters of this mixing from the solar, atmospheric and reactor neutrino data.

Keywords. Neutrino oscillations; three flavours; solar neutrinos; atmospheric neutrinos.

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The solar neutrino problem has been highlighted by the Homestake experiment [1] thirty years ago and the subsequent solar neutrino experiments [2–5] have confirmed it. The data from all the experiments together cannot be explained by changing the solar model. These experiments point towards solutions based on neutrino properties rather than those based on astrophysics [6]. Neutrino oscillations provide a very elegant solution to the solar neutrino problem and can easily account for the energy dependent suppression of the neutrino flux that is observed by the different experiments. The recent results of super-Kamiokande on atmospheric neutrinos give independent evidence for neutrino oscillations [7].

The energy and the distance scales of solar neutrinos and atmospheric neutrinos are widely different. The neutrino oscillation probability is a function of $(\delta m^2 L/E)$ where δm^2 is the mass-squared difference of neutrino mass eigenstates, L is the distance of travel and E is the energy of the neutrino. To explain the atmospheric neutrino problem in terms of neutrino oscillations, δm^2 of about 10^{-3} eV^2 [8] is needed whereas the neutrino oscillation solution to the solar neutrino problem requires $\delta m^2 \sim 10^{-5}$ eV^2. Hence both solar and atmospheric neutrino problems cannot be explained in terms of $\nu_e \leftrightarrow \nu_\mu$ oscillations. To generate two widely different mass-squared differences, we need at least three different mass eigenstates. Three different mass eigenstates can only arise through the mixing of three flavours.

The experiments at LEP have established that there are three light neutrino flavours [9]. Hence it is natural to consider the mixing among all the three flavours. As mentioned above, three flavour mixing has the added advantage that it can account for solar and atmospheric neutrino problems simultaneously. In three flavour mixing, neutrino oscillations depend on six parameters: two mass-squared differences $\delta_{21} = m_2^2 - m_1^2$ and $\delta_{31} = m_3^2 - m_1^2$ where m_i are the mass eigenvalues, three mixing angles ω, ϕ and ψ and a CP-violating phase. This large number of parameters can make the analysis very complicated [10]. However, because $\delta m^2_{\rm atm} \gg \delta m^2_{\rm sol}$, we can make the approximation $\delta_{31} \gg \delta_{21}$. Then $\delta_{31} \simeq \delta_{32}$ sets the oscillation scale for atmospheric neutrino problem and δ_{21} does so for the solar neutrino problem. Within this approximation, the algebra gets simplified considerably. It can be shown that the solar neutrino problem depends essentially on δ_{21}, ω and ϕ [11,12] and the atmospheric neutrino problem depends only on δ_{31}, ϕ and ψ [13]. In each case, features of three flavour ocillations are still present. The oscillation probability, though a function of a single mass-squared difference, depends on *two* mixing angles rather than on only one, which would have been the case for two flavour oscillations.

We assume that the three flavours of neutrinos ν_e, ν_μ and ν_τ mix with one another via the unitary matrix and form three mass eigenstates ν_1, ν_2 and ν_3

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = U \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}. \tag{1}$$

As in the case of quark sector, the 3×3 unitary matrix U can be parametrized by three angles and a phase [11]

$$U = U_{23}(\psi) U_{\rm phase} U_{13}(\phi) U_{12}(\omega), \tag{2}$$

where U_{ij} is the matrix which leads to mixing between ith and jth states and is parametrized by a single angle. $U_{\rm phase}$ is a diagonal complex matrix, which leads to CP violation. Without loss of generality we can assume that the mass eigenvalues satisfy the inequalitites $m_1 < m_2 < m_3$. Then the ranges for the mixing angles are

$$0 \leq \omega, \phi, \psi \leq \pi/2. \tag{3}$$

Now all the possibilities, such as whether ν_e contains more of ν_1 than ν_2 or vice versa, are taken into consideration. We make the further assumption that $(m_3^2 - m_1^2) \gg (m_2^2 - m_1^2)$ so that both solar and atmospheric neutrino problems can be solved simultaneously.

In the solar neutrino problem, the experiments are sensitive only to the electron neutrino survival probability: that is the probability for an electron neutrino produced in the sun to be detected as an electron neutrino on earth. This probability depends only on U_{ei}, the elements of the first row of the mixing matrix U. As can be seen from eq. (1), these elements depend only on the mixing angles ω and ϕ. Hence the angle ψ and the phase can be set to zero [11,12]. Due to the assumption $\delta_{31} >> \delta_{21}$, the terms containing δ_{31} get averaged out and the expression for the survival probability depends on δ_{21}, ω and ϕ only. The expression for the survival probability and the analysis of the data is described in [14,15]. The results of our analysis are presented in figure 1. On the x-axis, $\log(\sin^2 2\omega)$ is plotted and δ_{21} is plotted on the y-axis in $\log$ scale. Different panels correspond to different values of ϕ. These values are

- For panel (1): $\phi = 0$
- For panel (2): $\phi = 10°$
- For panel (3): $\phi = 20°$
- For panel (4): $\phi = 30°$
- For panel (5): $\phi = 40°$
- For panel (6): $\phi = 45°$

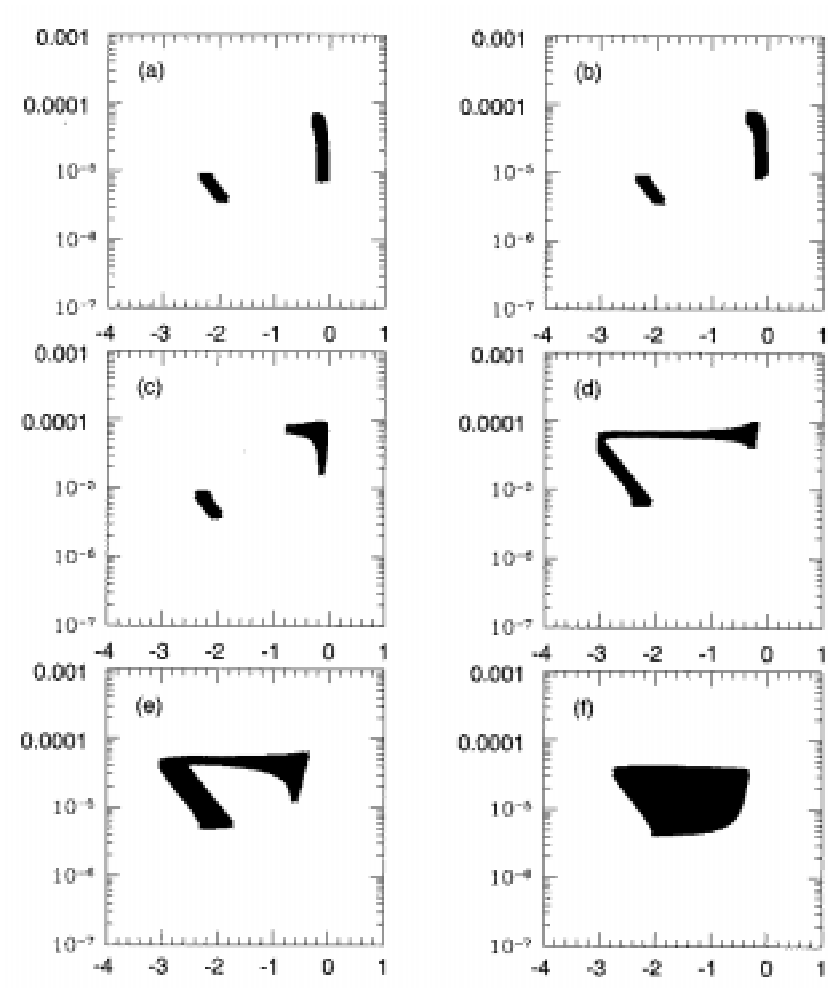

Figure 1. Allowed regions in the $\omega - \delta_{21}$ plane for various values of ϕ. The x-axis shows log $(\sin^2 2\omega)$ and the y-axis δ_{21} in eV^2.

From these, we find that the allowed values of δ_{21} are

$$4 \times 10^{-6}\ \text{eV}^2 \leq \delta_{21} \leq 8 \times 10^{-5}\ \text{eV}^2. \qquad (4)$$

The allowed values of ω are strong functions of ϕ. For smaller values of ϕ, there are two distinct allowed regions of ω: Small ω, with a range of $1°$–$3°$ and large ω with a range of $25°$–$45°$. For large values of ϕ the range of ω $(1°$–$23°)$ is continuous. The range of ϕ is

$$0 \leq \phi \leq 50°. \qquad (5)$$

The primary cosmic rays collide with the atomic nuclei in the atmosphere and produce a number of pions and kaons. The decays of these mesons and their decay products produce two muon neutrinos for every electron neutrino. Here we do not distinguish between neutrinos and anti-neutrinos. Detailed simulations of cosmic ray interactions show that the naive expectation is correct. However, it is impossible to predict the flux of muon neutrinos or electron neutrinos accurately because of the large uncertainty in the overall normalization of the cosmic ray flux. But their ratio can be predicted to good accuracy. IMB [16], Kamiokande [17] and super-Kamiokande [7] have measured the ratio of single ring μ events to single ring e events. Their measurements are presented in the form of the double ratio

$$R = \left(\frac{\Phi_\mu}{\Phi_e}\right)_{\rm data} \Big/ \left(\frac{\Phi_\mu}{\Phi_e}\right)_{\rm MC}. \tag{6}$$

The double ratio will be 1 if there are no neutrino oscillations. Super-Kamiokande measures this value to be $R = 0.68 \pm 0.05$ for their sub-GeV data and $R = 0.68 \pm 0.09$ for their multi-GeV data. Moreover, the multi-GeV muon data shows a zenith angle dependence with large suppression for upward going neutrinos and less suppression for downward going neutrinos. The electron data has no zenith angle dependence. The zenith angle dependence of muon data is precisely what one expects if $\nu_\mu \to \nu_x$ oscillations take place. Note, however, that $x \neq e$ because then electron data should show corresponding increase with zenith angle. Since no such effect is seen, we conclude that $x = \tau$ or $x =$sterile. A complete analysis of super-Kamiokande atmospheric neutrino data in the framework of two flavour oscillations shows that the mixing angle should be maximal $(\simeq 45^\circ)$ and the mass-squared difference should be in the range $(1\text{–}8) \times 10^{-3}$ eV^2. $\nu_\mu \to \nu_\tau$ oscillations are not affected by the matter term which arises due to the passage of neutrinos through the earth matter but $\nu_\mu \to \nu_{\rm sterile}$ oscillations are. The matter effects produce distortions in the zenith angle dependence of muon data. In the recent, more precise data [18] no such distortions are seen. Hence $\nu_\mu \to \nu_\tau$ oscillations are favoured over $\nu_\mu \to \nu_{\rm sterile}$ oscillations.

In interpreting the atmospheric neutrino data in terms of three flavour oscillations, we must keep in mind that one mass-squared difference δ_{21} has been set equal to about 10^{-5} eV^2 by the solar neutrino data. Since the mass-squared difference relevant for atmospheric neutrino problem is of the order of 10^{-3} eV^2, we set this equal to δ_{31}. Now we have $\delta_{31} \simeq \delta_{32} \gg \delta_{21}$ and can set δ_{21} equal to zero in the oscillation probabilities. In that case, mass eigenstates 1 and 2 become degenerate and cannot be distinguished. Hence the 12 mixing angle ω can be set zero. As in the case of quark sector, the CP violation is zero if two of the masses are degenerate. So the CP violating phase can also be set equal to zero. Thus the three flavour oscillation analysis of atmospheric neutrino data depends on the three parameters δ_{31}, ϕ and ψ. This analysis is performed in reference [19] and allowed range for δ_{31} is

$$10^{-4}\ {\rm eV}^2 \leq \delta_{31} \leq 8 \times 10^{-4}\ {\rm eV}^2, \tag{7}$$

which is the same that one obtains in the two flavour analysis. The mixing angle ψ once again is required to be maximal. Recall that only the muon data has significant zenith angle dependent suppression whereas the electron data seems to be consistent with the MC calculation and does not have any zenith angle dependence. This is a signal that $\nu_\mu \to \nu_e$ oscillations are *not* the dominant cause for muon neutrino depletion. If we calculate the $\nu_\mu \to \nu_e$ oscillation probability we find it to be

$$P(\nu_\mu \to \nu_e) = \sin^2\psi \sin^2 2\phi \sin^2\left(\frac{\delta_{31}L}{E}\right). \tag{8}$$

Given that $\psi \simeq 45°$, it would seem, from eq. (8), that ϕ should be small to prevent large $\nu_\mu \to \nu_e$ oscillations. However, it was pointed [20] that this need not be the case. Note that there are 2 muon neutrinos for every electron neutrino. Because $\psi \simeq 45°$, about half the muon neutrinos oscillate into tau neutrinos. So there will be equal numbers of remaining muon and electron neutrinos. Since there is no CP violation, we have $P(\nu_\mu \to \nu_e) = P(\nu_e \to \nu_\mu)$. So as many muon neutrinos oscillate into electron neutrinos as the electron neutrinos into muon neutrinos. Hence the electron neutrino number is unaffected and we will not be able to see any signal for $\nu_\mu \to \nu_e$ oscillation in this scenario. So no meaningful constraint can be set on ϕ from atmospheric neutrino data.

A very strong constraint on ϕ can be obtained, however, from the reactor experiment CHOOZ [21]. This experiment measured the probability $P(\bar{\nu}_e \to \bar{\nu}_e)$ and set a limit on the probability for disappearance. The experimental limit is

$$P(\bar{\nu}_e \to \text{anything}) \leq 0.08. \tag{9}$$

For the CHOOZ experiment, the distance between the source and the detector is 1 km and the average energy of the neutrinos is 3 MeV. Hence CHOOZ is sensitive to the same values of mass-squared difference which are relevant for the atmospheric neutrino problem. If we interpret the CHOOZ result in the above scenario of three flavour oscillations with one dominant mass, we get the constraint

$$\sin^2 2\phi \sin^2\left(\frac{\delta_{31}L}{E}\right) \leq 0.08. \tag{10}$$

This equation is independent of ψ and gives us the strong constraint [22]

$$\sin^2 2\phi \leq 0.2, \tag{11}$$

if the atmospheric data constraint $\delta_{31} \geq 10^{-3}$ eV^2 is imposed. Note that the above equation does not constrain ϕ to be small. It says that either $\phi \leq 13°$ or $\phi \geq 77°$. However, the solar neutrino data gives the constraint that $\phi \leq 50°$. Combining both constraints together, we obtain the range

$$0 \leq \phi \leq 13°. \tag{12}$$

A non-zero value for ϕ leads to $\nu_\mu \to \nu_e$ oscillations at the present and future long baseline experiments K2K and MINOS. This signal will be large enough to be observable if ϕ is a few degrees [23].

There is one neutrino oscillation experiment we must discuss. The LSND experiment observed a non-zero value for the oscillation probability

$$P(\nu_\mu \to \nu_e) = (3.3 \pm 1.1) \times 10^{-3}. \tag{13}$$

The distance and the energy scales of LSND are such that it is also sensitive to the mass-squared difference relevant for the atmospheric neutrino problem. In fact, we can obtain values of δ_{31}, ϕ and ψ which can simultaneously explain both LSND and atmospheric neutrino problem [24]. However, the values of ϕ we need in this case are in conflict with the range of ϕ given in eq. (12). Hence we cannot simultaneously account for CHOOZ, LSND and atmospheric and solar neutrino problems [22]. We need a minimum of four flavours to accomplish that [25,26].

In conclusion, we can say that the neutrino data restrict the neutrino parameters quite well. By analysing the data from solar neutrino experiments, atmospheric neutrino experiments and CHOOZ in the three flavour mixing scenario with one mass-squared much larger than the other, we get

$$4 \times 10^{-6}\ \mathrm{eV}^2 \leq \delta_{21} \leq 8 \times 10^{-5}\ \mathrm{eV}^2 \tag{14}$$

$$10^{-3}\ \mathrm{eV}^2 \leq \delta_{31} \leq 8 \times 10^{-3}\ \mathrm{eV}^2$$

$$35^\circ \leq \psi \leq 55^\circ$$

$$0 \leq \phi \leq 13^\circ. \tag{15}$$

We see that the mass-squared differences are determined to within an order of magnitude and ranges for ϕ and ψ are quite small. Only ω is not well constrained. In fact we have two allowed regions of ω for the allowed values of ϕ.

$$\begin{aligned} \mathrm{Small}\,\omega: &\ \ 1^\circ \leq \omega \leq 3^\circ \\ \mathrm{Large}\,\omega: &\ 25^\circ \leq \omega \leq 45^\circ. \end{aligned} \tag{16}$$

Super-Kamiokande data on day–night asymmetry and the electron spectrum distortion can help us in distinguishing between the small and the large ω solutions [27–29]. Future solar neutrino experiments such as SNO and Borexino will precisely determine which of the presently allowed solutions are relevant for the solar neutrino problem.

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References

[1] B T Cleveland *et al*, *Astrophys. J.* **496**, 505 (1998)
[2] Kamiokande Collaboration: Y Fukuda *et al*, *Phys. Rev. Lett.* **77**, 1683 (1996)
[3] SAGE Collaboration: V Gavrin *et al*, in *Neutrino 96*, Proceedings of the XVII International Conference on Neutrino Physics and Astrophysics, Helsinki, edited by K Huitu, K Enqvist and J Maalampi (World Scientific, Singapore, 1997) p. 14
[4] GALLEX Collaboration: P Anselman *et al*, *Phys. Lett.* **B342**, 440 (1995)
W Hampel *et al*, *Phys. Lett.* **B388**, 364 (1996)
[5] Super-Kamiokande Collaboration: Y Fukuda *et al*, *Phys. Rev. Lett.* **81**, 1158 (1998); erratum *ibid* **81**, 4279 (1998)
[6] N Hata, S Bludman and P Langacker, *Phys. Rev.* **D49**, 3622 (1994)
[7] Super-Kamiokande Collaboration: Y Fukuda *et al*, *Phys. Rev. Lett.* **81**, 1562 (1998)
[8] K Scholberg, 8th International Workshop on Neutrino Telescopes, Venice, February 1999 ep-ex/9905016
[9] Particle Data Group, *European J. Phys.* **3**, 23 (1998)

[10] V Barger, K Whisnant, S Pakvasa and R J N Phillips, *Phys. Rev.* **D22**, 2718 (1980)
[11] T K Kuo and J Pantaleone, *Rev. Mod. Phys.* **61**, 937 (1989)
[12] A S Joshipura and M V N Murthy, *Phys. Rev.* **D37**, 1374 (1988)
[13] J Pantaleone, *Phys. Rev.* **D49**, R2152 (1994)
[14] M Narayan, M V N Murthy, G Rajasekaran and S Uma Sankar, *Phys. Rev.* **D53**, 2809 (1996)
[15] Mohan Narayan, University of Madras, Physics Ph.D. Thesis; talk presented at the *Discussion meeting on Neutrino Physics*, Physical Research Laboratory, Ahmedabad, February 1999, to be published in the proceedings (*Pramana Suppl.*)
[16] IMB Collaboration: D Casper *et al*, *Phys. Rev. Lett.* **66**, 2561 (1989)
R Becker-Szendy *et al*, *Phys. Rev* **D46**, 3720 (1989)
T J Haines *et al*, *Phys. Rev. Lett.* **57**, 1986 (1986)
[17] Kamiokande Collaboration: K S Hirata *et al*, *Phys. Lett.* **B205**, 416 (1988); *ibid* **B280**, 146 (1992)
Y Fukuda *et al*, *Phys. Lett.* **B335**, 237 (1994)
[18] Chang Kee Jung, talk presented at 16th European Physical Society High Energy Physics Conference, Tampere, Finland, July 1999
[19] G L Fogli, E Lisi, A Marrone and G Scoscia, *Phys. Rev.* **D59**, 033001 (1999)
[20] J Pantaleone, *Phys. Rev. Lett.* **81**, 5060 (1998)
[21] CHOOZ Collaboration: M Apllonio *et al*, *Phys. Lett.* **B420**, 397 (1998)
[22] M Narayan, G Rajasekaran and S Uma Sankar, *Phys. Rev.* **D58**, 031301 (1998)
[23] Y Oyama, talk presented at *YITP workshop on flavour physics*, Kyoto, January 1998; preprint hep-ex/9803014
[24] M Narayan, G Rajasekaran and S Uma Sankar, *Phys. Rev.* **D56**, 437 (1997)
[25] S Goswami, K Kar and A Raychaudhuri, *Int. J. Mod. Phys.* **A12**, 781 (1997)
[26] S M Bilenky, C Giunti and W Grimus, hep-ph/9812360
[27] J N Bahcall, P I Krastev and A Yu Smirnov, *Phys. Rev.* **D58**, 096016 (1998)
[28] J N Bahcall, P I Krastev and A Yu Smirnov, hep-ph/9905220
[29] S Uma Sankar, talk presented at the *Discussion meeting on Neutrino Physics*, Physical Research Laboratory, Ahmedabad, February 1999, to be published in the proceedings (*Pramana Suppl.*)