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Review of recent results in deep inelastic scattering

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Abstract. This talk gives a summary of recent results in deep inelastic lepton hadron scattering. This includes structure functions from inclusive measurements as well as fragmentation in semi-inclusive processes, mainly with respect to data from colliders such as HERA at DESY, and their associated phenomenology.

Keywords. Deep inelastic scattering; structure function.

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1. Introduction

In this overview, I will selectively highlight some results (both experimental and theoretical) that I have found interesting. After a brief overview of theoretical results in basic deep inelastic scattering (DIS) including structure functions, and semi-inclusive processes, I will discuss the new results from HERA, including structure functions, fragmentation functions, as well as a determination of the W mass and α_s.

Theoretical issues in DIS broadly fall into two categories: pure DGLAP evolution of parton densities (whether of proton or photon, polarised or unpolarised), and deviations from pure DGLAP behaviour. The latter class includes the BFKL type of resummation at low-x, inclusion of higher twist effects, colour coherence (as in MLLA evolution of fragmentation functions), etc. Many QCD effects, especially in fragmentation, are included in Monte Carlos (where they are implemented according to different types of models) and I will discuss them briefly as well.

2. Deep inelastic scattering

2.1 *Notation and definitions*

Inclusive DIS where the proton breaks up after being struck by a lepton can be described in terms of two variables, $Q^2 = -q^2$, the momentum square transferred from the lepton to the proton vertex, and $x = Q^2/2\nu$, where the energy transfer, $\nu = p \cdot q$. Here q, p are the momenta of the photon and proton. The cross section can be expressed in terms of structure functions that describe the photon–proton vertex. We have (for unpolarised scattering)

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{2\pi\alpha^2}{xQ^4}\left[Y_+F_2(x,Q^2) - Y_-xF_3(x,Q^2) - y^2F_L(x,Q^2)\right], \qquad (1)$$

where $Y_\pm = 1\pm,(1-y)^2$, $Q^2 = sxy$, where s is the c.m. energy of the process. While F_3 is negligible for $Q^2 \ll m_Z^2$, F_L is important only at large y and so F_2 is so far the best studied structure function. Sometimes the reduced structure function, $\sigma_r = F_2 - (y^2/Y_+)F_L(x,Q^2)$, is also defined.

2.2 *Scaling violations*

As is well known, the structure functions can be expressed in terms of parton distributions in the parton picture, with partons having momentum fraction, x of the parent proton. The cross-section can then be expressed in terms of a convolution of these (perturbatively incalculable) parton distribution functions with the (perturbatively calculable) hard scattering lepton–parton cross-section. QCD then incorporates Q^2 dependence into the density distributions by scaling violations, traditionally calculated in the DGLAP framework. They are purely logarithmic and have been calculated to NLO for both the polarised and unpolarised densities. The equations, for the singlet parton densities, are coupled differential equations and can be expressed as

$$\frac{\partial q^{NS}}{\partial t} = \frac{\alpha_s}{2\pi}P_{qq}^{NS}q^{NS}\ ;\ \ \frac{\partial}{\partial t}\begin{pmatrix}\Sigma\\ g\end{pmatrix} = \frac{\alpha_s}{2\pi}\begin{bmatrix}P_{qq} & P_{qg}\\ P_{gq} & P_{gg}\end{bmatrix}\otimes\begin{pmatrix}\Sigma\\ g\end{pmatrix}. \qquad (2)$$

Here the Q^2 dependence is shown with respect to $t = \log(Q^2)$, $\Sigma = \sum_i q_i$, where q and g are the quark and gluon density distribution functions (expressible in terms of twist-2 operator matrix elements). The P_{ij} are the perturbatively calculable splitting functions that give the probability of finding an ith type parton in a j type one. Within the DGLAP approach, only ladder-type diagrams dominate at large Q^2, with the singlet Σ dominating at low-x.

In general, DGLAP predicts a rapidly rising gluon at small x due to a pole of the form $1/x$ in P_{gg}. Because the gluon is coupled to the singlet evolution, this implies also a rapidly increasing sea. Furthermore, the charm content of F_2, that is, F_2^c arises purely from the gluon, in $g \to c\bar{c}$. Hence F_2^c at low x must also reflect the (rising) gluon behaviour. Of course, the increase of Σ is crucially dependent on the increase of gluon. Finally, the F_L structure function occurs at a higher order than F_2 and turns out to be driven by the gluon as well. Hence DGLAP predicts that the low-x F_2 is dominated by the singlet densities, its slope, $\mathrm{d}F_2/\mathrm{d}\log Q^2$, is dominated by the gluon, as also are the charm structure function and the longitudinal structure function, F_L.

DGLAP resums at a fixed x, so that it resums terms of order $(\alpha_s \log Q^2)^n$. Another way of resumming, taking into account the fact that $\log(1/x)$ terms can be equally significant at low x, is the BFKL approach, which resums terms of order $(\alpha_s \log(1/x))^n$, at fixed Q^2. The difference arises in the ordering placed on the transverse k_T of the parton ladder (see figure 1): DGLAP assumes a strong ordering $k_T^n \gg k_T^{n-1} \gg \ldots \gg k_T^1$, where parton 1 is closest to the hadron vertex, while BFKL assumes no such ordering, but assumes

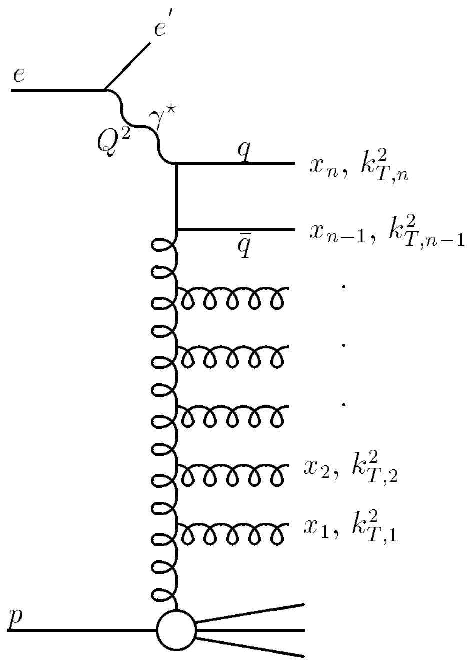

Figure 1. Typical (singlet) evolution at low x showing the ladder of gluons with momenta, $(x_i, k_{T,i})$ respectively. The DGLAP and BFKL approaches correspond to different orderings of these.

an ordering in x. This leads to an evolution equation in $\log 1/x$ at a fixed (but suitably large) value of Q^2,

$$xg(x, Q^2) \sim x^{-\lambda} \ , \tag{3}$$

where $\lambda = C_2(A)(\alpha_s/\pi)4\log 2 \sim 0.5$ in LO at fixed Q^2. The NLO corrections were computed recently and found to be large. However, the basic prediction is a rise of the gluon and hence F_2 at low x.

There are many other choices of evolution equation, valid in different kinematic regimes: the DLLA which resume terms of order $(\alpha_s \log Q^2 \log(1/x))^n$, and the GLR equation (and its variations) which include parton overlap in a kind of twist-4 term. I will not discuss them in detail here.

The basic questions in inclusive F_2 measurements are (1) what is the observed Q^2 dependence of F_2 and (2) what is the physics that decides the Q^2 behaviour. An accurate measurement of the Q^2 dependence may be able to distinguish the underlying physical phenomena at work. We will see, when we discuss the results from HERA, that the data is as yet inconclusive.

3. Parton branching and fragmentation

Parton branching applies in particle production in jets. For time-like branching, soft enhancements occur when the propagator goes on-shell. These soft enhancements can be factored out, and resummed, within the eikonal approximation. This results in angular ordering, so that soft emissions occur only within a small cone around the leading parton. The gluon coherence in fact destroys the enhancement by reducing the available phase space. An inclusion of this effect into the usual evolution equation for parton fragmentation functions results in the modified leading log (MLLA) approximation, in which the rise of the gluon at small x_p is in fact supressed. Here x_p is the fraction of the momentum of the parton carried by the hadron into which it fragments. It is related to the variable in space-like DIS by $x \to -1/x_p$. To LO, the splitting functions are the same as in DIS, with the replacement $P_{ij} \to P_{ji}$ for $i \neq j$. MLLA thus predicts a gaussian behaviour of the singlet fragmentation function:

$$D(x_p, t) \sim N \exp\left[-\left(\log 1/x - \log 1/x_p^0\right)^2 / (2\sigma^2)\right] \,, \tag{4}$$

where the peak and width x_p^0 and σ of the gaussian are also predicted. At large x_p, the usual DGLAP evolution holds, and predicts scaling violations similar to that in DIS. Finally, another question that one can ask for fragmentation functions is whether they are universal: for example, if they are target independent, then results from HERA must be similar to those from the $e^+ e^-$ machine, LEP.

4. Models of hadronisation

This is a brief list of current models of hadronisations of parton showers in jets, and associated Monte Carlos.

The Lund model is a string-based model of fragmentation, implemented in the Monte Carlo, JETSET. This includes both parton distribution functions and QED radiative corrections.

The LEPTO Monte Carlo is based on LO-DGLAP evolution, and should fail when $E^2_{T,\mathrm{jet}} \geq Q^2$. RAPGAP, on the other hand, includes photon structure contribution (resolved photon contribution), and uses the scale, $Q^2 + p_T^2$.

The ARIADNE colour dipole model assumes a chain of independently radiating dipoles formed by emitting gluons. The parton cascade is *not* k_T ordered; it assumes a scale p_T^2 and hence is popularly expected to reproduce BFKL-like behaviour.

There also exists an independent BFKL routine, that uses LO-BFKL, normalised to jet data (since BFKL cannot predict the overall normalisation, only the power of x, for F_2); this is then used to predict the particle spectrum, using standard fragmentation functions.

5. Comparison with data

5.1 *DIS at HERA*

HERA is an e-p collider, with 27.5 and 820 GeV energy of the electron and proton beam respectively. The results are from the two detectors, H1 and ZEUS. The rise of F_2 at small x is clearly seen at both detectors; also, the behaviour of the data agrees with standard DGLAP evolution when $Q^2 \gg 1$ GeV^2.

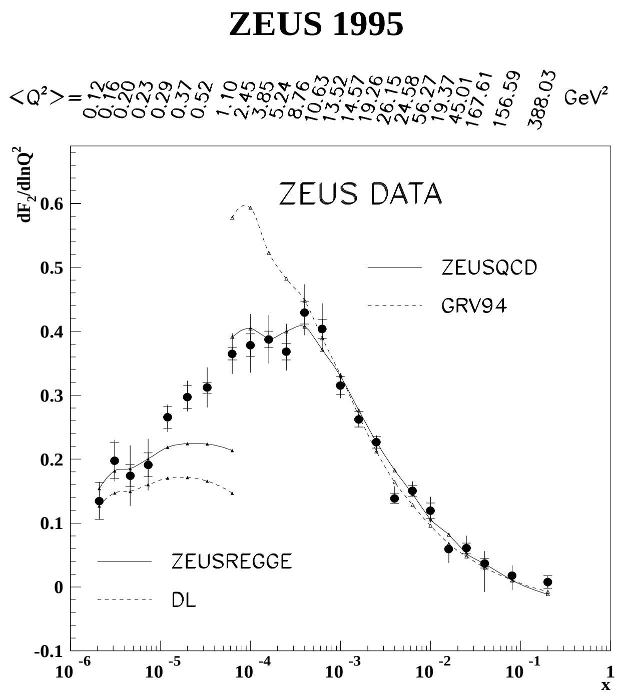

Figure 2. Slope of the F_2 structure function shown as a function of x. Data are from ZEUS [5] and correspond to different averaged Q^2 values. The GRV prediction is also shown.

The slope of the structure function however caused some excitement recently. This is because, as figure 2 shows, the slope is clearly decreasing with x, turning around at $x \sim 10^{-4}$, where Q^2 is still not small: $Q^2 > 2$ GeV^2. Martin *et al* [1] attribute this to a vanishing gluon and a rising sea at small x. Recall that F_2 is driven by the sea, while its slope is determined by the gluon. Such a non-traditional behaviour of the low x densities can then account for the data. This implies either a break-down of DGLAP or a rapidly fluctuating gluon at low Q^2. However, it must be noted that the same gluon densities drive F_2^c; it can be seen from figure 3 that, down to about $q^2 \sim 2$ GeV^2, the charm contribution to F_2 is seen to rise at low x, consistent with DGLAP, albeit with larger error bars. Hence it may be necessary to seek other solutions of the puzzle of the small-x slope of F_2. One possibility is to include higher twist terms, either by using a modified evolution equation [2], or by using a VMD-kind of approach [3].

It is also possible to analyse the data itself for possible higher twist effects, purely phenomenologically. The reduced cross-section, σ_r, has been described in terms of F_2 and F_L. Hence the slope $\mathrm{d}\sigma_r/\mathrm{d}y$ at constant x or Q^2 involves the slope of F_2 with respect to Q^2 or x. The large y data determines F_L, while the small y data is dominated by F_2. Hence at small y, $\mathrm{d}\sigma_r/\mathrm{d}y|_x \simeq \mathrm{d}F_2/\mathrm{d}\log Q^2$, while the large y data involves F_L as well. From data on σ_r and F_L, it is therefore possible to obtain the slope $\mathrm{d}F_2/\mathrm{d}\log Q^2$ at two different Q^2 for the same x values, and hence to obtain the Q^2 dependence of the slope of F_2. Such an analysis has been done [4]; however, while there are hints that pure twist-2 based analysis cannot fit the data, and a twist-4 component may be required, the

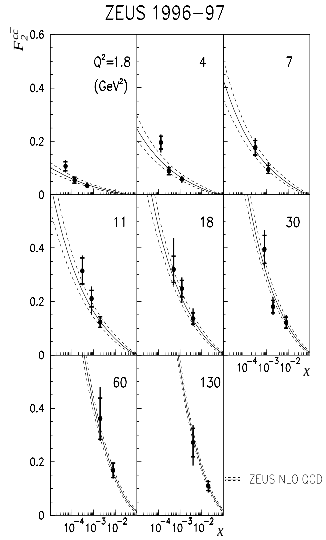

Figure 3. The charm structure function, F_2^{cc}, shown as a function of x. The data is from ZEUS [6].

error bars on the data preclude a firmer result. However, once more data is acquired, it may be possible to look for the higher twist effects.

5.2 *Fragmentation at HERA*

We now turn to results on fragmentation. The predicted MLLA behaviour at low x has indeed been seen at HERA (as in LEP), as seen in figure 4. The theoretical prediction of the peak of the gaussian is in agreement with data not only from HERA, but other e^+e^- experiments such as TASSO, OPAL and TOPAZ, at different $\sqrt{s}$ values, thus confirming universality of fragmentation. This is shown in figure 5, where the prediction from pure phase space arguments is seen to be in disagreement with data. At high x, typical scaling violations are seen, similar to those in e^+e^- experiments. Hence the universality of fragmentation is observed to hold for large x as well.

5.3 *Other measurements*

One of the best testing grounds for BFKL dynamics is from forward jets at low x. At large Q^2, and small transverse momentum, E_T, of the jet, DGLAP is expected to hold. At very small Q^2 the photon is almost real and the experiment is expected to probe the photon structure. In the intermediate regime, $Q^2/2 < E_T < 2Q^2$, where BFKL dynamics is expected to be dominant, it is seen from figure 6 that DGLAP based models fail to describe the data. While the ARIADNE Monte Carlo does fit better than the rest, it is not

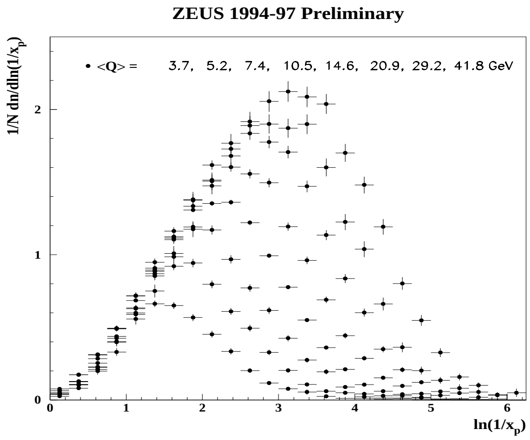

Figure 4. Semi-inclusive data from ZEUS [7], showing the evolution of the charged particle distribution with Q^2, as a function of $\log(1/x_p)$, where x_p is the fraction of momentum of the fragmenting quark that is carried by the hadron. The Gaussian behaviour predicted by MLLA is clearly seen.

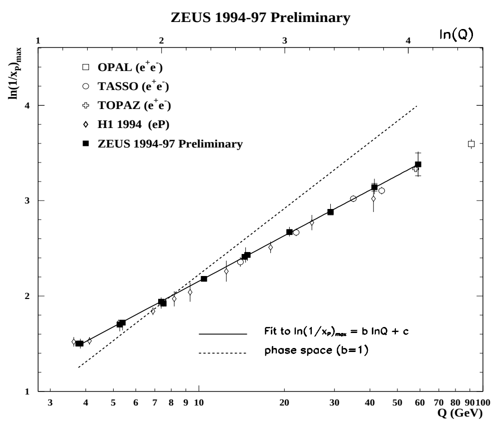

Figure 5. The evolution of the peak position of the gaussian in figure 4 with Q^2.

yet clear that this is indeed evidence for BFKL dynamics. In fact, it is seen that DGLAP dynamics along with a resolved photon structure function is able to fit the data as well. Similar results are seen in the inclusive particle spectrum, for both charged and neutral particles (pions).

A lot of data is also now available on photon structure functions. The resolved component is certainly required in order to match theory with data for dijet photoproduction as well as for charm photoproduction.

Charged current data is also available, for large q^2, where W exchange begins to become important [8]. The cross-section is sensitive to the mass of the W, and in fact, has the best

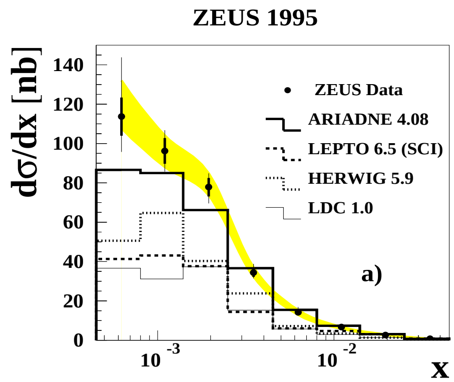

Figure 6. The forward jet cross-section at hadron level, in comparison with various Monte Carlos. The data correspond to $0.5 < E^2_{T,\mathrm{jet}}/Q^2 < 2$ [7].

errors on this quantity to date: $M_W = 78.6 \pm 2.5 \pm 3 \pm 1.5$ (parametrisation). It is also a good test of the d quark density at large x values. Both the neutral current and charge current cross-sections agree very well with data at large Q^2 upto 10,000 GeV^2. This precision data is now in fact claimed to be a bench-mark for the standard model.

In this same kinematical regime, the excess events (the 'HERA large Q^2 excess') that was much talked about seems to be less significant. Upon including the 1997 data, the excess events at $Q^2 > 15,000$ GeV^2 for the NC events remain, but the excess is not seen in the 1997 data alone. Also, the excess in the CC channel is compatible with the standard model, within errors.

A novel measurement that has been presented by H1 is that of the pion structure function [9]. This was done by studying events where a forward proton spectrometer detected the presence of a proton with almost the same energy as the incoming proton. There is still very little data, but it is likely that more would soon be accumulated, so that comparisons can be made with existing models.

A new determination of the strong coupling,

$$\alpha_s = 0.114 \pm 0.005 \pm 0.008 \pm 0.005 \text{ (theory)},$$

has been provided by the measurement of the parity violating process in νp scattering by CCFR and NuTev [10]. They obtain this from the sum rule for the F_3 structure function, which they verify:

$$\int_0^1 F_3(x, Q^2)\mathrm{d}x = 3\left[1 - \frac{\alpha_s}{\pi}\right].$$

The NuTeV collaboration [11] has a value for $\sin^2\theta_W$ from the ratio:

$$R(NC/CC) = \frac{\sigma^{\nu}_{NC} - \sigma^{\bar{\nu}}_{NC}}{\sigma^{\nu}_{CC} - \sigma^{\bar{\nu}}_{CC}}.$$

They have $\sin^2\theta_W = 0.2253 \pm 0.0019 \pm 0.001$, from which they also obtain M_W.

The NMC collaboration also has new measurements of the ratio d/u from proton and deuteron DIS [12]. They provide the best constraint on this ratio at different values of x.

6. Summary

There is a lot of new data from several colliders on inclusive and semi-inclusive deep inelastic scattering, both in the neutral current and charge current channel. I have highlighted mainly the results from HERA. While a general agreement with perturbative QCD predictions is emerging, the high quality of the data also allows for detailed predictions to be made on little-known quantities such as higher twist effects, low-x behaviour, and the dynamics of fragmentation. This will in turn help us to better understand the dynamics of hadrons in terms of the dynamics of their underlying constituents.

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References

[1] A D Martin, R G Roberts, W J Stirling and R S Thorne, *Europhys. J.* **C4**, 463 (1998)
[2] E Gotsman, E Levin, U Maor and E Naftali, *Nucl. Phys.* **B539**, 535 (1999)
[3] A D Martin, M G Ryskin and A M Stasto, *Europhys. J.* **C7**, 643 (1999)
[4] R Basu and D Indumathi, *Phys. Lett.* **B452**, 115 (1999)
[5] The ZEUS Collab.: *Europhys. J.* **C7**, 609 (1999)
[6] The ZEUS Collab.: *Europhys. J.* **C12**, 35 (2000)
[7] For the H1 and ZEUS Collab.: N H Brook, talk presented at the Second Latin American Symposium on high energy physics, SILAFAE98, San Juan, Puerto Rico, April 1998; hep-ex/9805031
[8] The ZEUS Collab.: hep-ex/9907010; and the H1 Collab: hep-ex/9908059, to appear in *Europhys. J.* (2000)
[9] The H1 Collab.: C Adloff *et al*, *Europhys. J.* **C6**, 587 (1999)
[10] The CCFR and NuTev Collabs.: J Yu *et al*, talk presented at the 33rd Recontres de Moriond, QCD and Hadronic interactions, France, 1998; hep-ex/9806030
[11] The NuTev and CDF Collabs.: J Yu *et al*, talk presented at the 6th international conference on deep inelastic scattering, Brussels, Belgium, 1998; hep-ex/9806032
[12] The NMC Collab.: *Nucl. Phys.* **B487**, 3 (1997)