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Dynamics of two coupled chaotic multimode Nd:YAG lasers with intracavity frequency doubling crystal

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Abstract. The effect of coupling two chaotic Nd:YAG lasers with intracavity KTP crystal for frequency doubling is numerically studied for the case of the laser operating in three longitudinal modes. It is seen that the system goes from chaotic to periodic and then to steady state as the coupling constant is increased. The intensity time series and phase diagrams are drawn and the Lyapunov characteristic exponent is calculated to characterize the chaotic and periodic regions.

Keywords. Chaos; multimode laser; coupled lasers; Lyapunov exponent.

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1. Introduction

Coupled chaotic systems have attracted much attention recently especially in connection with synchronization of chaotic systems for communication. The effect of coupling on the dynamics of chaotic systems have been studied for different systems like, coupled chaotic oscillators [1,2], coupled chaotic pendula [3], coupled lasers [4–7] etc. Chaos in an array of coupled systems has also been studied for coupled maps [8–10], coupled semiconductor lasers [11], neural networks [12] and in the Josephson junction [13]. In this paper we report the results of our numerical studies on the effect of coupling of two multimode Nd:YAG lasers with intracavity KTP crystal for frequency doubling. The chaotic behavior of such a laser has been studied extensively by many authors both experimentally and theoretically [14–22].

The coupling of two chaotic Nd:YAG lasers have been experimentally studied by Roy and Thornburg [6]. They made the coupling effective by generating two laser beams on the same crystal so that the intracavity laser fields overlap. The coupling strength was varied by changing the seperation between the two lasers. The lasers were driven chaotic through periodic modulation of the pumping laser. They observed synchronized chaotic intensity fluctuations when the coupling strength was sufficiently strong.

We have studied the coupling of two chaotic multimode Nd:YAG lasers with intracavity frequency doubling crystal by using a coupling scheme different from that used by Roy *et al*. We considered external electronic coupling of two separate lasers. We did the coupling in such a way that the pumping of each laser is modulated according to the output intensity of the other. The scheme is similar to the one we have used to couple two chaotic semiconductor lasers [7].

The paper is organized as follows. In §2 the model we have used for our numerical studies is reviewed. Section 3 describes our numerical studies on the chaotic behavior of the intracavity doubled Nd:YAG laser operating with three longitudinal modes. In §4 the effect of coupling of two such chaotic lasers is presented. The results are summarized in §5.

2. The model

The schematic diagram of a diode pumped Nd:YAG laser with intracavity frequency doubling crystal is shown in figure 1. The Nd:YAG laser normally lases at 1064 nm at the infrared. When KTP crystal is introduced into the cavity some of the infrared fundamental is converted to green light at 532 nm. It is possible to make the laser operate in a few number of modes by introducing etalons in to the laser cavity. In order to describe the multilongitudinal mode operation of the laser, Baer [14] modelled the system by a set of coupled nonlinear differential equations. He considered the frequency doubling in the KTP crystal by second harmonic generation and sum frequency generation. The sum frequency generation was assumed to give the mode–mode coupling necessary for the appearance of chaos. Later Bracikowski and Roy [17,18] observed that the intensity fluctuations can be eliminated by suitably changing the relative orientation between the fast axes of the Nd:YAG and KTP crystals. They modified the earlier model by introducing a geometric factor which is a measure of the relative orientation as well as the phase delays of Nd:YAG and KTP crystal, and obtained the following set of coupled differential equations for the intensity I_k and gain G_k associated with the kth longitudinal mode.

$$\tau_{\mathrm{c}} \frac{\mathrm{d}I_k}{\mathrm{d}t} = \left(G_k - \alpha - g\epsilon I_k - 2\epsilon \sum_{j \neq k} \mu_{jk} I_j \right) I_k, \tag{1}$$

$$\tau_{\mathrm{f}} \frac{\mathrm{d}G_k}{\mathrm{d}t} = \gamma - \left(1 + I_k + \beta \sum_{j \neq k} I_j \right) G_k, \tag{2}$$

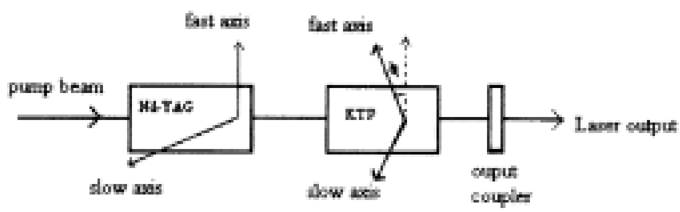

Figure 1. Schematic diagram for a diode pumped Nd:YAG laser with intracavity KTP crystal.

where τ_c is the cavity round trip time, τ_f is the fluorescence life time of Nd^{3+} ion, α is the cavity loss parameter, γ is the small signal gain related to the pump rate, ϵ is a nonlinear coefficient describing the conversion efficiency of the crystal and g is a geometric factor which depends on the phase delays of the Nd:YAG and KTP crystals as well as on the angle between their fast axes. The value of g can be calculated using the relation

$$g = \frac{4u_1^2 u_2^2}{(u_1^2 + u_2^2)^2} \quad \text{for} \quad y \neq 0, \tag{3}$$

where $u_1 = 2I_m(a) - 2\sqrt{1 - [\mathrm{Re}(a)]^2}$ and $u_2 = 2y$ with $a = \mathrm{e}^{i\xi}(\cos^2\phi \mathrm{e}^{i\delta} + \sin^2\phi \mathrm{e}^{-i\delta})$ and $y = \sin 2\phi \sin\delta$. Also

$$g = 0 \quad \text{for} \quad y = 0. \tag{4}$$

Here ϕ is the angle between Nd:YAG and KTP fast axes and ξ and δ are their respective phase delays which can be calculated using the relation $2\pi(n_f - n_s)/\lambda$, where n_f and n_s are fast and slow refractive indices, L is the length of the crystal and λ is the wavelength of light. The value of g varies from 0 to 1 [21]. Each cavity mode can be polarized only in one of the two orthogonal directions (X or Y). For modes j having the same polarization as the kth mode $\mu_{jk} = g$ and for modes having orthogonal polarization $\mu_{jk} = (1 - g)$.

3. Chaotic behavior of the laser

To study the chaotic behavior of the multimode Nd:YAG laser with intracavity KTP crystal we have numerically integrated eqs (1) and (2) for different values of g. The values of the various parameters used for the numerical study are listed in table 1. We assumed that the laser is operating with three coexisting longitudinal modes, with two modes polarized parallel to each other and the third mode polarized orthogonal. The case was experimentally studied by James and Harrel [15]. They found that the chaotic amplitude fluctuations in the system can be eliminated by proper rotary alignment of the angle between the fast axes of the Nd:YAG and KTP crystals. For the numerical study of the system we varied the value of g over the range 0 and 1. As mentioned earlier, the value of this parameter depends on the relative orientations as well as the phase delays of the crystals.

Table 1. Parameter values used for the numerical simulation.

Parameter	Value
τ_c the cavity round trip time	0.2 ns
τ_f the fluorescence life time of Nd^{3+} ion	240 μs
α the cavity loss parameter	0.01
γ the small signal gain related to the pump	0.05
β the cross saturation parameter	0.7
ϵ the nonlinear coefficient of the crystal	5 x 10^{-6}

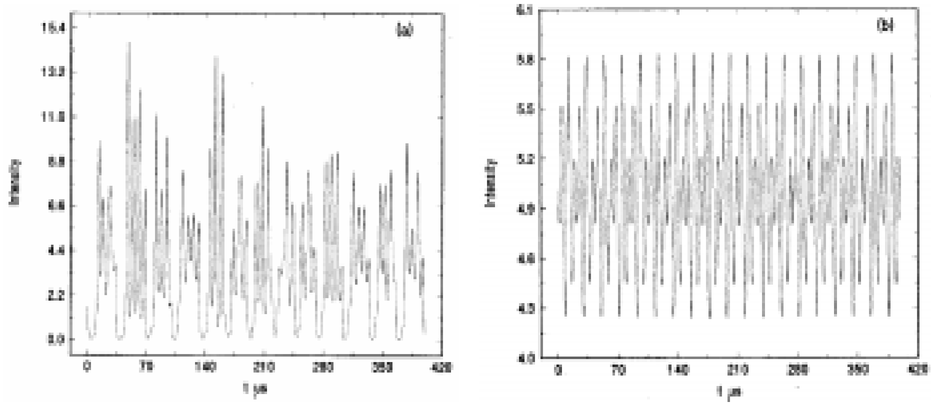

Figure 2. The total intensity time series for the laser (**a**) $g = 0.1$ (chaotic) (**b**) $g = 0.45$ (period 3).

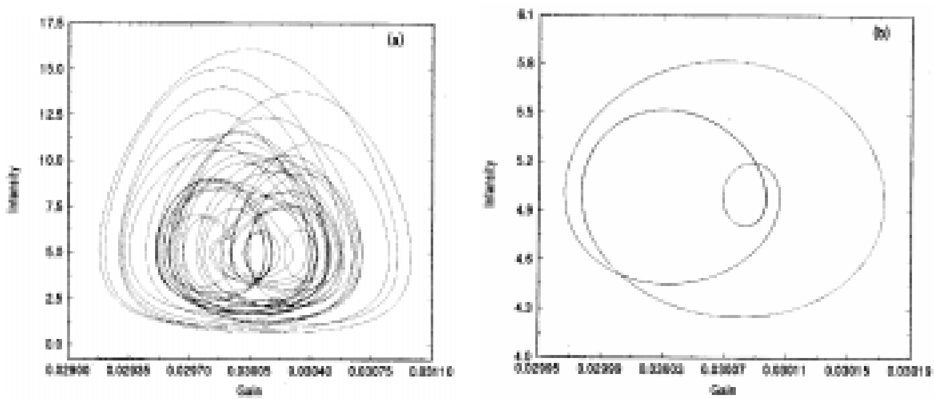

Figure 3. Phase portrait (total intensity vs total gain) (**a**) $g = 0.1$ and (**b**) $g = 0.45$.

As the value of g is increased the fluctuations in intensity of the laser showed chaotic and periodic variations. In figure 2 we plotted the time variation of intensity for different values of g. Figure 2a shows the intensity variations corresponding to $g = 0.1$. The intensity varies in a random manner, which indicates chaotic behavior. The intensity variations are chaotic up to $g = 0.43$ and after that it becomes periodic. When $g = 0.45$ the intensity variations become periodic with period 3. Figure 2b shows the period three oscillations of the laser intensity. In these figures we have considered the total infrared intensity in all the longitudinal modes because the green light produced by frequency doubling is proportional to it.

We have also plotted the phase diagrams for the laser. Here the total intensity is plotted against total gain for differrent values of g. The phase diagram for $g = 0.1$ is shown in figure 3a. It shows a complex structure indicating chaotic behavior. We may call it the strange attractor for the chaotic laser. When $g = 0.45$ we get three closed loops as shown in figure 3b. This indicates the period three oscillations in the laser intensity.

To get a quantitative measure of chaos in the laser system we also calculated the Lyapunov characteristic exponent (LCE) of the intensity time series for different values of g. The Wolf's *et al* algorithm is used for the computation of the maximum value of LCE [23]. The maximum LCE is $+3.6653323\times 10^{-5}$ bits/ns for $g = 0.1$ (chaotic) and it is equal to $-1.642305\times\ 10^{-5}$ bits/ns for $g = 0.45$ (periodic). It can be observed that the LCE is positive when the value of g is between 0 and 0.43. The positive value of LCE shows that the laser oscillations are chaotic in this region. When the value of g is above 0.43 the LCE is found to be negative. This shows the periodic behavior of the laser in this region.

4. Coupling of two chaotic lasers

In this section we consider the effect of coupling two chaotic Nd:YAG lasers with intracavity KTP crystal. It is assumed that the lasers are operating in three longitudinal modes with one mode polarized orthogonal to the other two modes. A schematic diagram of the coupling is shown in figure 4. The two lasers (Laser1 and Laser2) are coupled in such a way that the pump parameter of each laser is modulated according to the output intensity of the other. The corresponding rate equations for the intensity I_k and gain G_k associated with the kth longitudinal mode of Laser1 and Laser2 are given by eqs (5) through (8). Here the subscripts 1 and 2 denote Laser1 and Laser2 respectively.

$$\tau_{\mathrm{c}}\frac{\mathrm{d}I_{k1}}{\mathrm{d}t} = \left(G_{k1} - \alpha - g\epsilon I_{k1} - 2\epsilon \sum_{j1\neq k1} \mu_{j1k1} I_{j1}\right) I_{k1}, \tag{5}$$

$$\tau_{\mathrm{f}}\frac{\mathrm{d}G_{k1}}{\mathrm{d}t} = \gamma_1 - \left(1 + I_{k1} + \beta \sum_{j1\neq k1} I_{j1}\right) G_{k1}, \tag{6}$$

$$\tau_{\mathrm{c}}\frac{\mathrm{d}I_{k2}}{\mathrm{d}t} = \left(G_{k2} - \alpha - g\epsilon I_{k2} - 2\epsilon \sum_{j2\neq k2} \mu_{j2k2} I_{j2}\right) I_{k2}, \tag{7}$$

$$\tau_{\mathrm{f}}\frac{\mathrm{d}G_{k2}}{\mathrm{d}t} = \gamma_2 - \left(1 + I_{k2} + \beta \sum_{j2\neq k2} I_{j2}\right) G_{k2}. \tag{8}$$

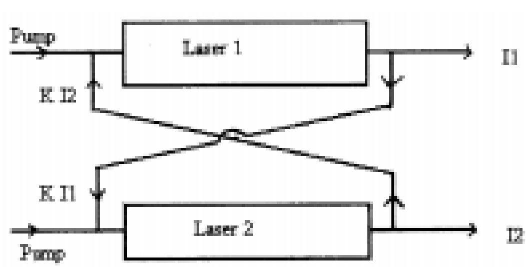

Figure 4. Schematic diagram for the coupling of two lasers.

Here γ_1 and γ_2 are the pump parameters for Laser1 and Laser2 respectively and each of them is modulated according to the output intensity of the other. Correspondingly we can write

$$\gamma_1 = \gamma_b + K_1 \sum_{k2} I_{k2}, \tag{9}$$

$$\gamma_2 = \gamma_b + K_2 \sum_{k1} I_{k1}. \tag{10}$$

Here γ_b is the bias value of the pump which is modulated. K_1 and K_2 are the coupling constants. $\sum I_{k1}$ and $\sum I_{k2}$ are the total output intensities in all the modes for Laser1 and Laser2 respectively. We consider two different cases of coupling, the symmetric bidirectional coupling and the asymmetric unidirectional coupling.

In the symmetric bidirectional coupling, two identical lasers operating in the chaotic region are mutually coupled. For this we put $K_1 = K_2 = K$ and $g = 0.1$. When the coupling constant K is varied from 0 to 0.01, we get the following interesting results. The lasers show chaotic behavior if the coupling constant is between 0 and 0.0073. The intensity pulse train of Laser1 corresponding to $K = 0.006$ is shown in figure 5a.

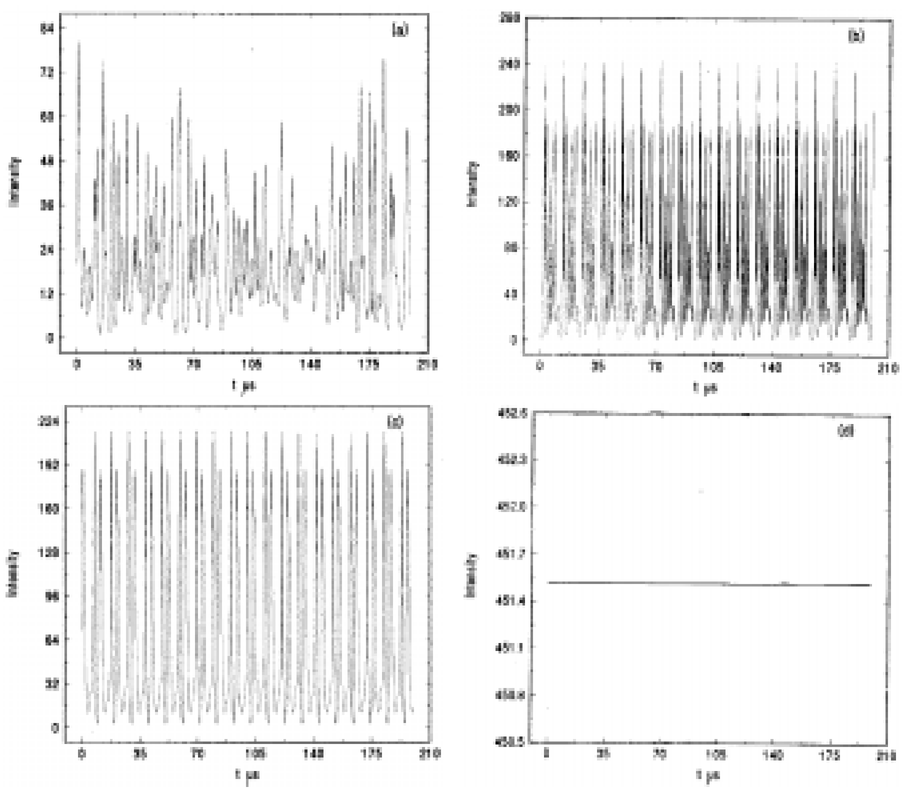

Figure 5. The total intensity time series for the Laser1 when the coupling constant (**a**) $K = 0.006$ (chaotic), (**b**) $K = 0.00736$ (period 6), (**c**) $K = 0.00748$ (period 2), (**d**) $K = 0.0087$ (steady state). Bi-directional coupling with $K_1 = K_2 = K$.

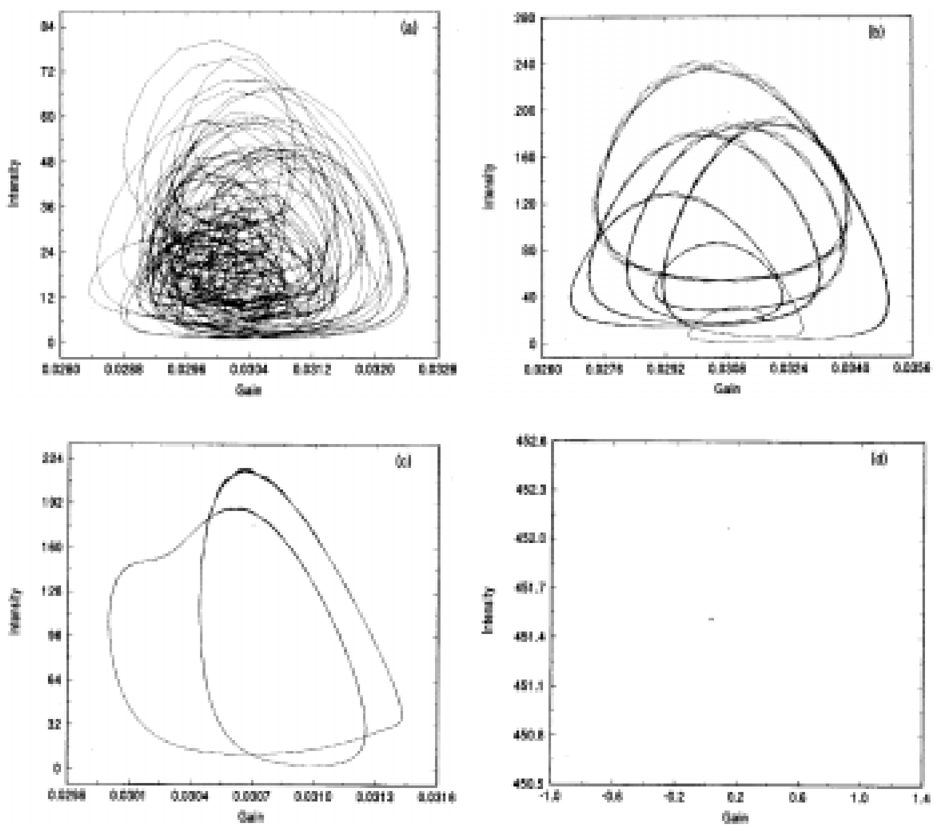

Figure 6. The phase diagram for the Laser1 when the coupling constant (**a**) $K = 0.006$ (chaotic), (**b**) $K = 0.00736$ (period 6), (**c**) $K = 0.00748$ (period 2), (**d**) $K = 0.0087$ (steady state). Bi-directional coupling with $K_1 = K_2 = K$.

The intensity varies in a random manner showing chaotic behavior. Figure 6a shows the phase diagram for the Laser1 corresponding to $K = 0.006$. The complex nature of the phase diagram also shows the chaotic behavior of the laser. We further computed the maximum LCE of the output intensity time series and found to be equal to $+1.01835\times10^{-4}$ bits/ns. The positive value of the LCE also proves the chaotic nature of the output intensity.

When the coupling constant is equal to 0.00736 the laser showed periodic behavior with period 6 oscillation of output intensity. The period 6 oscillations of output intensity corresponding to $K = 0.00736$ is shown in figure 5b. In figure 6b the phase diagram corresponding to $K = 0.00736$ is plotted. It consists of 6 loops which also shows the period 6 oscillations. The LCE of the periodic time series is -2.31543×10^{-6} bits/ns. The negative value of LCE also shows the periodic behavior of the laser. On further increase of the coupling constant the laser showed alternatively chaotic and periodic behavior.

When the coupling constant is equal to 0.00748 the laser oscillations become periodic with period 2. In figure 5c the intensity time series corresponding to $K = 0.00478$ is plotted. The coresponding phase diagram which is a two loop, is given in figure 6c.

Finally, when the value of the coupling constant $K = 0.0087$ the laser output intensity became steady. The intensity time series for Laser1 corresponding to $K = 0.0087$ is given in figure 5d. The phase diagram corresponding to this value of K is given in figure 6c, which is a single point. Since the lasers are identical and are symmetrically coupled, the Laser2 also shows similar type of behavior.

The second case we have considered is the asymmetric unidirectional coupling in which the Laser1 operating in the chaotic region is coupled to the Laser2 operating in the periodic region and there is no reverse coupling. Here we put $K_1 = K$ and $K_2 = 0$. Also we put $g = 0.1$ for the Laser1 (chaotic) and $g = 0.7$ for Laser2 (periodic). The intensity time series for Laser1 and Laser2 without coupling are given in figures 7a and b respectively. Figure 7a shows chaotic variation of intensity (Laser1) and figure 7b shows periodic variations in output intensity (Laser2).

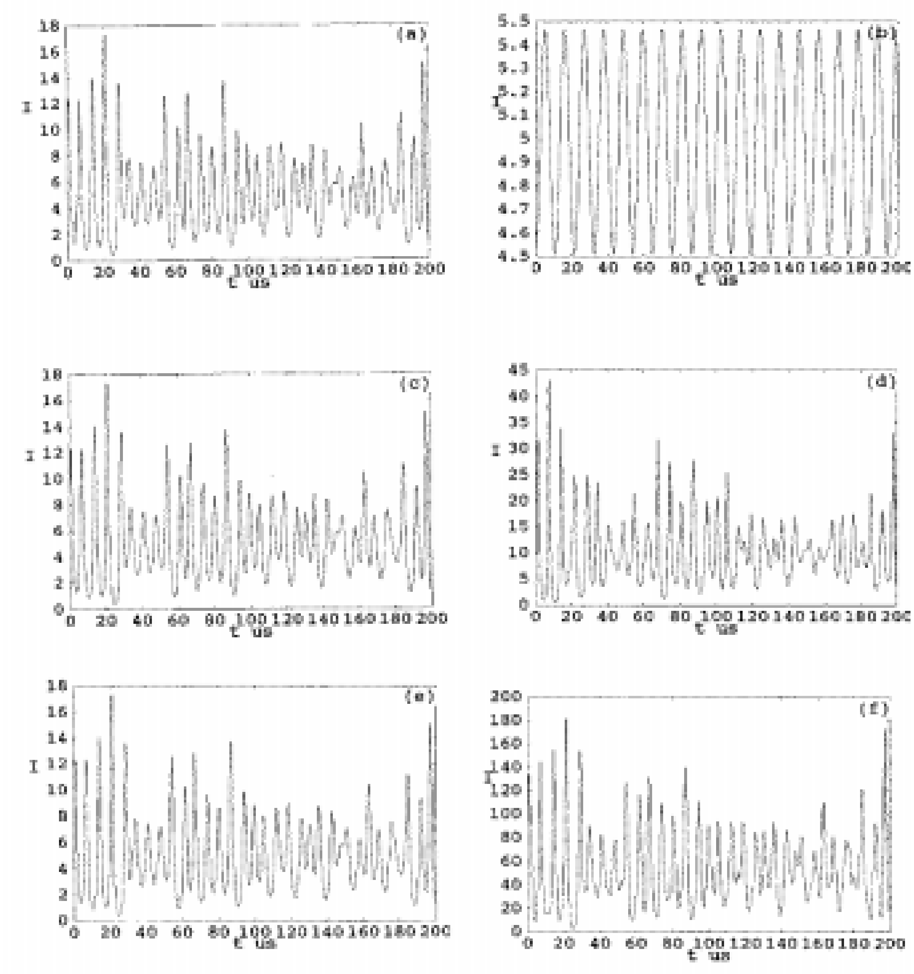

Figure 7. Intensity time series for the Laser1 (left) and the Laser2 (right) with $K_2 = 0$ and (**a** and **b**) $K_1 = 0$, (**c** and **d**) $K_1 = 0.001$, (**e** and **f**) $K_1 = 0.08$. Unidirectional coupling.

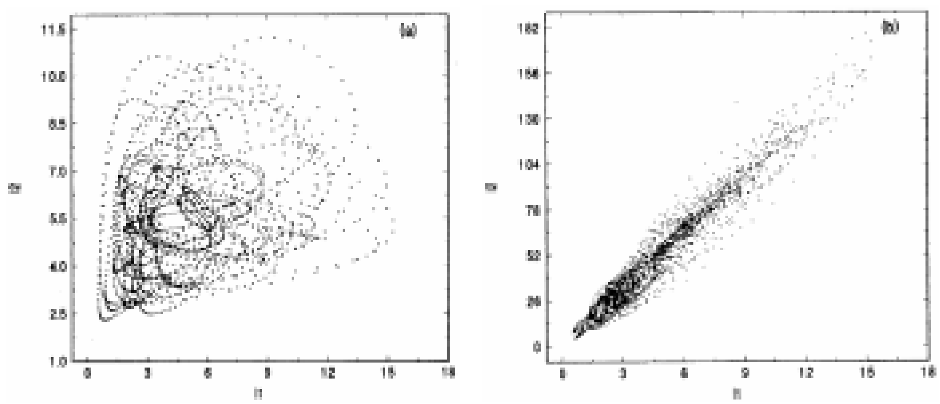

Figure 8. The plot of intensity of Laser1 against that Laser2 when $K_2 = 0$ and (**a**) $K_1 = 0.001$ and (**b**) $K_1 = 0.08$.

When we introduced a small coupling, both the lasers started oscillating chaotically. Figures 7c and d shows the chaotic variations of output intensities of Laser1 and Laser2 respectively, corresponding to $K_1 = 0.001$. The Lyapunov exponent for the intensity time series is computed to be equal to $+3.4566\times10^{-5}$ bits/ns and $+1.65340\times10^{-5}$ bits/ns respectively for Laser1 and Laser2 respectively. The intensity of Laser1 plotted against that of Laser2 for $K_1 = 0.001$ is given in figure 8a. It shows no linearity from which it is understood that there is no synchronization between the lasers.

When the coupling constant is increased to a sufficiently large value, both the lasers showed chaotic oscillations with synchronization. In figures 7e and f we plotted the intensity time series for Laser1 and Laser2 respectively for $K_1 = 0.08$. The LCEs of the time series are found to be $+1.68106\times10^{-4}$ bits/ns and $+1.707155\times10^{-4}$ bits/ns respectively. The intensity of Laser1 is plotted against that of Laser2 in figure 8b. It shows some linearity which means that the lasers are oscillating chaotically in a synchronized manner.

5. Conclusion

In this paper we have presented the results of our numerical studies on the coupling of two chaotic multimode Nd:YAG lasers with intracavity frequency doubling crystal. It can be seen that the lasers can be stabilized or synchronized by coupling them in a way we have described. The model we have used for our numerical study and the results of the numerical studies on the chaotic behavior of the laser are also given. The coupling scheme we have considered is external electronic coupling in which the pumping of each laser is modulated according to the output intensity of the other. In contrast to the earlier works we considered the coupling of two separate lasers externally. It is shown that in the symmetric case the lasers can be stabilized by coupling and in the asymmetric case they can be synchronized by coupling.

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