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Nonlinear wave propagation through a ferromagnet with damping in (2+1) dimensions

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Abstract. We investigate how dissipation and nonlinearity can affect the electromagnetic wave propagating through a saturated ferromagnet in the presence of an external magnetic field in $(2 + 1)$ dimensions. The propagation of electromagnetic waves through a ferromagnet under an external magnetic field in the presence of dissipative effect has been studied using reductive perturbation method. It is found that to the lowest order of perturbation the system of equations for the electromagnetic waves in a ferromagnet can be reduced to an integro-differential equation.

Keywords. Solitons; integro-differential equations; reductive perturbation method.

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1. Introduction

The phenomenon of propagation of electromagnetic waves in ferromagnets are not only interesting as itself but are also important in connection with the behaviour of ferrite devices at microwave frequencies such as ferrite loaded waveguides, magneto-optical recording systems, etc. The concept of magneto-optical recording has become technologically very important for the purpose of high storage and fast reading. Electromagnetic waves in ferromagnets have been investigated by several authors so far [1–7]. The propagation of electromagnetic waves in a ferromagnet obeys nonlinear equations with dispersion and dissipation. Recently several authors [8,9] gave a rigorous study of the propagation of long wavelength electromagnetic waves in a saturated ferromagnet taking into account nonlinearity and dissipation in $(1 + 1)$ dimensions. It may be interesting and worthwhile to examine the effect of dissipation on the propagation of electromagnetic waves of long wavelength in $(2 + 1)$ dimensions.

The purpose of the present work is to study the property of electromagnetic waves of long wavelength propagating through an isotropic ferromagnet in the classical continuum limit in (2+1) dimensions taking into account the dissipative effect. By using the long wave approximation of the reductive perturbation method (RPM) [10–12], it is found that the system of equations can be reduced to an integro-differential equation. It is shown that under certain conditions the equation admits steady state solutions.

2. Formulation of the problem

We are considering electromagnetic waves in a ferromagnet under an external magnetic field in the presence of dissipative effect in $(2+1)$ dimensions. The basic equations relevant to the present problem are the following :

$$\nabla \times \vec{E} = \frac{1}{c}\frac{\partial \vec{B}}{\partial t},\ \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}. \tag{1}$$

The magnetic induction $\vec{B}$ and the magnetic field $\vec{H}$ are related to the magnetization of the medium through the relation:

$$\vec{B} = \mu_0(\vec{H} + \vec{M}),$$

where μ_0 is the magnetic permeability in vacuum. The magnetization density M is governed by the 'torque' equation

$$\frac{\partial \vec{M}}{\partial t} = -\mu_0\gamma_0 \vec{M} \times \vec{H} - \delta \frac{\vec{M} \times (\vec{M} \times \vec{H})}{M^2}, \tag{2}$$

where γ_0 is the gyromagnetic ratio and δ is a positive constant. Following Landau and Lifshitz [13,14], we have assumed that the dissipative effect in the ferromagnet is incorporated by taking into account the second term on the right hand side of (2). Taking the time derivative of (1) and then substituting for $\vec{B}$ and $\vec{E}$ we can get

$$-\nabla(\nabla \cdot \vec{H}) + \nabla^2 \vec{H} = \frac{1}{c^2}\ \frac{\partial^2}{\partial t^2}(\vec{H} + \vec{M}), \tag{3}$$

where $c = 1/\sqrt{\mu_0 \varepsilon}$ is the speed of light based on the dielectric constant of the ferromagnet. Equations (2) and (3) are simultaneous equations for $\vec{M}$ and $\vec{H}$.

Let us consider the propagation of plane waves in $(2+1)$ dimensions. All the physical quantities are assumed to be functions of two space coordinates x, y and the time coordinate t. Let us seek a solution of these equations in the form of a Fourier expansion in harmonics of the fundamental $E = \exp i(kx - \omega t)$ as

$$\vec{M} = \sum_{n=-\infty}^{+\infty} \vec{M}^n E^n,\ \vec{H} = \sum_{n=-\infty}^{+\infty} \vec{H}^n E^n.$$

We assume here that the ferromagnet is in an uniform state at upstream infinity. Writing $\vec{M} = (M_x,\ M_y,\ M_z)$ and $\vec{H} = (H_x,\ H_y,\ H_z)$ and expressing the Fourier components of M and H in powers of a small parameter ε, we can write

$$M^n = \sum_{j=0}^{\infty} \varepsilon^j M_j^n(x,y,t), H^n = \sum_{j=0}^{\infty} \varepsilon^j H_j^n(x,y,t). \tag{4}$$

$\vec{M}$ and $\vec{H}$ satisfy the following boundary conditions,

$$M_{jx} \to 0, \quad \text{except that} \quad M_{0x} \to m_0 \cos\phi = m_{0x}, \tag{5}$$

$$M_{jy} \to 0, \quad \text{except that} \quad M_{0y} \to m_0 \sin\phi = m_{0y}, \tag{6}$$

$$M_{jz} \to 0, \tag{7}$$

$$H_{jx} \to 0, \quad \text{except that} \quad H_{0x} \to h_0 \cos\phi, \tag{8}$$

$$H_{jy} \to 0, \quad \text{except that} \quad H_{0y} \to h_0 \sin\phi, \tag{9}$$

$$H_{jz} \to 0, \tag{10}$$

as $x \to -\infty$, where m_0, h_0 and ϕ are positive constants and $0 \le \phi \le \pi/2$.

Before proceeding to the nonlinear problem, we will first examine the dispersion relation in the linearized limit. Assuming a sinusoidal waveform for the field variables the equations (2) and (3) can be cast in the form

$$\begin{aligned}\left[\frac{\partial}{\partial t} - in\,\omega\right] M^n = -\mu_0\gamma_0 &\left[\sum_{p+q=n} M^p \times H^q\right] \\ &-\delta\left[\sum_{p+q+r=n} M^p \times [M^q \times (H^r - \alpha M^r)\right],\end{aligned} \tag{11}$$

$$\begin{aligned}\left[\frac{\partial^2}{\partial t^2} + 2in\omega\frac{\partial}{\partial t} - n^2\omega^2\right][M_s^n + H_s^n] = c^2 &\left[\frac{\partial^2}{\partial x^2} + 2ink\frac{\partial}{\partial x} - n^2k^2\right] \\ &\times H_s^n(1 - \delta_{sx}),\end{aligned} \tag{12}$$

where $s = x, y, z$ and δ_{sx} is the Kronecker delta function. Taking the leading order terms for the order ($n = 1$) from both equations, we can write the components of M_s^1 as functions of H_s^1. Thus we can find a linear homogeneous system of equations for H_x^1, H_y^1 and H_z^1. Here we have assumed a linear relation between the zeroth components of M and H, i.e., $h_0 = \alpha m_0$. The determinant of this system of equations $\Delta(n)$ is

$$\begin{aligned}\Delta(n) = in\omega\,&[-n^2\gamma^2\omega^2 + \beta^2(m_{0x})^2 + \gamma\beta(1+\alpha)(m_{0y})^2 \\ &+\delta\beta\gamma(1+\alpha)(m_0)^2\,],\end{aligned} \tag{13}$$

where

$$\beta = (1 + \alpha\gamma), \tag{14}$$

$$\gamma = \left(1 - \frac{c^2k^2}{\omega^2}\right). \tag{15}$$

For $n = 1$, $\Delta(1)$ is zero if ω satisfies the dispersion relation

$$-\gamma^2\omega^2 + \beta^2(m_{0x})^2 + \gamma\beta(1+\alpha)(m_{0y})^2 + \delta\beta\gamma(1+\alpha)(m_0)^2 = 0. \tag{16}$$

Writing γ and β in terms of α, k and ω, we can obtain an equation which is cubic in ω^2:

$$\left(1+\alpha\left(1-\frac{c^2k^2}{\omega^2}\right)\right)^2(m_{0x})^2+\left(1-\frac{c^2k^2}{\omega^2}\right)$$
$$\left(1+\alpha\left(1-\frac{c^2k^2}{\omega^2}\right)\right)(1+\alpha)(m_{0y})^2+\delta\left(1-\frac{c^2k^2}{\omega^2}\right)$$
$$\left(1+\alpha\left(1-\frac{c^2k^2}{\omega^2}\right)\right)(1+\alpha)(m_0)^2=\omega^2\left(1-\frac{c^2k^2}{\omega^2}\right). \tag{17}$$

This shows that there are three kinds of waves, for small values of k, the three values of ω/k are given by

$$\frac{\omega}{k}=\sqrt{\frac{\alpha}{1+\alpha}}c, \tag{18}$$

$$\frac{\omega}{k}=\sqrt{\frac{\alpha+\sin^2\phi}{1+\alpha}}c, \tag{19}$$

$$\frac{\omega}{k}=\frac{1+\alpha}{\alpha}c\left(1-\frac{\delta\alpha l}{c}\right)\frac{1}{kl}, \tag{20}$$

where $l=c/(\mu_0\gamma_0H_0)$.

In the long wavelength approximation and for finite values of velocity we can exclude the wave corresponding to the third mode for which $\omega/k\to\infty$ as $k\to 0$. We assume that the y coordinate is orthogonal to the plane determined by the external magnetic field H^0 and the direction of propagation of the perturbation is along the x-axis. The system will have a solution only if the determinant of the augmented matrix is zero.

In the study of the asymptotic behaviour of nonlinear dispersive waves, Gardner and Morikawa [12] introduced a scale transformation

$$\xi=\varepsilon^a(x-Vt), \tag{21}$$

$$\tau=\varepsilon^b t \tag{22}$$

in the (1 + 1) dimensional case, where V stands for the velocity of the wave motion. Gardner and Morikawa combined this transformation with a perturbation expansion of the dependent variables so as to describe the nonlinear asymptotic behaviour of the system of equations and will arrive at a single tractable equation describing the asymptotic behaviour of a wave. This method is called the reductive perturbation method (RPM) which is the basic mechanism for carrying out this problem. In (2 + 1) dimensions we will introduce one more space co-ordinate y.

We will now introduce the stretching variables ξ, ζ and τ as,

$$\xi=\varepsilon(x-Vt), \tag{23}$$

$$\zeta=\varepsilon^2 y, \tag{24}$$

$$\tau=\varepsilon^3 t, \tag{25}$$

where ε is the small parameter measuring the weakness of dispersive effect or smallness of k. The expression for ζ represents weak dependence of the field parameters on the coordinate y. In the present case we consider the waves with velocity V given by (18)

$$V = \frac{\omega}{k} = \sqrt{\frac{\alpha}{1+\alpha}}c.$$

One can also proceed with velocity given by (19). Applying the scale transformations (23), (24) and (25) and substituting the expansions given by (4) in (2) and (3) and then collecting and solving coefficients of different orders of ε^j, for $n = 1$, we get the following:

At order ε^0:

$$\mu_0\gamma_0[M_{0y}H_{0z} - M_{0z}H_{0y}] - \delta[(1+\alpha)M_{0y}M_{0x} - (1+\alpha)M_{0z}M_{0x}] = 0, \tag{26}$$

$$\mu_0\gamma_0[M_{0z}H_{0x} - M_{0x}H_{0z}] - \delta[(1+\alpha)M_{0z}M_{0y} - (1+\alpha)M_{0x}M_{0y}] = 0, \tag{27}$$

$$\mu_0\gamma_0[M_{0x}H_{0y} - M_{0y}H_{0x}] - \delta[(1+\alpha)M_{0z}M_{0x} - (1+\alpha)M_{0z}M_{0y}] = 0, \tag{28}$$

$$\frac{\partial^2[H_{0x} + M_{0x}]}{\partial\xi^2} = 0, \tag{29}$$

$$V^2\frac{\partial^2}{\partial\xi^2}(\gamma H_{0y} + M_{0y}) = 0. \tag{30}$$

At order ε^1:

$$V\frac{\partial M_0}{\partial\xi} = \mu_0\gamma_0[m \times (H_1 - \alpha M_1)] - \delta\left[\frac{\vec{M} \times (\vec{M} \times \vec{H})}{M^2}\right]. \tag{31}$$

The x, y, z components of this equation are

$$-\mu_0\gamma_0(1+\alpha)M_{0z}M_{1x} = V\frac{\partial M_{0y}}{\partial\xi} - \delta[(1+\alpha)(M_{0z})^2M_{1y} - (1+\alpha)(M_{0x})^2M_{1y}], \tag{32}$$

$$\mu_0\gamma_0(1+\alpha)M_{0y}M_{1x} = V\frac{\partial M_{0z}}{\partial\xi} - \delta[(1+\alpha)(M_{0x})^2M_{1z} - (1+\alpha)(M_{0y})^2M_{1z}], \tag{33}$$

$$H_{1x} + M_{1x} = 0, \tag{34}$$

$$H_{1y} - \alpha M_{1y} = 0, \tag{35}$$

$$H_{1z} - \alpha M_{1z} = 0. \tag{36}$$

$M_0 = m_0$ and $H_0 = \alpha m_0$ are the constant vectors characterizing the initial static state of the system, where $m_0 = (m_x, m_y, 0)$. Using this fact in (31) we can show that $H_1 - \alpha M_1$ and m_0 are colinear vectors. Thus we can write

$$H_1 - \alpha M_1 = (1+\alpha)\beta g m_0, \tag{37}$$

where $g(\xi, \zeta, \tau)$ is an arbitrary function of ξ, ζ, τ. By using the relation given by (37) and using the fact that H_0 and M_0 are constant vectors and that for $\xi \to -\infty$, $M_1 \to 0$ and $H_1 \to h_1'$ (h_1' parallel to m) in first order equations (31)–(36), the components of M_{1y} and M_{1z} are found to be

$$M_{1y} = \gamma(1+\alpha) m_y g,$$
$$M_{1z} = 0, \quad \text{since} \quad m_z = 0.$$

At order ε^2:

$$V\frac{\partial M_{1x}}{\partial \xi} = -\mu_0\gamma_0[M_{0y}(H_{2z} - \alpha M_{2z}) - M_{0z}(H_{2y} - \alpha M_{2y})] -\delta[(1+\alpha)M_{1y}M_{0y}M_{1x} - (1+\alpha)M_{1x}M_{0z}M_{1z}], \tag{38}$$

$$V\frac{\partial M_{1y}}{\partial \xi} = -\mu_0\gamma_0[M_{0z}(H_{2x} - \alpha M_{2x}) - M_{0x}(H_{2z} - \alpha M_{2z})] -\delta[(1+\alpha)M_{1z}M_{1y}M_{0z} - (1+\alpha)M_{0x}M_{1x}M_{1y}], \tag{39}$$

$$V\frac{\partial M_{1z}}{\partial \xi} = -\mu_0\gamma_0[M_{0x}(H_{2y} - \alpha M_{2y}) - M_{0y}(H_{2x} - \alpha M_{2x})] -\delta[(1+\alpha)M_{1z}M_{1x}M_{0x} - (1+\alpha)M_{1z}M_{0z}M_{1y}], \tag{40}$$

$$H_{2x} + M_{2x} = 0, \tag{41}$$

$$\frac{\partial(H_{2y} - \alpha M_{2y})}{\partial \xi} = -2V\frac{(1+\alpha)^2}{c^2}\frac{\partial M_{0y}}{\partial \tau} - (1+\alpha)\frac{\partial^2 M_{0y}}{\partial \zeta^2}, \tag{42}$$

$$\frac{\partial(H_{2z} - \alpha M_{2z})}{\partial \xi} = -2V\frac{(1+\alpha)^2}{c^2}\frac{\partial M_{0z}}{\partial \tau} - (1+\alpha)\frac{\partial^2 M_{0z}}{\partial \zeta^2}. \tag{43}$$

Solving for H_{2y}, M_{2y} and H_{2z}, M_{2z}, from (42) and (43) we can get

$$(H_{2y} - \alpha M_{2y}) = \int_{-\infty}^{\xi} -2V\frac{(1+\alpha)^2}{c^2}\frac{\partial M_{0y}}{\partial \tau}\mathrm{d}\xi + \int_{-\infty}^{\xi}\int_{-\infty}^{\xi} \alpha(1+\alpha)\frac{\partial^2 M_{0y}}{\partial \zeta^2}(\mathrm{d}\xi)^2, \tag{44}$$

$$(H_{0z} - \alpha M_{0z}) = \int_{-\infty}^{\xi} -2V \frac{(1+\alpha)^2}{c^2} \frac{\partial M_{0z}}{\partial \tau} \mathrm{d}\xi - \int_{-\infty}^{\xi} \int_{-\infty}^{\xi} \alpha(1+\alpha) \frac{\partial^2 M_{0z}}{\partial \zeta^2} (\mathrm{d}\xi)^2. \quad (45)$$

Substituting for

$$(H_{2y} - \alpha M_{2y}) \quad \text{and} \quad (H_{2z} - \alpha M_{2z}) \quad (46)$$

in (38) we get

$$\begin{aligned} \frac{V}{\mu_0 \gamma_0} \frac{\partial M_{1x}}{\partial \xi} = M_{0y} \Bigg[& \int_{-\infty}^{\xi} \frac{-2V(1+\alpha)^2}{c^2} \frac{\partial}{\partial \tau} M_{0z} \mathrm{d}\xi \\ & + \int_{-\infty}^{\xi} \int_{-\infty}^{\xi} \alpha(1+\alpha) \frac{\partial^2 M_{0y}}{\partial \zeta^2} (\mathrm{d}\xi)^2 \Bigg] \\ & - M_{0z} \Bigg[\int_{-\infty}^{\xi} \frac{-2V(1+\alpha)^2}{c^2} \frac{\partial}{\partial \tau} M_{0y} \mathrm{d}\xi \\ & - \int_{-\infty}^{\xi} \int_{-\infty}^{\xi} \alpha(1+\alpha) \frac{\partial^2 M_{0y}}{\partial \zeta^2} (\mathrm{d}\xi)^2 \Bigg] \\ & - \frac{\delta}{\mu_0 \gamma} \delta[(1+\alpha) M_{1y} M_{0y} M_{1x} - (1+\alpha) M_{1x} M_{0z} M_{1z}]. \end{aligned} \quad (47)$$

Introducing two new variables A and $\theta = \theta(\xi, \zeta, \tau)$ defined by

$$M_{0y} = A\cos\theta, M_{0z} = A\sin\theta, A = m_0 \sin\phi,\ \theta \to 0 \text{ as } \xi \to -\infty. \quad (48)$$

Since we are considering the wave propagation along the x-direction, substituting (48) in (33), the δ term goes to zero since M_{1z} is zero, we have the expression for the M_{1x} component in terms of the new variables as

$$M_{1x} = \frac{V}{\mu_0 \gamma_0 (1+\alpha)} \frac{\partial \theta}{\partial \xi}. \quad (49)$$

Now substituting the value of M_{1x} and using the new variables given in (48), (47) can be written as

$$\begin{aligned} -\mu \frac{\partial^2 \theta}{\partial \xi^2} = & \cos\theta \frac{\partial}{\partial \tau} \int_{-\infty}^{\xi} \sin\theta d\xi + \sigma \cos\theta \frac{\partial^2}{\partial \zeta^2} \int_{-\infty}^{\xi} \int_{-\infty}^{\xi} \sin\theta (d\xi)^2 \\ & - \sin\theta \frac{\partial}{\partial \tau} \int_{-\infty}^{\xi} \cos\theta d\xi - \sigma \sin\theta \frac{\partial^2}{\partial \zeta^2} \int_{-\infty}^{\xi} \int_{-\infty}^{\xi} \cos\theta (\mathrm{d}\xi)^2 \\ & + \nu \sin^2\theta \frac{\partial \theta}{\partial \xi}, \end{aligned} \quad (50)$$

where

$$\mu = \frac{Vc^2}{2(1+\alpha)^3\mu_0\gamma_0},$$
$$\sigma = \frac{c^2}{2V(1+\alpha)\mu_0\gamma_0},$$
$$\nu = \frac{\delta c^2}{2(1+\alpha)}.$$

Differentiating (50) with respect to ξ and simplifying we obtain

$$\frac{\partial}{\partial\xi}\left[\frac{\frac{\partial\theta}{\partial\tau}+\mu\frac{\partial^3\theta}{\partial\xi^3}-\int_{-\infty}^{\xi}\sigma\frac{\partial^2\theta}{\partial\zeta^2}+\nu\sin^2\theta\frac{\partial^3\theta}{\partial\xi^3}}{\frac{\partial\theta}{\partial\xi}}\right] = -\mu\frac{\partial^2\theta}{\partial\xi^2}\left[\frac{\partial\theta}{\partial\xi}\right]-\nu\left[\frac{\partial\theta}{\partial\xi}\right]^2. \tag{51}$$

Equation (51) can be integrated with respect to ξ to give

$$\frac{\partial\theta}{\partial\tau}+\mu\frac{\partial^3\theta}{\partial\xi^3}-\sigma\int_{-\infty}^{\xi}\frac{\partial^2\theta}{\partial\zeta^2}+\nu\sin^2\theta\frac{\partial^2\theta}{\partial\xi^2} = -\mu\frac{\partial\theta}{\partial\xi}\int_{-\infty}^{\xi}\frac{\partial^2\theta}{\partial\xi^2}\frac{\partial\theta}{\partial\xi}\mathrm{d}\xi$$
$$-\nu\frac{\partial\theta}{\partial\xi}\int_{-\infty}^{\xi}\left[\frac{\partial\theta}{\partial\xi}\right]^2. \tag{52}$$

Putting $f=\partial\theta/\partial\eta$ the above equation becomes

$$\frac{\partial f}{\partial\tau}+\frac{3}{2}\mu f^2\frac{\partial f}{\partial\xi}+\mu\frac{\partial^3 f}{\partial\xi^3}-\nu\left[\sin^2\theta\frac{\partial^2 f}{\partial\xi^2}+\frac{\partial f}{\partial\xi}\int_{-\infty}^{\xi}\left(\frac{\partial f}{\partial\xi}\right)^2\right]$$
$$=\sigma\int_{-\infty}^{\xi}\frac{\partial^2 f}{\partial\zeta^2}, \tag{53}$$

where f is a function of ξ, ζ and τ.

Differentiating (53) with respect to ξ we obtain

$$\left[\frac{\partial f}{\partial\tau}+\frac{3}{2}\mu f^2\frac{\partial f}{\partial\xi}+\mu\frac{\partial^3 f}{\partial\xi^3}+\nu\left(\sin^2\theta\frac{\partial^2 f}{\partial\xi^2}+\frac{\partial f}{\partial\xi}\int_{-\infty}^{\xi}\left(\frac{\partial f}{\partial\xi}\right)^2\right)\right]_{\xi}$$
$$=\sigma\frac{\partial^2 f}{\partial\zeta^2}. \tag{54}$$

This equation is an integro-differential equation. In the absence of the ν term this equation reduces to the modified Kadomtsev–Petviashvili equation [15,16]. The ν term represents the effect of dissipation. It should be noted that this additional term is not only a higher derivative of f with respect to ξ but contains nonlocal nonlinear terms.

We first examine the existence of steady state solutions of this equation. We assume that $f(\xi,\zeta,\tau)$ is a function of η through $(\eta-\lambda\tau)$, where λ is a constant. That is we seek solutions of (54) in a moving frame of reference. Under this assumption, (54) can be integrated once with respect to η subject to the boundary conditions $f \to 0$ as $\eta \to -\infty$, giving rise to

$$-\lambda f + \frac{1}{2}\mu f^3 + \mu \frac{\partial^2 f}{\partial \eta^2} + \nu \frac{\partial^2 f}{\partial \eta^2} + \nu \left[\frac{\partial f}{\partial \eta} + f \int_{-\infty}^{\eta} f^2\right] = \int\int \sigma \frac{\partial f}{\partial \eta}(\mathrm{d}\eta)^2. \tag{55}$$

We can now show that in the special case of $-\mu << \nu$ that is, when the effect of dispersion can be neglected compared to that of dissipation, (55) admits solutions of the form

$$f(\eta, \tau) = \frac{-\lambda}{\nu} \sec h^2 \sqrt{\frac{-\lambda}{\nu}}(\eta - \lambda\tau) \tag{56}$$

provided $\lambda < 0$ and there is no bounded solution if $\lambda > 0$. For the case of $\nu = 0$ we can arrive at the modified Kadomtsev–Petviashvili equation as

$$\left[\frac{\partial f}{\partial \tau} + \frac{3}{2}\mu f^2 \frac{\partial f}{\partial \xi} + \mu \frac{\partial^3 f}{\partial \xi^3}\right]_\xi = \sigma \frac{\partial^2 f}{\partial \zeta^2}. \tag{57}$$

The steady state solution of this equation can be obtained by using elliptic integrals. Thus we find

$$f = 6k^2 \mathrm{sech}^2 k\eta,$$

where $\eta = \xi + \zeta - \lambda\tau$, and $k^2 = \mu(\sigma + \mu)$ and λ is a constant. Since $f = \partial\theta/\partial\eta$, θ is obtained as $\theta = \int_{-\infty}^{\eta} f \mathrm{d}\eta$. θ increases from 0 to 2π or decreases from 0 to -2π according as $k > 0$ or $k < 0$ as η goes from $-\infty$ to $+\infty$.

We can obtain explicit form of solutions for M_{0y}, M_{0z}, H_{0y}, H_{0z}, H_{1x}, M_{1x}, M_{1y} and H_{1y}. Thus we can obtain

$$M_{0y} = m_0 \sin\phi \cos(6k^2 \sqrt{[1 - \mathrm{sech}^2 k\eta]}), \tag{58}$$

$$M_{0z} = m_0 \sin\phi \sin(6k^2 \sqrt{[1 - \mathrm{sech}^2 k\eta]}), \tag{59}$$

$$H_{0y} = \alpha m_0 \sin\phi \cos(6k^2 \sqrt{[1 - \mathrm{sech}^2 k\eta)]}) \tag{60}$$

$$H_{0z} = \alpha m_0 \sin\phi \sin(6k^2 \sqrt{[1 - (\mathrm{sech}^2 k\eta)]}) \tag{61}$$

$$H_{1x} = \frac{6Vk^2 \mathrm{sech}^2 k\eta}{(1+\alpha)},$$

$$M_{1x} = -\frac{6Vk^2 \mathrm{sech}^2 k\eta}{(1+\alpha)}, \tag{62}$$

$$M_{1y} = V\left(\frac{1+\sin\theta}{\delta(1+\alpha)}\right)\sec h^2\eta, \tag{63}$$

$$H_{1y} = -\alpha V\left(\frac{1+\sin\theta}{\delta(1+\alpha)}\right)\sec h^2\eta. \tag{64}$$

We have found that $M_{1x}, H_{1x}, M_{0y}, H_{0y}, H_{1y}$ and M_{1y} components give soliton solutions while M_{0z} and H_{0z} components give kink solutions.

Any localized static (time-independent) solution is a solitary wave. All these solutions have localized energy density and finite total energy. The energy expression for the mKP solitary wave is obtained by putting

$$f = U_\xi$$

in (57) [17]. The function f can be considered as the derivative of the angle θ and can also be considered as the amplitude of the first-order term M_{1x} of the longitudinal component of the magnetization density. That is,

$$E = \int_{-\infty}^{\infty}\left[\frac{1}{2}\mu(U_{\xi\xi})^2 - \frac{3}{24}\mu(U_\xi)^4 - \frac{1}{2}\sigma(U_\zeta)^2\mathrm{d}\eta.\right. \tag{65}$$

On evaluating this integral with

$$f = 6k^2\mathrm{sec}h^2 k\eta,$$

we obtain

$$\begin{aligned} E = \left(\frac{1}{6720}\right)\sec h^7[kx]\,[&-315\,k\ \sin h[kx] + 45360\,k^3\ \sin h[kx] \\ &-840\,k^7\ \sin h[kx] - 525\,k\ \sin h[3kx] - 21168\,k^3\ \sin h[3kx] \\ &-504\,k^7\ \sin h[3kx] - 245\,k\ \sin h[5kx] + 3024\,k^3 \sin h[5kx] \\ &-168\,k^7\ \sin h[5kx] - 35\,k\ \sin h[7kx] \\ &+432\,k^3\ \sin h[7kx] - 24\,k^7\ \sin h[7kx]]. \end{aligned} \tag{66}$$

The momentum for this solitary wave is given by

$$P = \int_{-\infty}^{\infty}\left[\frac{1}{2}\mu(U_\xi)^3 + \sigma U_\zeta\right]\mathrm{d}\eta \tag{67}$$

and the momentum expression is given by

$$\begin{aligned} P = \cos h[kx]\,[12\,k^2\ \sec h^2[kx] + 12\,k^6\ \sec h^6[kx]][&30\ \sin h[kx] \\ &+80\,k^4\ \sin h[kx] + 45\ \sin h[3kx] \\ &+40\,k^4\ \sin h[3kx] + 15\ \sin h[5kx] \\ &+8\,k^4\ \sin h[5kx]]/[30k(3 + 8k^4 + 4\ \cos h[2kx] + \cos h[4kx])]. \end{aligned}$$

3. Conclusion

We have attempted to find out what happens to the dynamics of the magnetization of a ferromagnet when an electromagnetic wave propagates through it in the presence of dissipation in higher dimensions. A perturbation analysis is carried out to understand the nature of excitations due to the interaction between the intensity of magnetic field H and the magnetization M of the ferromagnet. We have analysed the resulting equations separately for the dissipation dominant case and the dispersion dominant case and obtained solutions in both cases.

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