PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 54, No. 2
February 2000
pp. 215–225

Static charged spheres with anisotropic pressure in general relativity*

J KRISHNA RAO*, M ANNAPURNA† and M M TRIVEDI
Department of Mathematics, Bhavnagar University, Bhavnagar 364 002, India
†Department of Mathematics, Vasavi Engineering College, Hyderabad 500 031, India
*Address for correspondence: 302, Surya Enclave, Asif Nagar, Mehdipatnam, Hyderabad 500 028, India

MS received 14 December 1998; revised 11 November 1999

Abstract. We report a generalization of our earlier formalism [*Pramana*, **54**, 663 (1998)] to obtain exact solutions of Einstein–Maxwell's equations for static spheres filled with a charged fluid having anisotropic pressure and of null conductivity. Defining new variables: $w = (4\pi/3)(\rho + \epsilon)r^2$, $u = 4\pi\xi r^2$, $v_{\rm r} = 4\pi p_{\rm r} r^2$, $v_\perp = 4\pi p_\perp r^2[\rho, \xi(= -(1/2)F_{14}F^{14})$, $p_{\rm r}$, $p_\perp$ being respectively the energy densities of matter and electrostatic fields, radial and transverse fluid pressures whereas ϵ denotes the eigenvalue of the conformal Weyl tensor and interpreted as the energy density of the free gravitational field], we have recast Einstein's field equations into a form easy to integrate. Since the system is underdetermined we make the following assumptions to solve the field equations (i) $u = v_{\rm r} = (a^2/2\kappa)r^{n+2}$, $v_\perp = k_1 v_{\rm r}$, $w = k_2 v_{\rm r}$; $a^2, n(> 0), k_1, k_2$ being constants with $\kappa = ((k_1 + 2)/3 + k_2)$ and (ii) $w + u = (b^2/2)r^{n+2}$, $u = v_{\rm r}$, $v_\perp - v_{\rm r} = k$, with b and k as constants. In both cases the field equations are integrated completely. The first solution is regular in the metric as well as physical variables for all values of $n > 0$. Even though the second solution contains terms like k/r^2 since $Q(0) = 0$ it is argued that the pressure anisotropy, caused by the electric flux near the centre, can be made to vanish reducing it to the generalized Cooperstock–de la Cruz solution given in [14]. The interior solutions are shown to match with the exterior Reissner–Nordstrom solution over a fixed boundary.

Keywords. Charged static spheres; energy density of the free gravitational field; anisotropic pressure.

PACS No. 04.20 Jb

1. Introduction

Spherically symmetric static charged dust/perfect fluid distributions of null conductivity have been studied extensively under various assumptions by several authors including

*Dedicated to Prof. F A E Pirani.

Cooperstock and de la Cruz [1], Patino and Rago [2], Tikekar [3,4], Xingxiang [5], Nduka [6], Mehra [7] and Singh *et al* [8]. On the other hand charged analogues of Einstein's self-gravitating cluster of particles were studied by Banerjee and Som [9], Florides [10], Gron [11], Ponce de Leon [12], Vaidya and Patel [13], Krishna Rao *et al* [14–16], where the radial component of the energy momentum tensor vanishes identically $(T_1^{\ 1} = 0)$ so that the charged particles move under tangential stresses only.

It is well known that in spherically symmetric static systems the electric field is in the radial direction only affecting the fluid pressure and hence causing pressure anisotropy. Even though the study of such charged fluid distributions, with anisotropic pressure, filling in the static spheres has not received much attention, some notable contributions in this direction have been made by Gron [11], Ponce de Leon [12], Roy *et al* [17]. We may mention here that our system has six variables, viz; the fluid density ρ, the radial and transverse pressure $p_{\rm r}$ and $p_\perp$ respectively, the electrostatic energy density $\xi(= -(1/2)F_{14}F^{14})$ besides the two functions λ, ν coming from the metric tensor. However, for a spherically symmetric static system there are just three equations connecting these six variables. Hence, a physically vivable choice of assumptions to solve the problem becomes difficult.

However, Gron [11] and Ponce de Leon [12] both assumed Florides' condition $(T_1^{\ 1} = 0)$. While the former obtained only a general relation between $p_{\rm r}, p_\perp, \rho$ and λ, the latter obtained an analytical solution by taking $p_\perp = kp_{\rm r}$. The solution given by Roy *et al* [17] is based on the assumptions: (i) exp $(\lambda/2)\sigma(r) = \sigma_0/r^2$, and (ii) exp $(\lambda)r(\rho + p_{\rm r}) = A$, σ_0 and A being constants. Besides it contains an undetermined mass function $m(r)$. The derivation of this solution is essentially tailored to match it with the exterior Reissner–Nordstrom solution. In the present paper we wish to extend the formalism given by us earlier [14–16] to the case when the fluid pressures along the radial and tangential directions differ and obtain solutions of charged static spherical distributions with anisotropic pressure.

In §2 we shall recast Einstein's field equations with the help of variables $u, v_{\rm r}, v_\perp$ and w. The alternative form of the field equations suggests certain linear relationships between the variables $u, v_{\rm r}, v_\perp$ and w, which are used to obtain new solutions. In §3 we have derived two such solutions with detailed discussion. In §4 we have established the boundary conditions for the interior space-times to be matched with the exterior Reissner–Nordstrom solution and the concluding remarks are given in §5.

2. Field equations

Using the Schwarzschild coordinates we choose the metric for a spherically symmetric static space-time as

$$\mathrm{d}s^2 = -\exp(\lambda)\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2) + \exp(\nu)\mathrm{d}t^2. \tag{1}$$

Assuming that the space-time given by (1) is filled with a charged fluid of anisotropic pressure and of null conductivity, the non-vanishing components of the energy–momentum tensor are given by [11,12]

$$T_1^1 = -p_{\rm r} + \xi,\ T_2^2 = T_3^3 = -(p_\perp + \xi),\ T_4^4 = \rho + \xi. \tag{2}$$

In (2), $\xi = ((-1/2)F_{14}F^{14})$ and ρ are respectively the energy densities of the electrostatic field and the fluid, whereas $p_{\rm r}$ and $p_\perp$ denote the radial and transverse fluid pressures.

The skew-symmetric Maxwell tensor F_{ab} satisfies the equations

$$F_{ab,c} + F_{bc,a} + F_{ca,b} = 0 \tag{3}$$

$$\{(-g)^{1/2} F^{ab}\}_{,b} = (-g)^{1/2} J^a, \tag{4}$$

$$J^a = \sigma u^a, \tag{5}$$

where $u_a = (0, 0, 0, \exp(-\nu/2))$ and σ denotes the charge density. In view of spherically symmetric and static nature of the space-time the only non-vanishing component of F_{ab} is F_{14}. We express F_{14} and J_4 as

$$F_{14} = -\exp[(\lambda + \nu)/2] Q(r)/r^2, \tag{6}$$

$$J_4 = -\exp[(\lambda - \nu)/2] Q'(r)/r^2, \tag{7}$$

where at present $Q(r)$ is an arbitrary function. We connect (1) and (2) through Einstein's field equations resulting in the following:

$$8\pi(p_{\rm r} - \xi) = -r^{-2}\{1 - \exp(-\lambda)\} + r^{-1}\exp(-\lambda)\nu', \tag{8}$$

$$8\pi(p_{\perp} + \xi) = -8\pi\epsilon + r^{-2}\{1 - \exp(-\lambda)\} + r^{-1}\exp(-\lambda)(\nu' - \lambda'), \tag{9}$$

$$8\pi(\rho + \xi) = r^{-2}\{1 - \exp(-\lambda)\} + r^{-1}\exp(-\lambda)\lambda', \tag{10}$$

where

$$8\pi\epsilon = r^{-2}[1 - \exp(-\lambda)] - (1/4)\exp(-\lambda)[2\nu'' + (\nu' - \lambda')(\nu' - 2r^{-1})], \tag{11}$$

denotes the eigenvalue of the conformal Weyl tensor [18]. Here and what follows a prime denotes a differentiation with respect to the radial coordinate r.

Now, making the combination of $\{(9) + (10) - (8)\}$, we get

$$\exp(-\lambda) = 1 - \frac{8\pi}{3}(\rho + \epsilon + 3\xi + p_{\perp} - p_{\rm r})r^2. \tag{12}$$

Since ϵ is coupled to ρ and ξ we call it the energy density of the free gravitational field [19–22] and it plays an important role in our present reformulation of Einstein's field equations. Comparing the expression (12) with that obtained by integrating (10), we note that the total energy inside a sphere of radius r is given by

$$M(r) = \frac{4\pi}{3}(\rho + \epsilon + 3\xi + p_{\perp} - p_{\rm r})r^3, \tag{13}$$

where

$$M(r) = \int_0^r 4\pi r^2(\rho + \xi)\mathrm{d}r, \quad M(0) = 0. \tag{14}$$

Thus, from (13) we note that ϵ, the energy density of the free gravitational field, as well as pressure anisotropy $(p_\perp - p_\mathrm{r})$ both contribute positively for the total energy of the system. If $p_\perp - p_\mathrm{r} < 0$, then pressure anisotropy contributes negatively to $M(r)$.

The form of (12) suggests that we define new variables $u, v_\mathrm{r}, v_\perp$ and w as below:

$$u = 4\pi\xi r^2, \tag{15}$$

$$v_\mathrm{r} = 4\pi p_\mathrm{r} r^2, \tag{16}$$

$$v_\perp = 4\pi p_\perp r^2, \tag{17}$$

$$w = \frac{4\pi}{3}(\rho + \epsilon)r^2, \tag{18}$$

so that (12) is rewritten as

$$\exp(-\lambda) = 1 - 2\left\{(w + u) + \frac{1}{3}(v_\perp - v_\mathrm{r})\right\}. \tag{19}$$

The above expression suggests a linear relationship between $u, v_r, v_\perp$ and w.

Also we can write (8) with the help of (15), (16) and (19) as

$$r\nu' = \frac{2\{w + v_\mathrm{r} + (1/3)(v_\perp - v_\mathrm{r})\}}{1 - 2\{(w + u) + (1/3)(v_\perp - v_\mathrm{r})\}}. \tag{20}$$

Now making the combination of $\{2 \times (10) + (8) - (9)\}$ we get

$$\frac{1}{r}\left\{(w' + u') + \frac{1}{3}(v'_\perp - v'_\mathrm{r})\right\} = \frac{4\pi}{3}\{(2\rho - \epsilon) - (p_\perp - p_\mathrm{r})\}. \tag{21}$$

These equations play a crucial role in the derivation of new solutions since the left hand side can be easily integrated. The radial equation of motion $(T^j_{1;j} = 0)$ on simplification takes the form

$$(\rho + p_\mathrm{r})\nu' = \frac{4}{r}(p_\perp - p_\mathrm{r}) - 2p'_\mathrm{r} + \frac{(Q^2(r))'}{r^4}, \tag{22}$$

where $Q(r)$ denotes the total charge on the sphere and is given by

$$Q(r) = \int_0^r 4\pi r^2 \sigma \exp(\lambda/2)\mathrm{d}r. \tag{23}$$

In (22) we note that on the right hand side the last two terms are positive so that the pressure gradient and electrostatic forces are away from the centre of the sphere. However, $(p_\perp - p_\mathrm{r})$ term can be either positive or negative. We have noted earlier pressure anisotropy $(p_\perp - p_\mathrm{r})$ contributes for the total energy $M(r)$ of the sphere. Thus if $(p_\perp - p_\mathrm{r}) > 0$, then to balance the additional mass contribution to $M(r)$ a radially outward pointing force $4(p_\perp - p_\mathrm{r})/r$ is available. Similarly if $(p_\perp - p_\mathrm{r}) < 0$, the decrease in the energy $M(r)$ is compensated by an inward pointing force $4(p_\perp - p_{\rm r})/r$. Hence, in either case the hydrostatic balance is maintained.

We now eliminate ν' from (20) and (22) to get

$$\frac{{\rm d}r}{r} = \frac{\begin{array}{c}[(3w+2v_{\rm r}+v_\perp)\{3({\rm d}w+{\rm d}u)+({\rm d}v_\perp-{\rm d}v_{\rm r})\}\\ +3\{3-6(w+u)-2(v_\perp-v_{\rm r})\}({\rm d}v_{\rm r}-{\rm d}u)]\end{array}}{[6\{3-6(w+u)-2(v_\perp-v_{\rm r})\}-(3w+2v_{\rm r}+v_\perp)^2]}. \tag{24}$$

Since (24) contains linear combination of $u, v_{\rm r}, v_\perp, w$ and their derivatives, we can assume physically plausible relationships between these four functions and integrate (24).

Finally, eliminating λ' from (10) with the help of (19) and then using (24) we get,

$$4\pi(\rho+\xi)r^2 = \left\{(w+u)+\frac{1}{3}(v_\perp - v_{\rm r})\right\} + \frac{\begin{array}{c}[({\rm d}w+{\rm d}u)+(1/3)({\rm d}v_\perp-{\rm d}v_{\rm r})][6\{3-6(w+u)\\ -2(v_\perp-v_{\rm r})\}-(3w+2v_{\rm r}+v_\perp)^2]\end{array}}{\begin{array}{c}[(3w+2v_{\rm r}+v_\perp)\{3({\rm d}w+{\rm d}u)+({\rm d}v_\perp-{\rm d}v_{\rm r})\}\\ +3\{3-6(w+u)-2(v_\perp-v_{\rm r})\}({\rm d}v_{\rm r}-{\rm d}u)]\end{array}}. \tag{25}$$

When $v_\perp = v_{\rm r}$, these equations reduce to those given earlier [14–16]. As pointed out earlier we have only three equations (19), (21) and (24) to solve for the six unknown quantities $u, v_\perp, v_{\rm r}, w, \lambda$ and ν and hence the system is underdetermined. In the next section we present two solutions of these equations based on some simplifying assumptions as well as relationship between $u, v_{\rm r}, v_\perp, w$.

3. Solutions of the field equations

Solution (i): To solve the equations (19), (21) and (24) given in the previous section, we assume the functional relationships

$$v_\perp = k_1 v_{\rm r}, \tag{26}$$

$$w = k_2 v_{\rm r}, \tag{27}$$

and

$$u = v_{\rm r} = \frac{a^2}{2\kappa} r^{(n+2)}, \tag{28}$$

where k_1, k_2, a are constants and $\kappa = (k_2 + (k_1/3) + (2/3))$. Equation (27) is the modified equation of state for isothermal gas spheres [23–25] whereas from the first equality of (28) we get $T_1^1 = 0$. Equation (26) is as in Ponse de Leon [12]. Thus from (19), after using (26), (27) and (28), we get

$$\exp(-\lambda) = 1 - a^2 r^{(n+2)}. \tag{29}$$

Also, using (28) in (26) and (27), we get

$$v_{\perp} = \frac{k_1 a^2}{2\kappa} r^{(n+2)}, \tag{30}$$

$$w = \frac{k_2 a^2}{2\kappa} r^{(n+2)}. \tag{31}$$

Now, with the help of (28), (30) and (31) we solve (20) for ν to get

$$\exp(\nu) = k_3 \{1 - a^2 r^{(n+2)}\}^{-1/(n+2)}, \tag{32}$$

where k_3 is a positive constant of integration. Also using (20), from (6), (7) and (5) we can write expressions for Q, F_{14}, J_4 and σ as follows:

$$Q = \frac{a r^{(n+4)/4}}{(4\pi\kappa)^{1/2}}, \tag{33}$$

$$F_{14} = -\frac{a k_3 r^{n/2}}{2\pi\kappa} (1 - a^2 r^{(n+2)})^{-(n+3)/2(n+2)}, \tag{34}$$

$$J_4 = -(k_3/16\pi\kappa)^{1/2} \{(n+4) a r^{(n/2)-1}\} \{1 - a^2 r^{(n+2)}\}^{(n+1)/2(n+2)} \tag{35}$$

and

$$\sigma = -\left(\frac{n+4}{4(\pi x)^{1/2}}\right) a r^{(n/2)-1} \{1 - a^2 r^{(n+2)}\}^{1/2}. \tag{36}$$

With the help of (34) and (35) we write the expressions for physical components of F_{14} and J_4 as follows:

$$F_{(14)} = F_{14} \lambda^1_{(1)} \lambda^4_{(4)} = -\frac{a}{2} \left(\frac{r^n}{\pi x}\right)^{1/2}, \tag{37}$$

$$\sigma = J_{(4)} = J_4 \lambda^4_{(4)} = -\left(\frac{n+4}{4(\pi x)^{1/2}}\right) a r^{(n/2)-1} \{1 - a^2 r^{(n+2)}\}^{1/2}, \tag{38}$$

where the orthonormal tetrad $\lambda_{(\alpha)} = \text{diag.}\ (\mathrm{e}^{-\lambda/2}, 1/r, 1/r \sin\theta, \mathrm{e}^{-\nu/2})$.

Also we write explicit expressions for $\xi, \rho, p_{\mathrm{r}}, p_{\perp}$ and ϵ as follows:

$$4\pi\xi = 4\pi p_{\mathrm{r}} = \frac{a^2}{2\kappa} r^n, \tag{39}$$

$$4\pi\rho = \left(\frac{nx + 3x - 1}{2\kappa}\right) a^2 r^n, \tag{40}$$

$$4\pi p_{\perp} = \frac{k_1 a^2}{2\kappa} r^n, \tag{41}$$

and

$$4\pi\epsilon = \frac{a^2}{2\kappa}(1 + 3k_2 - (n+3)\kappa)r^n. \tag{42}$$

From (39) to (42) we note that all the physical variables are non-singular for $n > 0$.
Solution (ii): Once again to solve (19), (21) and (24) we assume the following relationships:

$$u = v_{\rm r}, \tag{43}$$

$$v_\perp - v_{\rm r} = k, \tag{44}$$

$$w + u = \frac{b^2 r^{(n+2)}}{2}. \tag{45}$$

We note that (43) implies $T_1^1 = 0$ whereas (45) is similar to the assumption made by us [14] to generalize the Cooperstock–de la Cruz [1] solution. The assumption (44) simplifies the structure of the field equations (10), (21) and (24). The anisotropy can be made very small by taking the constant k appropriately so that near the centre the physical variables $\xi, \rho, p_\perp, p_{\rm r}$ as well as ϵ do not take very large values. Then from (10) we get,

$$\exp(-\lambda) = \gamma - b^2 r^{(n+2)}, \tag{46}$$

where $\gamma = 1 - (2k/3)$ is a constant. Using (43) to (45) in (20) and integrating for ν we get,

$$\exp\nu = \frac{\alpha r^{(1-\gamma)/\gamma}}{\{\beta^2 - r^{(n+2)}\}^{1/\gamma(n+2)}}, \tag{47}$$

where $\beta^2 = b^{-2/\gamma(n+2)}$ and α is a positive constant of integration. From (46) and (47) we note that the space-time is regular for $\gamma < 1$ (or $k > 0$) and hence $v_\perp - v_{\rm r} > 0$ suggesting that the pressure anisotropy in this case contributes positively to the total energy as defined by (13). Also using (8)–(11), and with the help of (43) to (45) we get the following expressions:

$$u = v_{\rm r} = v_\perp - 3(1-\gamma)/2, \tag{48}$$

where

$$v_\perp = \frac{(1-\gamma)(1+11\gamma)}{16\gamma} + \frac{b^2 r^{(n+2)}}{2(\gamma - b^2 r^{(n+2)})} - \frac{(\gamma^2 n + 3\gamma^2 - \gamma n + 6\gamma - 1)b^2 r^{(n+2)}}{16\gamma^2} + \frac{(\gamma n - 6\gamma + 1)b^4 r^{2(n+2)}}{16\gamma^2(\gamma - b^2 r^{(n+2)})}, \tag{49}$$

and

$$w = \frac{(1-\gamma)(13\gamma - 1)}{16\gamma} - \frac{b^2 r^{(n+2)}}{2(\gamma - b^2 r^{(n+2)})} + \frac{(\gamma^2 n + 11\gamma^2 - \gamma n + 6\gamma - 1)b^2 r^{(n+2)}}{16\gamma^2} - \frac{(\gamma n - 6\gamma + 1)b^4 r^{2(n+2)}}{16\gamma^2(\gamma - b^2 r^{(n+2)})}. \tag{50}$$

Thus, from (49) and (50), we note that $u, v_\perp, v_{\rm r}$ and ω are regular for $n < 2$.

With the help of (48) and (6) we can write $Q(r)$ as

$$Q(r) = \frac{1}{(2\pi)^{1/2}} \left[\frac{(1-\gamma)(1-13\gamma)r^2}{16\gamma} + \frac{b^2 r^{(n+4)}}{2(\gamma - b^2 r^{(n+2)})} - \frac{(\gamma^2 n + 3\gamma^2 - \gamma n + 6\gamma - 1) b^2 r^{(n+4)}}{16\gamma^2} + \frac{(\gamma n - 6\gamma + 1) b^4 r^{2(n+3)}}{16\gamma^2 (\gamma - b^2 r^{(n+2)})} \right]^{1/2} \tag{51}$$

and we note that $Q(0) = 0$.

For this solution the expressions for F_{14} and J_4 are as follows:

$$F_{14} = -\frac{(\alpha)^{1/2} b^{1/\gamma(n+2)} r^{(1-5\gamma)/2\gamma} Q(r)}{\{\gamma - b^2 r^{n+2}\}^{1/2} \{1 - b^2 r^{n+2}\}^{1/2\gamma(n+2)}}, \tag{52}$$

$$J_4 = -\frac{(\alpha)^{1/2} b^{1/\gamma(n+2)} r^{(1-5\gamma)/2\gamma} \{\gamma - b^2 r^{(n+2)}\}^{1/2} Q'(r)}{\{1 - b^2 r^{(n+2)}\}^{1/2\gamma(n+2)}}, \tag{53}$$

where $Q(r)$ is given by (51).

With the help of (52) and (53) we write the expressions for physical components of F_{14} and J_4 as follows:

$$F_{(14)} = -\frac{1}{(2\pi)^{1/2}} \left[\frac{(1-\gamma)(1+11\gamma)}{16\gamma r^2} + \frac{b^2 r^n}{2(\gamma - b^2 r^{(n+2)})} - \frac{(\gamma^2 n + 3\gamma^2 - \gamma n + 6\gamma - 1) b^2 r^n}{16\gamma^2} + \frac{(\gamma n - 6\gamma + 1) b^4 r^{2(n+1)}}{16\gamma^2 (\gamma - b^2 r^{(n+2)})} \right]^{1/2}, \tag{54}$$

$$\sigma = J_{(4)} = -\{\gamma - b^2 r^{(n+2)}\}^{1/2} \frac{Q'(r)}{r^2}, \tag{55}$$

where $Q(r)$ is given by (51).

Also we write explicit expressions for $\xi, \rho, p_{\rm r}, p_\perp$ and ϵ which are as follows:

$$4\pi\xi = 4\pi p_{\rm r} = \frac{(1-\gamma)(1-13\gamma)}{16\gamma r^2} + \frac{b^2 r^n}{2(\gamma - b^2 r^{(n+2)})} - \frac{(\gamma^2 n + 3\gamma^2 - \gamma n + 6\gamma - 1) b^2 r^n}{16\gamma^2} + \frac{(\gamma n - 6\gamma + 1) b^4 r^{2(n+1)}}{16\gamma^2 (\gamma - b^2 r^{(n+2)})}, \tag{56}$$

$$4\pi\rho = \frac{(1-\gamma)(21\gamma - 1)}{16\gamma r^2} - \frac{3 b^2 r^n}{2(\gamma - b^2 r^{(n+2)})} + \frac{(9\gamma^2 n + 27\gamma^2 - \gamma n + 22\gamma - 1) b^2 r^n}{16\gamma^2} + \frac{(22\gamma - \gamma n - 1) b^4 r^{2(n+1)}}{16\gamma^2 (\gamma - b^2 r^{(n+2)})}, \tag{57}$$

$$4\pi p_{\perp} = \frac{(1-\gamma)(1+11\gamma)}{16\gamma r^2} + \frac{b^2 r^n}{2(\gamma - b^2 r^{(n+2)})} - \frac{(\gamma^2 n + 3\gamma^2 - \gamma n + 6\gamma - 1) b^2 r^n}{16\gamma^2} + \frac{(\gamma n - 6\gamma + 1) b^4 r^{2(n+1)}}{16\gamma^2(\gamma - b^2 r^{(n+2)})}, \quad (58)$$

$$4\pi\epsilon = \frac{(1-\gamma)(9\gamma-1)}{8\gamma r^2} - \frac{(3\gamma^2 n - 3\gamma^2 + \gamma n + 2\gamma + 1) b^2 r^n}{8\gamma^2} - \frac{(\gamma n + 2\gamma + 1) b^4 r^{2(n+1)}}{8\gamma^2(\gamma - b^2 r^{(n+2)})}. \quad (59)$$

We note that in the physical variables given by (56) to (59) the terms falling as inverse square of distance has k (which can be chosen as small as we please) terms in the numerator. We have already mentioned that $Q(0) = 0$ and hence at the centre the pressure anisotropy caused by the electric flux vanishes. Thus when $k = 0$ $(\gamma = 1)$ the solution reduces to the generalized Cooperstock–de la Cruz solution given by us [14] earlier.

4. Boundary conditions

In this section we show that the two solutions derived in §3 can be matched with the exterior Reissner–Nordstrom solution given by the metric

$$\mathrm{d}s^2 = -\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)^{-1} \mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2) + \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)\mathrm{d}t^2; \quad (r > r_b), \quad (60)$$

where m and q denote respectively the material energy and charge enclosed within a constant radius r_b.

The metric for solution (i) is expressed with the help of (29) and (32) as

$$\mathrm{d}s^2 = -(1 - a^2 r^{n+2})^{-1}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2) + k_3(1 - a^2 r^{n+2})^{-1/(n+2)}\mathrm{d}t^2; \quad (r < r_1), \quad (61)$$

where we have selected, for convenience, the boundary of the sphere as r_1; (i.e. $r_b = r_1$). Thus, the continuity of the metric tensor over the boundary $r = r_1$ gives the following involved relationships between m, q, a and k_3 through r_1:

$$a^2 r_1^{n+4} - 2mr_1 + q^2 = 0, \quad (62)$$

$$r_1 = \left[\frac{1 - k_3^{(n+2)/(n+3)}}{a^2}\right]^{1/n+2}. \quad (63)$$

Similarly, in the case of the second solution with the interior metric taking the form

$$\mathrm{d}s^2 = -(\gamma - b^2 r^{n+2})^{-1}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2) + \frac{\alpha b^{2/\gamma(n+2)} r^{(1/\gamma)-1}}{(1-b^2 r^{n+2})^{1/\gamma(n+2)}}\mathrm{d}t^2,\ \gamma < 1,\ r < r_2, \tag{64}$$

the boundary conditions give the following relations between the constants m, q, α and β through r_2:

$$\beta^{-2\gamma(n+2)} r_2^{n+4} + (1-\gamma) r_2^2 - 2mr_2 + q^2 = 0, \tag{65}$$

$$(2mr_2 - q^2 - r_2)^{\gamma/(n+2)} + \frac{r_2^{(\gamma/(n+2))+2}\alpha^{\gamma/(n+2)}}{(2-\gamma)r_2^2 - 2mr_2 + q^2} = (r_2\alpha)^{\gamma/(n+2)}. \tag{66}$$

5. Conclusion

In the present paper we have considered spherically symmetric static charged distributions with anisotropic fluid pressure. The Einstein field equations when recast in terms of $u, v_\perp, v_\mathrm{r}$ and w still show that these variables and their derivatives occur in linear relationship with each other even though the resulting equations are pretty lengthy. However, we have been able to exploit this linear relationship between $u, v_\perp$, v_r and w to arrive at some mathematically simplifying as well as physically plausible assumptions in deriving two new solutions. The first solution is regular for $n > 0$ in its metric coefficients as well as physical variables $\xi, p_\mathrm{r}, p_\perp, \rho$ and ϵ. In the second solution, even though the physical variables have (k/r^2) terms, in view of the fact that the total charge $Q(r)$ vanishes at $r = 0$, the pressure anisotropy dies down as we approach the centre. In other words once the isotropy of the fluid pressure is restored near the centre, the solution tends to be regular, a property satisfied by the generalized Cooperstock and de la Cruz solution given in [14]. We hope that more exact solutions with anisotropic fluid pressure will be derivable using the present formulation.

Acknowledgement

The authors thank the referee for his comments on an earlier version of the paper and to Dr A H Hasmani and R B Shah for their help in the preparation of the manuscript.

References

[1] F I Cooperstock and V de la Cruz, *Gen. Relativ. Gravit.* **9**, 835 (1978)
[2] A Patino and H Rago, *Gen. Relativ. Gravit.* **21**, 637 (1989)
[3] R Tikekar, *Gen. Relativ. Gravit.* **15**, 445 (1984a)
[4] R Tikekar, *J. Math. Phys.* **25**, 1481 (1984b)
[5] Xingxiang Wang, *Gen. Relativ. Gravit.* **19**, 729 (1987)
[6] A Nduka, *Gen. Relativ. Gravit.* **7**, 493 (1976)

[7] A L Mehra, *Gen. Relativ. Gravit.* **12**, 187 (1980)
[8] T Singh, G P Singh and R S Srivastava, *Int. J. Theor. Phys.* **31**, 545 (1992)
[9] A Banerjee and M M Som, *Prog. Theor. Phys.* **65**, 1281 (1981)
[10] P S Florides, *J. Phys.* **A16**, 1419 (1983)
[11] O Gron, *Gen. Relativ. Gravit.* **18**, 591 (1986)
[12] J Ponce de Leon, *Gen. Relativ. Gravit.* **19**, 797 (1987)
[13] S K Vaidya and L K Patel, *Math. Today* (Ahmedabad) **15**, 55 (1997)
[14] J Krishna Rao and M M Trivedi, *Pramana – J. Phys.* **54**, 663 (1998)
[15] J Krishna Rao, M Annapurna and M M Trivedi, *Indian J. Pure Appl. Math.* (Delhi) (to appear)
[16] J Krishna Rao, M Annapurna and M M Trivedi, Communicated to *Math. Today* (Ahmedabad) **A16**, 29 (1998)
[17] R Roy, A A Rangwala and N C Rana, *Indian J. Pure Appl. Math.* **27**, 1119 (1996)
[18] J Krishna Rao, *Curr. Sci.* **35**, 589 (1966)
[19] J Krishna Rao, *Gen. Relativ. Gravit.* **4**, 385 (1971)
[20] J Krishna Rao, *J. Phys.* (*London*) **A5**, 479 (1972)
[21] J Krishna Rao, *Gen. Relativ. Gravit.* **5**, 351 (1973)
[22] J Krishna Rao, *Pramana – J. Phys.* **34**, 423 (1990)
[23] J Krishna Rao and M Annapurna, in *A random walk in relativity and cosmology* edited by N Dadhich *et al* (Wiley Eastern Ltd., New Delhi, 1985)
[24] J Krishna Rao and M Annapurna, *Pramana – J. Phys.* **36**, 95 (1991)
[25] J Krishna Rao and M Annapurna, *Math. Today* (Ahmedabad) **A11**, 57 (1993)