PRAMANA
— journal of
physics

Vol. 54, No. 1
January 2000
pp. 203–214

Introduction to a field-theoretical treatment of neutrino oscillations

P STOCKINGER
Theory Group, Physical Research Laboratory, Ahmedabad 380 009, India
Email: stocki@prl.ernet.in

Abstract. We discuss the main features of the field-theoretical approach to neutrino oscillations where one combines neutrino production and detection processes in a single Feynman graph. The 'oscillating neutrinos' are represented by inner lines of this graph and appear in the calculation of the cross section of the total process as propagators of the neutrino mass eigenfields. We show that this field-theoretical approach leads to a transparent treatment of neutrino oscillations without ambiguities and provides the correct answer in cases where the standard approach fails.

Keywords. Neutrino mass; laboratory experiments on neutrino mass; field theory.

PACS Nos 14.60 Pq; 03.65 -w

1. Introduction

The standard treatment of neutrino oscillations provides a beautiful and simple picture of this important phenomenon. However, after a closer look one discovers that the derivation of the standard neutrino oscillation formula raises some conceptional questions and needs clarification in several points. The first attempt towards a better understanding of the physics involved in neutrino oscillations was to describe neutrinos in terms of wave packets. But also this approach is not totally satisfactory either without knowing size and form of the wave packets. Furthermore, one should not care too much about the state of the neutrinos since they are never directly prepared or observed in an experiment.

This fact is best taken into account in the quantum field-theoretical approach where one combines neutrino production and detection process in a single Feynmann graph such that the oscillating neutrinos are associated with the inner line of this graph and are represented by the propagators of the neutrino mass eigenfields. Then the corresponding cross section of the total process may exhibit an oscillatory dependence on L, the distance between neutrino source and detection. The final formula involves only quantities pertaining to the source and detector which are, at least in principle, directly measurable and determined by the experimental set-up. In this paper we want to give an overall view of what distinguishes the field-theoretical treatment from the standard approach and which new effects can be calculated with this new method. In order not to lose track of things we often refer for details to the literature.

2. The standard treatment of neutrino oscillations and its shortcomings

In order to dicuss the ambiguities associated with the standard treatment of neutrino oscillations [1,2] we want to summarize briefly the main assumptions and approximations which have to be made there.

For massive neutrinos, the left-handed neutrino fields are in general linear combinations of mass eigenfields, i.e.

$$\nu_{\alpha L}(x) = U_{\alpha i}\nu_{iL}(x) \tag{1}$$

with a unitary mixing matrix U and neutrino fields ν_i with mass m_i. First of all one assumes that the neutrinos are produced in 'flavour eigenstates', i.e. the states are linear combinations of mass eigenstates with the simple form

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}|\nu_i\rangle \ . \tag{2}$$

Secondly, it is supposed that the mass eigenstates have the same momentum $\vec{p}$ but different energies E_i given by the relativistic energy-momentum relation. Thirdly, one confines oneself to the case of extremely relativistic neutrinos where one can use

$$E_i = \sqrt{\vec{p}^{\,2} + m_i^2} \simeq p + \frac{m_i^2}{2p} \tag{3}$$

and where the time evolution for the produced neutrino ν_α is given by

$$|\nu_\alpha(t)\rangle = \sum_i U^*_{\alpha i}\mathrm{e}^{-ipt}\mathrm{e}^{-i\frac{m_i^2}{2p}t}|\nu_i\rangle \ . \tag{4}$$

Finally, since neutrinos are ultrarelativistic, the time is replaced by L, where L is the distance between the production and the detection of the neutrinos, and hence the transition probability

$$P_{\alpha\beta} = |\langle\nu_\beta|\nu_\alpha(t)\rangle|^2 = \left|\sum_i U_{\beta i}U^*_{\alpha i}\mathrm{e}^{-i\frac{m_i^2}{2p}L}\right|^2 \tag{5}$$

becomes a function of this distance L. The derivation of the oscillation formula (5) is quite simple and transparent and gives the right answer under certain conditions. However, this standard approach also immediately raises a number of conceptional questions which give the motivation for a more comprehensive and thorough study of this important phenomenon. The most important questions and shortcomings are (see also ref. [3]):

- Why have the neutrinos a definite momentum rather than a definite energy?
- How can we change t into L when a neutrino with definite momentum must have a wave function of infinite extend?
- The weak eigenstates are in general not given by eq. (2) but depend on the special production process and are actually given by a superposition of mass eigenstates weighted by their transition amplitude [4].

- The different mass eigenstates must be produced and detected coherently and consequently the use of plane waves seems to be questionable.

Obviously the first question can only be answered when one investigates the certain production and detection process of the neutrinos. The second point expresses the fact that one actually has combined in this standard approach classical particle and quantum mechanical descriptions which usally leads to inconsistencies. The fourth point means that the weak states (2) do not describe in general correctly the neutrinos produced and detected in weak interaction processes. This can be understood best by taking a look at an certain example. Let us consider the weak charged current process $\nu + X_i \to X_f + e^-$ in which a neutrino is detected through the production of an electron. If the neutrino were correctly described by the weak states $|\nu_\alpha\rangle$ the transition amplitude

$$\begin{aligned} \mathcal{A} &\sim \langle e^-|\bar{e}\gamma^\lambda(1-\gamma_5)\nu_e|\nu_\alpha\rangle \; M_\lambda(X_i, X_f) \\ &= \sum_j U_{ej}U^*_{\alpha j}\langle e^-|\bar{e}\gamma^\lambda(1-\gamma_5)\nu_j|\nu_j\rangle \; M_\lambda(X_i, X_f) \end{aligned} \quad (6)$$

should vanish for $\alpha \neq e$ (M_λ are the matrix elements of the X part of the process). But although the mixing matrix is unitary, the factors $\langle ....\rangle M_\lambda$ which depend on the index j through the different masses m_j, spoil the diagonality in the flavour indices. Only in the ultrarelativistic limit, or when the masses m_j are almost degenerate, the factors can be taken out of the sum and the transition amplitude vanishes for $\alpha \neq e$. Finally (addressing the last point in the list), if the particles involved in the neutrino production process are assumed to have definite four momenta, the neutrino is forced to have a definite four momentum too and is hence in one of its mass eigenstates. Therefore, in order to observe neutrino oscillations one needs a sufficient spread of momentum or energy of the particles involved in the neutrino production and detection process. This suggests that the propagating flavour neutrino should rather be described as a superposition of wave packets than of plane waves. But in the wave packet approach [6–14] the size and form of the neutrino wave packet are not determined and have to be estimated (see, e.g., [2]). It has been shown in the literature [3,5] that the correct way to avoid all difficulties and ambiguities associated with neutrino oscillations is to concentrate on those things which can really be manipulated or observed like the particles responsible for neutrino production and the target responsible for neutrino detection. This can be done within the field-theoretical approach in a quite natural way.

3. The field-theoretical approach

In this section we study the conditions for neutrino oscillations in the field-theoretical approach by taking into account that only the neutrino production and detection processes, which are localized in space around the coordinates $\vec{x}_P$ and $\vec{x}_D$, respectively, can be manipulated.

For definiteness we discuss the features of this approach on the basis of an example where antineutrinos are produced by the β-decay of the fission products in a reactor and assume that the antineutrinos are detected by elastic electron scattering (see figure 1). As already mentioned, the purpose of the paper is to get

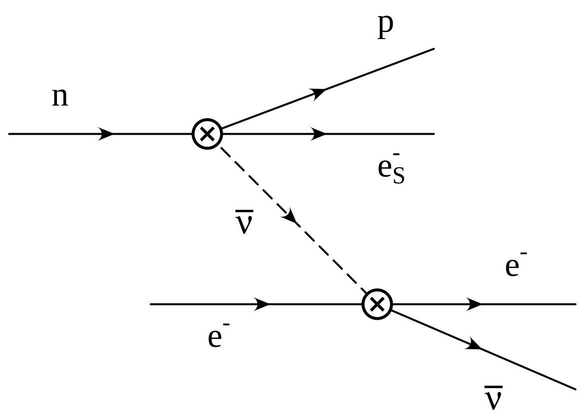

Figure 1. Feynman graph for the process under consideration.

a feeling for this method to describe neutrino oscillations and hence we try to concentrate on the main points and their connections and refer for the details to the literature. We simplify our model reaction by taking only the β-decay of the neutron and assuming that the neutron is bound to nuclei at rest. The most important points in our treatment of neutrino oscillations are the following (for more details see ref. [15]):

- The neutron is bound to nuclei in a stationary state localized at the coordinate $\vec{x}_P$ whereas the target electron is bound in an atom at the point $\vec{x}_D$. Hence, neutrino production and detection are macroscopically separated by a distance $L \equiv |\vec{L}|$ with $\vec{L} \equiv \vec{x}_D - \vec{x}_P$.
- All particles in the final state are described by plane waves.
- The internal (anti)neutrino is represented by its Feynman propagator and the neutrino oscillation amplitude is determined through the neutrino production and detection interaction Hamiltonians.

Thus we consider the weak process

$$\begin{array}{l} n \to p + e^- + \bar{\nu} \\ \qquad\qquad\qquad \searrow \\ \qquad\qquad\qquad\quad \bar{\nu} + e^-_D \to \bar{\nu} + e^-_D \end{array} \tag{7}$$

occurring through the intermediate propagation of an antineutrino, where n (neutron) and e^-_D (electron) are the initial particles. Since neutrino production and detection are localized at $\vec{x}_P$ and $\vec{x}_D$, respectively, the spinors of the initial particles can be written in position space as

$$\begin{aligned} \psi_n(x) &= \psi_n(\vec{x} - \vec{x}_P)\exp(-iE_n t), \\ \psi_{eD}(x) &= \psi_{eD}(\vec{x} - \vec{x}_D)\exp(-iE_{eD} t), \end{aligned} \tag{8}$$

respectively. The wave functions $\psi_{n,eD}(\vec{y})$ are peaked at $\vec{y}=0$ and stationary. The amplitude for the process (7) with an antineutrino of mass m_k in the final state is given by

$$\mathcal{A}_k = \Big\langle p, \bar{\nu}_k(\vec{p}_\nu'), e^-(\vec{p}_e'), e_D^-(\vec{p}_{eD}') \\ \times \Big| T\left[\int \mathrm{d}^4x_1 \int \mathrm{d}^4x_2\, \mathcal{H}_S(x_1)\mathcal{H}_D(x_2)\right]\Big| n, e_D^- \Big\rangle, \tag{9}$$

where $\mathcal{H}_S$ and $\mathcal{H}_D$ are the relevant Hamiltonian densities for the production and the detection of the neutrinos, describing neutron decay and electron–neutrino scattering, respectively. With the neutrino propagators of the mass eigenstate neutrinos

$$\langle 0|T[\nu_j(x_1)\bar{\nu}_j(x_2)]|0\rangle = i\int \frac{\mathrm{d}^4q}{(2\pi)^4}\frac{\not{q}+m_j}{q^2-m_j^2+i\epsilon}\mathrm{e}^{-iq\cdot(x_1-x_2)}\ , \tag{10}$$

we obtain the amplitude

$$\mathcal{A}_k = \sum_j \int \mathrm{d}^4x_1 \int \mathrm{d}^4x_2 \int \mathrm{d}^4q\ \mathrm{e}^{-iq\cdot(x_1-x_2)} \\ \bar{U}_j^S(\vec{x}_1-\vec{x}_P)(\left(\frac{\not{q}+m_j}{q^2-m_j^2+i\epsilon}\right)U_{jk}^D(\vec{x}_2-\vec{x}_D)\ , \tag{11}$$

where U_j^S and U_{jk}^D denote the corresponding part of the matrix element associated with the production and detection process, respectively (for the full expressions see ref. [15]). After the integrations over x_1 and x_2, which are rather straightforward, the integration over q remains. Since our initial and final states are all energy eigenstates, the integration over q^0 leads to the usual δ-function expressing that the initial energy is equal to the final energy of the total process (7). The analogous δ-functions corresponding to momentum are smeared out by the initial momentum distributions and therefore we have a non-trivial d^3q integration in the amplitude of the form

$$\int \mathrm{d}^3q\,\Phi(\vec{q})\,\mathrm{e}^{-i\vec{q}\cdot\vec{L}}\frac{1}{A-\vec{q}^{\,2}+i\epsilon}\ , \tag{12}$$

where $A=q_0^2-m_j^2$ and ϕ represents simply the remaining part of the amplitude. We can, however, take advantage of the fact that this integration actually amounts to calculating a Fourier transform and evaluating it at the coordinate $\vec{L}$, the macroscopic distance between neutrino production and detection. Hence, we can apply the theorem proved in ref. [15] which enables us to calculate the leading term of the amplitude for large L. In this limit the integral (12) is given by (for $A>0$)

$$-\frac{2\pi^2}{L}\Phi(-\sqrt{A}\vec{L}/L)\,\mathrm{e}^{i\sqrt{A}L}+\mathcal{O}(L^{-3/2}) \tag{13}$$

(for $A<0$ the integral decreases like L^{-2}) and the amplitude (11) can be written as

$$\mathcal{A}_k^\infty = \frac{1}{L}\,\delta(E_S-E_D)\sum_j \mathrm{e}^{iq_jL}\mathcal{A}_j^S\mathcal{A}_{jk}^D\ , \tag{14}$$

where $\mathcal{A}_j^S$ denotes the amplitude for the production of a neutrino with mass m_j and $\mathcal{A}_{jk}^D$ is the amplitude for the detecting scattering process for an initial neutrino with m_j and a final neutrino with m_k. The parts of these amplitudes which are important for the further discussion are given by

$$\mathcal{A}_j^S = ... \tilde{\psi}_n(\vec{p}_1 + q_j\vec{l}\,) .... \tag{15}$$

and

$$\mathcal{A}_{jk}^D = ... \tilde{\psi}_{eD}(\vec{p}_2 - q_j\vec{l}\,) . \tag{16}$$

For the full expressions see ref. [15]. The kinematical quantities occurring in eq. (14) are defined by

$$\begin{aligned} E_S &\equiv E_n - E_p - E'_e, & \vec{p}_1 &\equiv \vec{p}\,'_e, \\ E_D &\equiv E'_\nu + E'_{eD} - E_{eD}, & \vec{p}_2 &\equiv \vec{p}\,'_{eD} + \vec{p}\,'_\nu , \end{aligned} \tag{17}$$

and

$$\vec{l} \equiv \frac{\vec{L}}{|\vec{L}|}, \quad q_0 = -E_S = -E_D \quad \text{and} \quad q_j \equiv \sqrt{q_0^2 - m_j^2} . \tag{18}$$

The fact that the amplitude (14) can be written as a product of production and detection amplitude arises from the fact that in our limit the numerator in (11) of the neutrino propagator is given by

$$q_0\gamma^0 + q_j\vec{l}\cdot\vec{\gamma} + m_j = -\not{p}_j + m_j = -\sum_{\pm s} v_j(\vec{p}_j, s)\bar{v}_j(\vec{p}_j, s) , \tag{19}$$

where the 4-vector p_j defined by

$$p_j^0 \equiv E_\nu \equiv -q^0 = E_D = E'_{eD} + E'_\nu - E_{eD} \geq 0 \quad \text{and} \quad \vec{p}_j \equiv q_j\vec{l} \tag{20}$$

can be identified as the 4-momentum of this antineutrino. Hence $\mathcal{A}_k^\infty$ is a sum over terms which contain a real antineutrino of mass m_j, or in other words, in the limit $L \to \infty$ the virtual neutrinos become 'real'.

Considering this asymptotic amplitude $\mathcal{A}_k^\infty$ we can make the following remarks and observations.

- In order to obtain $\mathcal{A}_k^\infty$ we have performed the asymptotic limit $L \to \infty$. Looking at eqs (12) and (13) it is evident that 'L large' means $\bar{E}_\nu L/\hbar\, c \approx \bar{E}_\nu L/2\cdot 10^{-13}$ MeV $\cdot\, m \gg 1$ where $\bar{E}_\nu$ is an average antineutrino energy and L is measured in meters. This seems to be very well fulfilled for all neutrino experiments.
- The factor $1/L$ in the asymptotic amplitude corresponds to the geometrical decrease of the neutrino flux by $1/L^2$ in the cross section.
- The exponential factors in (14) give rise to neutrino oscillations (see eq. (22) below). The exponent suggests that neutrino oscillations take place between neutrinos with the same energy but different momenta q_j. This is a consequence of our assumption that the wave function of the electron in the detector is stationary and consequently we identify $E_\nu = E_D$ [16].

- We want to emphasize that in the field-theoretical approach the notion of a neutrino wave packet does not exist and that only the final formulas may be interpreted with respect to the state of the neutrino.
- Looking at eq. (14) we conclude that neutrino oscillations with masses m_j, m_k can only take place if

$$|q_j - q_k| \lesssim \sigma_S \quad \text{and} \quad |q_j - q_k| \lesssim \sigma_D, \tag{21}$$

where σ_S and σ_D are the widths of the functions $\tilde{\psi}_n$ and $\tilde{\psi}_{eD}$, respectively. We call these conditions amplitude coherence conditions (ACC). In coordinate space this simply means that the widths of ψ_n and ψ_{eD} must both be smaller than the oscillation length $L_{\text{osc}} = 4\pi E_\nu / |m_j^2 - m_k^2| \approx 2.48\,\text{m} \cdot E_\nu(\text{MeV})/\Delta m^2(\text{eV}^2)$.

We expect now that under certain conditions the cross section or event rate for the reaction (7) ($n + e_D^- \to e^- + e_D^- + \bar{\nu}$) exhibits an oscillatory dependence on the distance L. If $|q_j - q_k| \ll \min(\sigma_S, \sigma_D)$ is fulfilled $\forall\, j,k$ in addition to the assumption that all neutrinos are ultrarelativistic, then we can take the limit $m_j \to 0\, \forall\, j$ in all terms of $\mathcal{A}_k^\infty$ except the exponential factors $\exp(iq_j L)$. Then calculating the cross section one arrives at the same result as can be obtained by the standard treatment of neutrino oscillations and the following heuristic consideration: The probability of finding a neutrino $\bar{\nu}_\ell$ at a distance L from the source is given by $P_\ell = |\sum_j U_{ej} U_{\ell j}^* \mathrm{e}^{iq_j L}|^2$. Thus the number of events in elastic $\bar{\nu}e^-$ scattering at a certain neutrino energy E_ν is proportional to $\sum_\ell P_\ell \sigma(\bar{\nu}_\ell e^-; E_\nu)$ where the $\sigma(\bar{\nu}_\ell e^-; E_\nu)$ ($\ell = e, \mu, \tau$) are the elastic scattering cross sections as given by the SM. Thus the event rate is given by

$$\frac{\mathrm{d}N}{\mathrm{d}E_\nu} = N_0 \Big\{ \Big| \sum_j |U_{ej}|^2 \mathrm{e}^{iq_j L} \Big|^2 \Big(\sigma(\bar{\nu}_e e^-; E_\nu) - \sigma(\bar{\nu}_\mu e^-; E_\nu) \Big) + \sigma(\bar{\nu}_\mu e^-; E_\nu) \Big\} . \tag{22}$$

Hence, in the extremely relativistc limit the oscillation probability does not depend on the details of the production and detection process and the results of the standard treatment of neutrino oscillations can be confirmed. However, note that the field-theoretical treatment provides the correct answer also in cases where the standard approach to neutrino oscillations fails.

4. Applications

We are now endowed with a method to calculate such ‘macroscopic Feynman diagrams’ and can apply it in order to calculate corrections to the ultrarelativistic limit which are proportional to the mass of the neutrinos. For illustration we consider as above antineutrinos produced in a nuclear reactor which are detected via elastic $e^-\bar{\nu}$-scattering and take into account the possible effects of neutrino masses and mixing and of neutrino magnetic moments [17]. Without showing any calulations or formulas we want to dicuss briefly the results and make some remarks to this example which are of general interest. First of all, only within the field-theoretical approach it is possible to investigate in a clear and systematic way effects of neutrino oscillations and neutrino magnetic moments on elastic $e^-\bar{\nu}$-scattering without restriction to the extremely relativistic limit. If we confine ourselves to an investigation of terms in the cross section which are at most linear in the mass m_ν of the neutrino (first order corrections to the ultrarelativistic limit), we make following observations:

- Only the weak-electromagnetic interference part in the cross section exhibits terms of the order m_ν, for the pure weak and electromagnetic cross section the first non-trivial corrections to the massless case are of the order m_ν^2.
- For the usual experimental set-up of experiments with reactor neutrinos it turns out that the cross section factorizes into a product of a production cross section and a detection cross section.

Hence, in that case, not only the amplitude but also the cross section has a simple form with respect to the considered production and detection process of the neutrino. This means that production and detection are no longer entangled and we are able to define a scattering cross section which can directly be compared with the well known cross section for massless neutrinos. Note that although one never cares about the state of the intermediate neutrino in this treatment the obtained interference term in the cross section for elastic $e^-\bar{\nu}$-scattering takes into account the special production process and the propagation of the neutrino (for more details please see ref. [17]).

Furthermore, we want to mention that, in contrast to the standard approach, the field-theoretical treatment provides also the suitable framework to investigate all possible conditions for the existence of neutrino oscillations since one can, at least in principle, take into account the specific experimental situation. All neutrino oscillation experiments are evaluated with the formula (5) which can be rewritten as

$$P_{\nu_\alpha\to\nu_\beta} = \sum_j |U_{\beta j}|^2|U_{\alpha j}|^2 + 2\,\mathrm{Re}\left\{\sum_{j>k} U_{\beta j}U^*_{\alpha j}U^*_{\beta k}U_{\alpha k}\exp\left(-i\frac{\Delta m^2_{jk}L}{2E_\nu}\right)\right\} \tag{23}$$

with $\Delta m^2_{jk} \equiv m_j^2 - m_k^2$ and where $m_1 \leq m_2 \leq \ldots$ denote the neutrino masses. The aim of theoretical treatments of neutrino oscillations is to work out thoroughly under which conditions formula (23) is valid. Apart from corrections relevant for non-extremely relativistic neutrinos there are always several conditions for the validity of (23) associated with the certain production and detection process. We have already got to know two of such conditions (see eq. (21)) which are related to the momentum spread of the particles responsible for the production and the detection of the neutrino and have to be fulfilled for the transition amplitude. The transition probability (23) is given by the square of the sum over the amplitudes of the neutrino mass eigenstates, i.e., by a coherent summation over the mass eigenstates. The first term in the second line of eq. (23) represents the purely incoherent summation over the mass eigenstates whereas the second term denotes the interference terms. Equation (23) does not take into account the details of the specific neutrino production and detection processes, i.e., it is a theoretical expression without regard to an actual experimental situation. However, the experimental conditions may cause that some

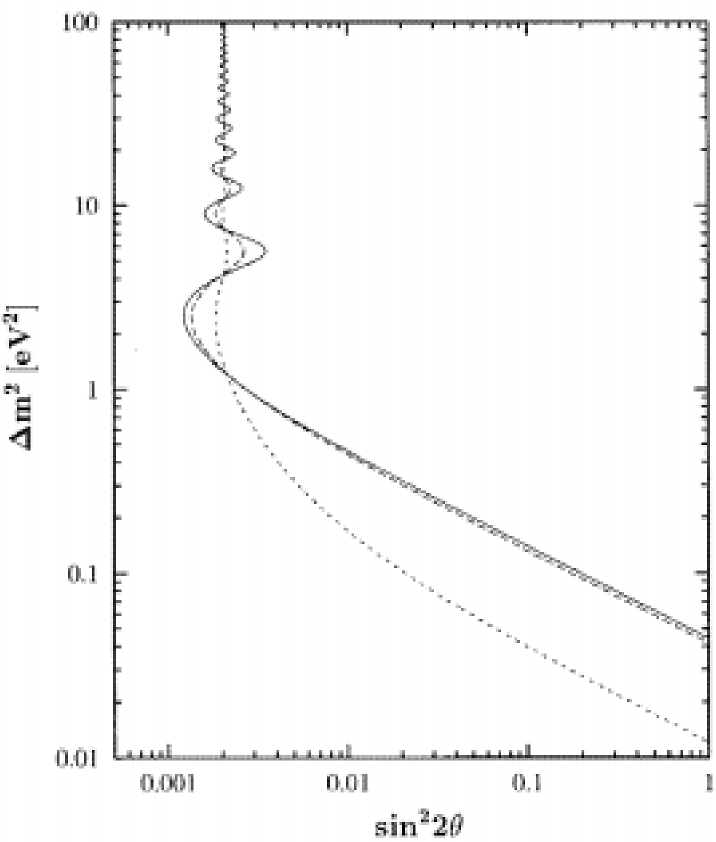

Figure 2. The $\Delta m^2 - \sin^2 2\theta$ plot showing the effect of the lifetime and the momentum spread σ_S of the source particle as given in eq. (27). The dashed and dotted line correspond to $\sigma_S = 1$ and 10 MeV, respectively. The solid line represents the standard oscillation formula.

or all of the interference terms drop out as a consequence of certain averaging or suppression mechanisms. One such mechanism which is always present stems from the inability to measure the energy of the neutrino E_ν better than at a certain realistic experimental accuracy leading to an averaging over E_ν.

Another example is the condition for the existence of neutrino oscillations which derives from the finite lifetime τ_S of the neutrino source particle (for a comprehensive discussion of this effect see [18]). This condition is present if the neutrino source is a free particle such that its wave function is non-stationary. This seems to be the case in the important experiments of the LSND [19] and KARMEN [20] Collaborations, which study $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations with μ^+ decay at rest as $\bar{\nu}_\mu$ source. It turns out that for these experiments the transition probability is given by

$$P_{\bar{\nu}_\mu \to \bar{\nu}_e} = \sum_j |U_{ej}|^2 |U_{\mu j}|^2 + 2\,\mathrm{Re}\left\{ \sum_{j>k} U_{ej}^* U_{\mu j} U_{ek} U_{\mu k}^* \; g_{jk} \; \exp\left(-i \frac{\Delta m_{jk}^2 L}{2E_\nu} \right) \right\} , \tag{24}$$

where in contrast to the formula derived within the standard treatment of neutrino oscillations (eq. (23)), the quantities g_{jk} appear as correction factors in eq. (24). These factors depend on the quantities

$$\rho_{jk} \equiv \frac{\Delta m_{jk}^2 \sigma_S}{2 m_\mu E_\nu \Gamma} , \tag{25}$$

where σ_S is the width of the muon (source) wave function in momentum space and m_μ and Γ are the mass and the decay width of the muon, respectively. The correction factors in eq. (24) are only sizeably different from one if $\rho_{jk} \gtrsim 1$. Thus a convenient formulation of this 'source wave packet – finite lifetime condition' (SFC) is $\rho_{jk} < 1$. Note that by defining a spread in velocity of the source wave packet by $\Delta v_S = \sigma_S / m_\mu$ the last formulation of SFC can be rewritten as [18]

$$\Delta v_S \tau_S \lesssim \frac{1}{4\pi} L^{\mathrm{osc}} . \tag{26}$$

It can be shown that SFC is well fulfilled for the LSND and KARMEN experiments, however, the margin for violation of SFC is only two orders of magnitude, in contrast to ACC (21) where the margin is at least ten orders of magnitude. This motivates us to have a look at the influence of ρ_{jk} (g_{jk}) on exclusion curves obtained, for instance, in the KARMEN experiment. In order to simplify matters we now confine ourselves to oscillations between two neutrino flavours where the transition probability becomes

$$P_{\bar{\nu}_\mu \to \bar{\nu}_e} = \frac{1}{2} \sin^2 2\theta \left(1 - g_{12} \cos \frac{\Delta m^2 L}{2E_\nu} \right) . \tag{27}$$

θ is the 2-flavour mixing angle. In comparison with the standard formula the factor g_{12} in (27) lowers for a given transition probability the upper bounds on Δm^2 and $\sin^2 2\theta$. This can be seen in figure 2 where we use $L = 17.7$ m, average in eq. (27) over the energy spectrum of $\bar{\nu}_\mu$ and as a Gedanken experiment vary the momentum spread σ_S. The corresponding exclusion curves are shown in the Δm^2 – $\sin^2 2\theta$ plane for the cases of $\sigma_S = 0$, 1 and 10 MeV corresponding to $g_{12} \simeq 1$, 0.9 and 0.3, respectively. Note that for $\sigma_S = 10$ MeV the exclusion curve has changed noticeably.

In general, the field theoretical treatment provides a solid method to locate all possible conditions and allows to separate unambiguously their different origins [18]. In contrast to discussions within the 'wave packet treatment', where conditions always depend on parameters of the neutrino wave packet which is difficult to ascertain experimentally, the final formulas derived within the field-theoretical treatment depend only on parameters supplied, at least in principle, by the experimental set-up and the neutrino mass squared difference. This is natural within the field-theoretical treatment which enables the study of the dependence on those quantities which are really observed or manipulated in oscillation experiments.

In order to summarize this paper we finally want to recapitulate the most important points of a field theoretical treatment of neutrino oscillations.

- In the field-theoretical approach the whole process of neutrino production and neutrino detection is represented by a single Feynman graph such that the neutrinos are associated with the inner line of this graph.
- The notion of a neutrino wave packet does not exist, only parameters associated with particles of the exterior legs of the Feynman graph, i.e., with those particles which are manipulated in the experiment, determine neutrino oscillations.
- Neutrino oscillations take place between neutrinos with the same energy but with different momenta. This arises only from the detection process when one assumes that the detector wave function is in a stationary state.
- Using the field-theoretical approach it is possible to calculate precisely corrections to the ultrarelativistic limit in the cross section.
- Furthermore, it allows to analyse all possible conditions for the existence of neutrino oscillations without referring to neutrino wave packets.

Acknowledgements

This review is largely based on work done in collaboration with W Grimus and S Mohanty.

References

[1] B Pontecorvo, *Sov. Phys. JETP* **26**, 984 (1968)
S M Bilenky and B Pontecorvo, *Phys. Rep.* **41**, 225 (1978)
S M Bilenky and S T Petcov, *Rev. Mod. Phys.* **59**, 671 (1987)
B Kayser, F Gibrat-Debu and F Perrier, *The Physics of Massive Neutrinos* (World Scientific, Singapore, 1989)
R N Mohapatra and P B Pal, *Massive Neutrinos in Physics and Astrophysics*, World Scientific Lecture Notes in Physics (World Scientific, Singapore, 1991) vol. 41
[2] C W Kim and A Pevsner, *Neutrinos in Physics and Astrophysics*, Contemporary Concepts in Physics (Harwood Academic Press, Chur, Switzerland, 1993) vol. 8
[3] J Rich, *Phys. Rev.* **D48**, 4318 (1993)
[4] C Giunti, C W Kim and U W Lee, *Phys. Rev.* **D45**, 2414 (1992)
[5] C Giunti, C W Kim, J A Lee and U W Lee, *Phys. Rev.* **D48**, 4310 (1993)
[6] S Nussinov, *Phys. Lett.* **B28**, 201 (1976)
[7] B Kayser, *Phys. Rev.* **D24**, 110 (1981)
[8] C Giunti, C W Kim and U W Lee, *Phys. Rev.* **D44**, 3635 (1991)
[9] K Kiers, S Nussinov and N Weiss, *Phys. Rev.* **D53**, 537 (1996)
[10] J Lowe, B Bassalleck, H Burkhardt, A Rusek, G J Stephenson Jr and T Goldman, *Phys. Lett.* **B384**, 288 (1996)
H Burkhardt, J Lowe, G J Stephenson Jr and T Goldman, hep-ph/9803365
[11] B Ancochea, A Bramon, R Muñoz-Tapia and M Nowakowski, *Phys. Lett.* **B389**, 149 (1996)
[12] Y Grossman and H Lipkin, *Phys. Rev.* **D55** (1997) 2760

[13] C Giunti, C W Kim and U W Lee, *Phys. Lett.* **B421**, 237 (1998)
[14] C Giunti and C W Kim, *Phys. Rev.* **D58**, 017301 (1998)
[15] W Grimus and P Stockinger, *Phys. Rev.* **D54**, 3414 (1996)
[16] The identification $E_\nu = E_D$ allows to determine, at least in principle, the neutrino energy with arbitrary accuracy by measuring the energies of the particles in the final state of the detector process with arbitrary accuracy
[17] W Grimus and P Stockinger, *Phys. Rev.* **D57**, 1762 (1998)
[18] W Grimus, S Mohanty and P Stockinger, *Phys. Rev.* **D59**, 013011 (1999); hep-ph/9904285
[19] LSND Coll: C Athanassopoulos *et al Phys. Rev. Lett.* **77**, 3082 (1996)
[20] KARMEN Coll: G Drexlin *et al*, *Prog. Part. Nucl. Phys.* **32**, 375 (1994)