PRAMANA
— journal of
physics

© Indian Academy of Sciences

Vol. 54, No. 1
January 2000
pp. 119–132

Neutrino spectrum from theory and experiments

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Abstract. The observed deficits in the solar and atmospheric neutrino fluxes along with the accelerator results on neutrino oscillations significantly constrain possible mass and mixing patterns among neutrinos. We discuss possible patterns emerging from the experimental results and review theoretical attempts to understand them.

Keywords. Neutrino masses and mixing; seesaw model.

PACS Nos 14.60 Pq; 11.30 Pb

1. Introduction

The precision results on the deficit in the muon neutrinos of atmospheric origin obtained at super-Kamioka [1] may be regarded as confirmation of the long standing suspicion that the neutrino has a mass [2]. The most likely interpretation of these results is that at least one neutrino is massive. Results from different solar neutrino experiments taken over many years are quite compelling for assuming that one more neutrino is massive. The mass scales probed in these two sets of experiments are quite distinct [3]. Important information on the neutrino spectrum is already provided by the presence of these two distinct scales, by the mixing patterns required to understand neutrino deficits and by the negative search results of the laboratory experiments, notably at CHOOZ [4]. This information can be judiciously combined with the well-motivated theoretical assumptions to arrive at constrained picture for neutrino masses and mixing. We wish to review recent developments in constraining neutrino masses and mixings and discuss most likely scenarios emerging as a result of these attempts.

We summarize the salient features of the well-known neutrino mass generation mechanisms [2] in the next section. Then we discuss the patterns of neutrino masses and mixing needed in order to understand the solar and atmospheric deficits in terms of oscillations between three neutrinos. In §4, we discuss how far the models of §2 can accommodate the neutrino spectrum suggested in §3. The next section discusses consequences of adding a fourth light neutrino in the spectrum. Summary is given in §6.

2. Neutrino mass generation: Theoretical summary

The gauge symmetry and the matter content of the standard model (SM) does not allow neutrino to have any mass. It is possible to extend the SM in a way that leads to neutrino masses. Some of these extensions such as SO(10) and supersymmetry (SUSY) are motivated for reasons different from neutrino masses while some have been proposed with a desire to incorporate neutrino mass in the standard picture. Different mechanisms for the neutrino mass generation are quite well-known [2] and we summarize the salient features of these below. All these mechanisms share a common goal of explaining the smallness of neutrino masses in comparison to masses of the charged fermions.

Seesaw model: In this mechanism [5], the light neutrinos obtain their masses through couplings to heavy right handed (RH) neutrinos. The neutrino mass matrix in these models is given by

$$m_\nu \sim -m_D M_R^{-1} m_D^T. \tag{1}$$

The Dirac mass m_D couples the light neutrinos to RH neutrinos and the mass matrix M_R for the latter suppresses the light neutrino masses compared to typical scales present in m_D. The quark lepton symmetry inherent in grand unified models such as SO(10) makes the identification of m_D with the (up) quark mass matrix natural. As a result, not only the overall scale but even the hierarchy among the neutrino masses can be a prediction in such theories. The simplest version of SO(10) identifies m_D with the up quark masses and this supplemented with the assumption of a diagonal M_R implies the following mass relation among the three neutrino masses m_i:

$$m_1 : m_2 : m_3 :: m_u^2 : m_c^2 : m_t^2. \tag{2}$$

The overall scale of these masses is set by the (common) RH neutrino mass M which break the $B - L$ symmetry. If M is near the grand unification (GU) scale, then m_3 is around the scale probed in atmospheric neutrino experiments. Alternatively, if the $B - L$ symmetry is broken at an intermediate scale, $M \sim 10^{12}$ GeV then $m_3 \sim$ O(eV) can provide the cosmic dark matter scale [6].

The generic seesaw model based on the left right symmetry also leads to direct majorana masses m_{LL} for the left handed neutrinos themselves [2]. As a result, the following equation describes neutrino masses in these theories instead of eq. (1):

$$m_\nu \sim m_{LL} - m_D M_R^{-1} m_D^T\ . \tag{3}$$

The overall scale of m_{LL} is inversely related [2,7] to the scale of the $B - L$ breaking as in the second term. But the structure of m_{LL} is not expected to be related to other fermion masses.

The simplest version of SO(10) would also imply small mixing angles in the leptonic sector as in the Cabibbo Kobayashi Maskawa (CKM) matrix. It is however possible to obtain very different leptonic mixing matrix without sacrificing the quark lepton symmetry. This can be done if M_R has non-trivial texture [8]. Large mixing can also co-exist with SO(10) symmetry if neutrino masses are given by eq. (3) instead of eq. (1) [9]. Alternatively, it is possible to obtain large (small) mixing among the left handed charged leptons (down quark) in the context of the SU(5) models [3,10]. One can also imagine extensions of the grand unified picture to obtain large mixing [11,12]. A comprehensive summary of different possibilities is given in [3].

Radiatively generated neutrino masses: It is possible to imagine extensions of the SM in which lepton number violation occurs but one or more neutrinos remain massless at tree level. If these masses are not protected by any other symmetry then they would be generated radiatively. They are calculable and small in this case. This scenario can be implemented by extending the fermionic sector [13] or by extending the Higgs sector of the SM [13–15]. This mechanism has the advantage of explaining smallness of neutrino mass without invoking heavy scales as in the seesaw model. Many of the radiative schemes [13] are based on the Zee model [14] which invokes a charged singlet Higgs and an extra doublet Higgs. The charged lepton masses induce the neutrino masses radiatively in this model and the neutrino mass is typically given (assuming that the scale associated with the new physics is also the weak scale) by

$$m_\nu \sim \frac{f m_l^2}{16\pi^2 M_W}. \tag{4}$$

Thus $m_l \sim m_\tau$ and $f \sim 10^{-5}$ can account for the masses as small as required to explain the atmospheric anomalies. Different versions of this mechanism have been proposed in recent years [16–18] to account for neutrino anomalies and we will summarize them in the next section.

Supersymmetry and neutrino masses: Supersymmetric extension of the SM automatically contains lepton number violation and hence the source of neutrino masses. This comes due to the presence of additional scalars and fermions in this model. The lepton number violation comes from the following terms in the superpotential written in the standard notation [20]:

$$W = \epsilon_i L_i H_2 + \lambda'_{ijk} L_i Q_j D_k^C + \lambda_{ijk} L_i L_j E_k^c. \tag{5}$$

The term linear in L_i induces vacuum expectation value (vev) for the scalar partner (sneutrino) of neutrino. This mixes neutrinos with neutralinos. If the sneutrino vev is suppressed then neutrinos acquire seesaw type mass [21]. One combination of neutrinos obtains its mass this way. The others can obtain their masses radiatively [22] through 1-loop diagrams involving scalar partners of the d-type quarks and charged leptons. This mechanism thus combines both the previous mechanisms.

The supersymmetry provides particularly interesting and economical framework for neutrino masses in the context of the minimal supergravity theory [23,24] or in the context of theory involving SUSY breaking by gauge interactions [25]. If ϵ_i are the only source of lepton number breaking then neutrino masses get directly linked to the charged lepton and the down quark masses in such theories. In fact the parameters ϵ_i become unphysical in the limit of taking the charged lepton and down quark masses to zero [23]. As a consequence, all the effects associated with lepton number violation are suppressed by these masses. This has three advantages: (i) The neutrino mass gets suppressed compared to typical weak scales in the theory, e.g. for $\epsilon_i \sim \mu \sim 100\,\text{GeV}$ one gets neutrino mass in the keV–MeV range, (ii) neutrinos obtain hierarchical masses and (iii) neutrino mixing is determined by the ratios of ϵ_i for a large range in the SUSY parameter space [23,25]. As a result, large neutrino mixing becomes natural in these theories in spite of the hierarchical neutrino masses. Detailed patterns of neutrino masses based on eq. (5) are worked out in a number of papers [23–26] and have been summarized in this issue [27]. We shall therefore not discuss this scenario any further.

Higher dimensional operators and flavour symmetry: This approach uses the basic mechanism due to Froggatt and Nielson [28]. In this approach, the light fermion masses and their mixing arise from effective non-renormalizable operators which would arise after spontaneous breaking of a U(1) flavor symmetry at a very high scale $\sim M$ by the vev of a singlet field θ. The occurrence of small parameters in the theory is explained in this approach in terms of a small parameter $\langle\theta\rangle/M$. The detailed textures of fermion masses are determined by the U(1) charges of the fermionic generations. Variety of schemes using this mechanism have been used to understand the quark spectrum. The family quantum numbers consistent with the required quark spectrum can also be used to constrain neutrino masses. It is possible to utilize this approach to understand neutrino mass hierarchy and in particular the large mixing angles. We refer to [3,29] for more details and original references.

3. Experimental implications within the minimal framework

We shall confine ourselves in this section to the minimal framework involving three active neutrinos $\nu_\alpha \;\; (\alpha = e, \mu, \tau)$. This framework is sufficient to account for the solar as well as atmospheric anomalies. It can also allow neutrinos to provide significant component in the dark matter provided they are nearly degenerate [30]. The only direct experimental indication in favour of the non-minimal framework with more than three neutrinos comes from the LSND experiment [3]. We shall discuss this case in the next section.

Atmospheric neutrino anomaly: The oscillations of the muon neutrinos appear to be most likely explanation of the observed deficits at the super-Kamioka. The data moreover also imply that ν_μ oscillates to ν_τ in the context of the minimal framework [31–33] with the following parameters

$$\begin{aligned} 10^{-3}\ \mathrm{eV}^2 \leq \Delta_A \leq 8 \cdot 10^{-3}\,\mathrm{eV}^2 \\ 0.86 \leq \sin^2 2\theta_A \leq 1.0\ , \end{aligned} \tag{6}$$

where Δ_A, θ_A refer to (mass)^2 differences and mixing among ν_μ, ν_τ. The above constraints allow two possibilities

$$\text{(A)} \qquad m_2 \leq m_3 \sim \sqrt{\Delta_A}\ . \tag{7}$$

$$\text{(B)} \qquad m_2 \sim m_3 \gg \sqrt{\Delta_A}\ ;\ \Delta_{32} \equiv m_3^2 - m_2^2 \sim \sqrt{\Delta_A}\ . \tag{8}$$

The canonical possibility (A) has the advantage that the second mass could be chosen in the range required for solution to the solar neutrino problem. It has the disadvantage that one needs to invoke additional neutrino in order to get significant dark matter [6] in the universe. In comparison, the possibility (B) provides the dark matter in the universe easily but can lead to the observed solar deficit only if all three neutrinos are almost degenerate. In either case, one could explain all neutrino anomalies if additional sterile state is invoked.

The simultaneous requirement of the large mixing and hierarchical masses forces the following generic form for the 2×2 neutrino mass matrix in the basis with the diagonal charged lepton:

$$\mathcal{M}_\nu \sim \frac{\sqrt{\Delta_A}}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} . \tag{9}$$

SUSY theory with bilinear R violation provides a simple realization of this scheme [23–25]. Seesaw model with only one RH neutrino coupling to light neutrinos gives another example of this matrix [34]. The form (9) can also be obtained in a radiative model [16] using the Zee type mechanism.

The form in eq. (9) has been obtained assuming that the third neutrino does not couple significantly to $\nu_{\mu,\tau}$. The large coupling with the third generation can allow one to realize the possibility (A) and large mixing without [35] the 2×2 $\nu_\mu - \nu_\tau$ block displaying the structure as in eq. (9).

Possibility (B) seems more constrained and in general would require additional symmetries to understand near degeneracy of neutrino masses. However such symmetry in the effective mass matrix for two light neutrinos may follow accidentally even when the underlying theory does not possess specific symmetry. An example is provided by the conventional seesaw model. Assume that the 2×2 matrix M_R is of the Fritzch form, i.e.

$$M_R = \begin{pmatrix} 0 & M \\ M & M' \end{pmatrix} .$$

Then $m_D = m_U$ leads to

$$\mathcal{M}_\nu \sim \frac{1}{M^2} \begin{pmatrix} m_c^2 M' & -m_c m_t M \\ -m_c m_t M & 0 \end{pmatrix} . \tag{10}$$

The M_R above is not invariant under any combinations of L_μ and L_τ. Still the effective mass matrix $\mathcal{M}_\nu$ for light neutrinos obtained from this M_R displays an approximate $L_\mu - L_\tau$ symmetry when $M \sim M'$. In fact, the choice $M \sim M' \sim 10^{11}$ GeV simultaneously leads to (1) almost degenerate masses in the eV range (2) splitting between these states at the atmospheric neutrino scale Δ_A and (3) almost maximal mixing between the relevant states. This simple example based on realistic assumptions thus illustrates that it is quite easy to obtain large mixing and a solution to the atmospheric neutrino anomaly in the conventional seesaw model.

Combined solutions for the solar and atmospheric neutrino anomalies: Unlike in the case of atmospheric neutrinos, demanding an oscillation solution to the solar neutrino problem does not completely fix the ranges of the allowed neutrino parameters as the solar neutrino problem can be solved in more than one ways through neutrino oscillations. Three different possibilities exist which explain the presently available results on the solar neutrinos but differ in their predictions for the experimentally measurable parameters like distortion of the neutrino energy spectrum, day–night asymmetry in the neutrino flux and their seasonal variations etc [36]. The parameter ranges required according to the analysis in [37] are:

$$5\cdot 10^{-11}\ \mathrm{eV}^2 \le \Delta_S \le 5\cdot 10^{-10}\,\mathrm{eV}^2$$
$$0.65 \le \sin^2 2\theta_S \le 1.0 \qquad (\mathrm{VO})\ , \tag{11}$$

$$4\cdot 10^{-6}\ \mathrm{eV}^2 \le \Delta_S \le 9\cdot 10^{-6}\,\mathrm{eV}^2$$
$$10^{-3} \le \sin^2 2\theta_S \le 10^{-2} \qquad (\mathrm{SAMSW})\ , \tag{12}$$

$$2\cdot 10^{-5}\ \mathrm{eV}^2 \le \Delta_S \le 2\cdot 10^{-4}\,\mathrm{eV}^2$$
$$0.65 \le \sin^2 2\theta_S \le 0.95 \qquad (\mathrm{LAMSW})\ . \tag{13}$$

The Δ_S, θ_S above refer to the (mass)^2 difference and mixing among two neutrinos participating in the solar neutrino oscillations. The three different possibilities correspond to the vacuum [38] oscillations (VO), small angle MSW (SAMSW) and the large angle MSW (LAMSW) solutions [39] to the solar neutrino problem [37].

Solution of the solar neutrino problem can be incorporated in schemes A and B discussed in the earlier section. The third neutrino can be added to the scheme A with two different mass hierarchies:

$$(\mathrm{A1}) \qquad m_1 \sim m_2 \ll m_3\ ;$$
$$(\mathrm{A2}) \qquad m_1 \ll m_2 \ll m_3\ , \tag{14}$$

while possibility B can be extended to include the solar neutrino problem only if the third mass state is also almost degenerate [30] with the first two, i.e.

$$(\mathrm{B}) \qquad m_1 \sim m_2 \sim m_3 \gg \sqrt{\Delta_A}\ ;\ \ \Delta_{32} \sim \Delta_A\ \ ;\Delta_{21} \sim \Delta_S\ . \tag{15}$$

In addition to these three possibilities, a fourth possibility exists [35] which is a genuine three generation possibility. This corresponds to two of the neutrinos having mass at the atmospheric scale and (mass)^2 difference at the solar scale, i.e.

$$(\mathrm{C}) \qquad m_1 \sim m_2 \sim \sqrt{\Delta_A} \gg m_3\ \ \text{with}\ \ \Delta_{12} \sim \Delta_S\ . \tag{16}$$

This scheme seems to require somewhat unconventional mass hierarchy but it is nevertheless interesting as it can be argued [35,40] to follow from a symmetry namely $L_e - L_\mu - L_\tau$. Imposition of this symmetry implies the following texture for the neutrino mass matrix:

$$\sqrt{\Delta_A}\begin{pmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & 0 & 0 \\ \sin\theta & 0 & 0 \end{pmatrix}\ . \tag{17}$$

This structure gives rise to the mass pattern displayed in eq. (16) to zeroth order. In addition, it can provide large angle solution to the atmospheric neutrino anomaly if $\theta \sim 45^\circ$. Very small breaking [17,18,40] of the $L_e - L_\mu - L_\tau$ symmetry can lead to a solution of the solar neutrino problem in this scheme.

Mixing patterns: The allowed mixing among three generations is very significantly constrained if both the neutrino anomalies are to be solved [3]. The mixing matrix relating the flavour $\alpha = e, \mu, \tau$ and mass $i = 1, 2, 3$ eigenstates can be written as follows:

$$\nu_\alpha = U_{\alpha i}\nu_i \; . \tag{18}$$

After appropriate redefinition of the charged lepton states, the matrix U contains three mixing angles and three physical phases and can be parameterized as [41]

$$U = W_{23}(\theta_{23})\; W_{13}(\theta_{13})\; W_{12}(\theta_{12}\mathrm{e}^{i\delta})P(\lambda_i)\;, \tag{19}$$

where $P(\lambda_i)$ =diag.$(\mathrm{e}^{i\lambda_1}, \mathrm{e}^{i\lambda_2}, 1)$; W_{ij} denote complex rotation in the ij plane. For example,

$$W_{12}(\theta_{12}\mathrm{e}^{i\delta}) \equiv \begin{pmatrix} c_{12} & s_{12}\mathrm{e}^{-i\delta} & 0 \\ -s_{12}\mathrm{e}^{i\delta} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{20}$$

The δ in the above equation is analogous to the Kobayashi Maskawa phase while $\lambda_{1,2}$ arise due to Majorana nature of neutrinos.

The non-observation of the $\bar{\nu_e}$ oscillations at CHOOZ places an important constraint. This result requires [4]

$$|U_{e3}| \leq 0.1 \quad \text{for} \quad \Delta_{31} \geq 10^{-3}\,\mathrm{eV}^2\;.$$

The following two possible mixing patterns emerge when the constraint from CHOOZ is imposed on U. These can be written as follows to leading orders in relevant parameters:

$$U_S \approx \begin{pmatrix} 1 & \lambda^2\mathrm{e}^{i\delta} & A\lambda \\ -A\lambda s_{23} + O(\lambda^2) & c_{23} & s_{23} + O(\lambda^2) \\ -A\lambda c_{23} + O(\lambda^2) & -s_{23} & c_{23} + O(\lambda^2) \end{pmatrix} \begin{pmatrix} \mathrm{e}^{i\lambda_1} & 0 & 0 \\ 0 & \mathrm{e}^{i\lambda_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{21}$$

$$U_L \approx \begin{pmatrix} c_{12} & s_{12}\mathrm{e}^{i\delta} & A\lambda \\ -s_{12}c_{23}\mathrm{e}^{-i\delta} & c_{12}c_{23} & s_{23} \\ s_{12}s_{23}\mathrm{e}^{-i\delta} & -c_{12}s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} \mathrm{e}^{i\lambda_1} & 0 & 0 \\ 0 & \mathrm{e}^{i\lambda_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{22}$$

In the above equation, $\lambda \sim 0.2$ and $A \sim O(1)$. The s_{12} in eq. (22) is large and accounts for either the LAMSW or VO solution. The form (21) applies only to the SAMSW solution. Interestingly enough, this form can also be characterized by the Wolfenstein parameter [42] λ due to fortuitous result that $|U_{e3}| \leq \lambda$ and small angle needed for the SAMSW solution is $\sim \lambda^2$. But the detailed form of U in terms of this parameter is characteristically different from the analogous CKM matrix. Equation (22) reduces to the bimaximal mixing [44] if $s_{12} = \frac{1}{\sqrt{2}}$ and $A = 0$. Note that demanding a simultaneous solution to the solar and the atmospheric neutrino anomalies fixes the allowed ranges of the mixing angle θ_{12}, θ_{23}. The phases δ, $\lambda_1\lambda_2$ remain undetermined. The phases $\lambda_{1,2}$ are not probed in neutrino oscillation experiments but they govern lepton number violating phenomena, e.g. lepton asymmetry [45] which may be produced by the decay of a RH neutrino in the early universe.

The mass hierarchies displayed in eq. (14) are consistent with either of the mixing patterns in eqs (21), (22). In contrast, the possibility B is considerably constrained if the (common) almost degenerate mass m_0 is of O(eV). This comes from the observed limit, $\leq 0.2\,\mathrm{eV}$, on the effective mass m_{eff} seen in the $0\nu\beta\beta$ decay. In particular,

- the SAMSW solution corresponding to the matrix (21) is inconsistent with this limit.
- If A in eq. (22) is zero then common mass $m_0 \sim 2\,\mathrm{eV}$ can be consistent with the limit on m_{eff} [43] only for $\lambda_1 - \lambda_2 - \delta \sim \pi$ and

  $$\sin^2 2\theta_{12} \geq 0.99.$$

  This makes LAMSW solution also difficult and only the VO solution may be permitted in this case.
- For $A \sim 1$, the scheme B allows both the LAMSW and VO solution.

The possibility C, eq. (16), goes more naturally with LAMSW or the VO solution for the solar neutrino problem and hence with the mixing matrix in eq. (22). In particular, explicit realization of this possibility displayed in eq. (17) implies $A = 0$ and $\theta_{12} = \theta$.

4. Theoretical realizations

We now discuss possible realizations of the above mentioned mass and mixing patterns among the three neutrinos. Many different possibilities have been put forward in the context of the conventional mechanisms for neutrino mass generation discussed in the previous section. We shall be specific and restrict ourselves to those possibilities which also accommodates well-motivated theoretical assumptions or expectations like supersymmetry or quark lepton symmetry etc.

Grand unified seesaw model: The hierarchical neutrino masses with hierarchies determined by the (up) quark mass is one of the main features of this class of models. The simplest version of this scheme will also relate the leptonic mixing matrix U to the standard CKM matrix. However there exist number of ways exemplified by large class of models in which one could retain the basic quark lepton symmetry and still get the large leptonic mixing. These have been summarized in [3] and we mention some of these in the following.

The neutrino mass pattern obtained in the simplest SO(10) type of models simultaneously accounts for the vacuum solutions to neutrino anomalies since

$$\frac{\Delta_S}{\Delta_A} \sim (\frac{m_c}{m_t})^4 \sim 10^{-8}\ . \tag{23}$$

The correct value for Δ_A can be reproduced if the RH neutrino mass scale is taken around the grand unification scale. The seesaw model with intermediate masses for the RH neutrinos can easily account for the dark matter and the atmospheric neutrino scale as exemplified in eq. (10). Seesaw model can also accommodate scenarios with almost degenerate masses if neutrino masses are described by eq. (3). Natural expectation in this case is [30]

$$\frac{\Delta_S}{\Delta_A} \sim \left(\frac{m_c}{m_t}\right)^2 \sim 10^{-4} \tag{24}$$

which simultaneously incorporates the MSW mechanism and a solution to the atmospheric neutrino anomaly.

It should be emphasized that the seesaw model which reproduces the mass relation (23) cannot in its simplest version account for the large mixing angle. This requires either departures from the relation $m_D = m_U$ or texture in the RH masses. Doing this would change the predicted mass relation (23) itself. It is possible to obtain a mixing pattern different than in quark sector by adding additional singlet neutrino [11] at a high scale. A concrete example which retains prediction in eq. (23) but also has large mixing is presented in [12].

Seesaw neutrino masses can be generated in SU(5) GU model also if additional heavy SU(5) RH neutrinos are added. Unlike in case of SO(10), the existence of the RH neutrinos does not follow from group theoretical arguments but the resulting scheme is less constrained and can accommodate large mixing naturally [3,10].

SU(5) theory can lead to seesaw type mass relation even in the absence of the RH neutrinos provided a (heavy) 15-plet Higgs, Δ is introduced. A coupling $5_{\rm H}5_{\rm H}15_{\rm H}$ in the scalar potential induces [46] a very small vev of $\sim O(m_W^2/M_{\rm GU})$ for the triplet Higgs. As a consequence, the neutrino masses are 'seesaw' suppressed in these models even in the absence of the RH neutrinos. It is possible to build a realistic model [17] implementing this scenario and accounting for parameters needed to explain neutrino anomalies.

Radiative mechanisms: Many different versions of the radiative schemes have been proposed in the context of the minimal framework. It has been argued [47] that Zee model can lead to the simultaneous solution of the solar and the atmospheric neutrinos only if additional $L_e - L_\mu - L_\tau$ symmetry is imposed. In the presence of this symmetry, Zee model can account for the bimaximal pattern as well as the atmospheric mass scales with proper (and not very unnatural) choice of its parameters. It cannot account for the solar neutrino deficit. It is possible to add a doubly charged Higgs [14,15] to the model in a way which accounts for mass scales and mixing angles needed to understand solar and atmospheric anomalies [18].

Radiative models have also been used to understand smallness of mass splittings among neutrinos rather than the masses. Basic idea is to obtain small and degenerate masses at tree level through some softly broken symmetry [17,19]. This can generate mass splitting radiatively. Advantage here is that the mass splitting is calculable if the symmetry in question is broken softly. Realistic models which offer a simultaneous solution to the solar and atmospheric neutrino anomalies have been presented recently [17,19].

5. Going beyond minimal structure: Sterile neutrino

There are three possible motivations to add a light sterile neutrino to the minimal scheme discussed above.

- Just three neutrinos are not enough to provide three different (mass)^2 differences Δ_A, Δ_S and the $\Delta_{\rm LSND}$ corresponding to the scale of the $\nu_\mu - \nu_e$ oscillations reported by the LSND experiment.

- Sterile state may be needed irrespective of the LSND result if neutrinos are to provide the hot dark matter and if the degenerate spectrum is not supported by future neutrinoless double beta decay experiments.

- Theoretically, a sterile neutrino provides an attractive means to understand large mixing without sacrificing the idea of quark lepton symmetry [48].

Masses and mixing in four neutrino framework: Introduction of a sterile neutrino opens up many different possibilities for mixing between the sterile and the active states. The neutrino mass hierarchy is quite restrained [49] if one wants to incorporate the LSND results in addition to solving the solar and atmospheric neutrino problems. Two simple possibilities which find natural realization in the grand unified framework are the following:

(a) Solar neutrino deficit is due to $\nu_e - \nu_s$ oscillations [50]. The $\nu_\mu - \nu_\tau$ mixing can offer a solution to the atmospheric neutrino through scheme (A) in eq. (7) if their masses are hierarchical or through scheme (B), eq. (8) with almost degenerate masses. Small mixture of ν_e with $\nu_\mu - \nu_\tau$ system can account for the LSND results in the latter case [50].
(b) ν_μ mixes strongly with ν_s to account for the atmospheric neutrino anomaly. The solar neutrino problem can be solved by mixing of the ν_e with ν_μ, ν_s [51]. ν_τ in this case play the role of providing hot dark matter. Solution to the LSND result cannot be achieved in this case. Such a scenario can be realized in seesaw model with intermediate masses [51]. Alternatively, the solar anomaly may be resolved due to $\nu_e - \nu_\tau$ oscillations. This scenario also includes an explanation of the LSND result and can be naturally incorporated in the GU seesaw model with all the RH neutrino masses at the GU scale [48].

The gauge symmetries of the SM cannot protect the mass of a sterile state. Hence understanding of small mass of the sterile state poses a more acute problem than understanding the origin of the small neutrino mass compared to other fermions. One could impose some chiral symmetry associated with sterile state to forbid its mass. But the justification of the very existence of this symmetry and sterile state itself needs to be provided. Three different mechanisms have been proposed.

- *Incomplete seesaw:* This mechanism follows within the conventional seesaw model. If the RH matrix M_R in eq. (1) is singular then the seesaw mechanism is incomplete and one of the RH neutrinos remains light. Typical mass for this is however of O(m_D). Thus this mechanism cannot provide the sterile state needed to understand neutrino anomaly unless some fine tuning is done [52]. It can however provide the warm dark matter candidate.

- *Mirror world:* If one postulates the existence of a mirror world [53] which is replication of the standard model with its own mirror gauge interactions, then the mirror neutrinos behave as sterile neutrinos with respect to the normal gauge interactions. Mixing between two worlds can occur through gravitational interactions. Viable scenarios involving this alternative are discussed in [53]. They require that the SU(2) symmetry in the mirror sector should be broken at somewhat higher scale than in the visible sector.

- *Quasi goldstone fermion:* Supersymmetry may provide a viable explanation for the lightness and the existence of a sterile state. If MSSM is extended by imposing a U(1) Peccei Quinn symmetry to solve the strong CP problem then the supersymmetric partner of the axion can be an ideal candidate to describe the sterile neutrino. The breaking of R parity can generate mixing between the active and sterile states in this picture. The mass of the sterile state can be protected even after breaking of supersymmetry in a number of situations based on supergravity [54] or on gauge interaction induced SUSY breaking [55]. Such a scenario can also be realized in string based models [56]. It is possible [54] to realize either the scheme (a) or (b) discussed above using this approach.

Lastly, several radiative models [57] incorporating a sterile neutrino have also been proposed. These models invoke a sterile state and additional symmetry which keeps it massless. The mass for the sterile state and its mixing with active neutrinos is generated radiatively using singly and doubly charged Higgs scalars.

6. Summary

The presently available information on the solar and the atmospheric neutrinos is fully consistent with the standard picture with only three light neutrinos mixing with each other. The masses and mixing required to understand these data follow different pattern than in the quark sector. But it is possible to understand these within conventional ideas of neutrino mass generation like quark lepton symmetry, supersymmetry etc. It is quite difficult to decide at this stage which of the popular scenarios is more appropriate in describing the data.

At the phenomenological level, four different mass patterns eqs (14)–(16) and two mixing patterns eqs (21), (22) describe the data. One will be able to fix the mixing pattern once the correct solution for the solar neutrino problem is identified. The diagnostic tools needed for this purpose are the study of the recoil energy spectrum, the day–night and seasonal variation of the flux and the zenith angle dependence of the events averaged over the year. The present information is inconclusive and different data set favour different possibilities [36].

The other key question is to determine the presence of a sterile neutrino into spectrum. This crucially depends on the verification of the LSND result in experiment like the Mini-Boon [58]. The presence of large $\nu_\mu - \nu_s$ mixing can be detected at the super-Kamioka. Such mixing will reduce [36] the number of pions produced in the reaction $\nu N \to \nu N \pi^0$ compared to the $\nu_\mu - \nu_\tau$ oscillations. The presently available information is inconclusive due to large systematic errors. The zenith angle distribution of the high energy events is different in case of the $\nu_\mu - \nu_\tau$ and $\nu_\mu - \nu_s$ mixing and the present data seem to favour the former at 2σ level [36].

In summary, while different possibilities still exist, information on the neutrino masses and mixing has become more focused now than few years ago. More data in present experiments and the new experiments for the solar neutrinos and the long baseline experiments will be a positive step in the direction of sharpening the knowledge of the neutrino spectrum further.

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