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Transmission of neutrinos through matter

L WOLFENSTEIN
Carnegie Mellon University, Pittsburgh, PA 15213, USA

Abstract. Neutrinos travel through matter with negligible absorption except in very extreme situations. However, the index of refraction of neutrinos can play an important role in the oscillation of one type of neutrino to another when passing through matter.

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After Fermi proposed his theory of nuclear beta decay, Bethe and Peierls, in 1934, calculated the cross-sections for the neutrino processes

$$\begin{aligned}&\nu_e + n \rightarrow p + e^-,\\ &\overline{\nu}_e + p \rightarrow n + e^+.\end{aligned} \tag{1}$$

For beta-decay neutrinos, the cross-sections were of the order $10^{-43}\ \mathrm{cm}^2$. If a beam of 10^{10} neutrinos was aimed at the earth, all but one would emerge on the other side. As far as neutrinos were concerned, it seemed the matter did not matter.

It was proposed in the 1930's that the origin of solar energy was in nuclear reactions occurring near the center of the sun. Most of this energy is slowly transported to the surface by radiative transfer taking the order of 10^4 years. However, detailed calculations in the 1960's showed that 3% of the energy comes out in the form of neutrinos. These emerge directly without significant absorption and arrive on earth after seven minutes. In a pioneering experiment [1], starting in 1968 and continuing to the present day, Raymond Davis detected solar neutrinos via their capture on ^{37}Cl leading to ^{37}A. Thus, it was possible with neutrinos to look into the very center of the sun. Subsequent experiments using water-Cerenkov detection (Kamiokande and super-Kamiokande) and capture on gallium (GALLEX and SAGE) have also detected these neutrinos. The detected rate, however, has been of order 50% or less of the calculated rate [2]. We will return to this solar neutrino problem later.

There do exist extreme situations in which neutrino absorption is important. A type II supernova results from the collapse of a large star (of order or greater than 10 solar masses) when its nuclear fuel is exhausted. The central region of the star, with a mass of about 1.4 solar mass, contracts suddenly to a density of order $10^{13}\ \mathrm{g/cm^3}$ and temperatures greater than 20 MeV. Neutrinos of all varieties are produced by reactions at these temperatures. At this high density, the mean free path is only about 1 km, whereas the size of the core is of order 30 km so that the neutrinos are scattered and captured and re-emitted as they escape. In this case, one must calculate neutrino opacity [3] just as one calculates optical opacity for radiative transport in ordinary stars like the sun. Nevertheless, it is much easier for neutrinos to escape from the hot core of the collapsed star than photons, so that 99% of the energy of the collapse is emitted in the form of neutrinos. The detection of 19 out of about 10^{58} neutrinos from supernova 1987a was one of the great events of modern science.

The possibility of seeing very high energy neutrinos that might be emitted by sources like active galactic nuclei (AGN) is the goal of proposed experiments like NESTOR in the Mediterranean Sea and ICE CUBE at the South Pole. Energies up to 10^9 GeV have been predicted in some models. The neutrino cross-section rises with energy, roughly linearly above 1 GeV. For energies greater than 10^5 GeV, neutrinos are absorbed in going through the earth and by energies of 10^7 GeV only one in a million would come through [4]. There is one interesting exception for the case of tau-neutrinos ν_τ. When these are absorbed, they yield τ leptons, which then decay back into ν_τ; because the τ lifetime is only 3×10^{-13} s, these decays will occur after a short distance in spite of the large time dilation factor. Thus, by a sequence of absorptions and decays, an incident ν_τ of 10^7 GeV could emerge on the other side of the earth [5], although degraded in energy.

In 1973, neutral current interactions were discovered, as predicted by the spontaneously-broken gauge theory of Weinberg and Salam. For neutrinos scattering from nuclei, the important reaction is:

$$\nu_x + n \to \nu_x + n, \tag{2}$$

where the cross-section is the same for $\nu_x = \nu_e, \nu_\mu$ or ν_τ. There is also scattering from electrons; while the neutral-current amplitude is the same for all ν_x, in the case of ν_e the scattering

$$\nu_e + e \to \nu_e + e \tag{3}$$

can take place as well by the charged current so the total amplitude is the sum of the two. As a result, the cross-section for elastic $\nu_e - e$ scattering is 6–7 times larger than for ν_μ or ν_τ.

After the experimental discovery of neutral currents, it was of great interest to see if these indeed agreed with the theoretical predictions. One prediction that intrigued me was that the neutral current was diagonal in flavor: the neutrino emerging had the same flavor as the neutrino coming in. In the detection of reaction (2), all one saw was the recoil neutron; observing the outgoing neutrino would require a detector much larger than the earth.

However, I realized in 1978 that there was the possibility of detecting the coherent interference between the forward scattered neutrino and the incident beam [6]. This interference determines the index of refraction n in accordance with the optical theorem

$$p(n-1) = 2\pi N f(0)/p, \tag{4}$$

where p is the neutrino momentum and N is the number of scatterers per unit volume. In the case of the off-diagonal neutral current, the index of refraction effect would 'rotate' the flavor just as an optically active medium rotates the plane of polarization.

The imaginary part of the index of refraction is proportional to the total cross-section

$$\sigma = 4\pi \text{ Im } f(0)/p \tag{5}$$

which we have emphasized is negligible. However, the real part is much larger since it is proportional to the weak coupling G_F, whereas σ is proportional to G_F^2. Thus for the neutral current we have

$$p(n-1) \approx G_\mathrm{F} N_n = 2\pi/L\,, \tag{6}$$

where N_n is the neutron number density and L is the distance over which the phase factor changes by 2π. For earth matter, L is of the order of the radius of the earth. (Note: Fermi's constant times Avogadro's number defines a length about the size of the earth; the fundamental reason for this has not yet been determined.) Thus, if the neutral current were to change ν_μ to ν_τ, massless neutrinos traveling through the earth would oscillate from ν_μ to ν_τ. Recently, it was discovered with the super-Kamiokande detector that ν_μ with energies of a few GeV produced by cosmic rays were reduced in flux by a factor of 2 when observed coming upward through the earth. The only explanation was that on average half of the ν_μ oscillated into ν_τ on their way through the earth. This could indeed be explained by off-diagonal currents with massless neutrinos and such an explanation has been given by Gonzalez–Garcia *et al* [7]. Since the standard model has been shown to work extremely well, this requires neutral currents outside the standard model.

The simpler explanation of the results is that neutrinos have mass and that two eigenstates are approximately equal mixtures of ν_μ and ν_τ. The standard model can easily accommodate neutrino masses and grand unified theories like SO(10) suggest small non-zero neutrino masses. In the case of ν_μ–ν_τ oscillations due to neutrino mass and assuming the neutral current is diagonal in flavor the matter does not matter and the results are just the same as if the earth were a vacuum. For these vacuum oscillations, the oscillation length increases with energy unlike eq. (6) where it is energy independent. The super-K data shows such an energy dependence, but the authors of ref. [7] claim that the results are not yet accurate enough to distinguish the two possibilities [8].

Oscillations due to neutrino mass and mixing are affected by the material medium when they involve ν_e. This was discussed in the second part of the 1978 paper [6]. Neutrino oscillations occur in vacuum because the relative phase of two components (mass eigenstates) ν_1 and ν_2 changes with time. Since this is a matter of phases, the phase associated with the index of refraction could be important. For the neutral current of the standard model, the amplitude $f(0)$ is the same for all neutrinos so it does not matter. However, as noted, the ν_e has an elastic amplitude due to the charged current (W-exchange) that the other neutrinos do not have. Thus, the ν_e part of the wave function has a phase change due to

$$p(n_e-1) = -\sqrt{2} G_\mathrm{F} N_e = 2\pi/l_0\,, \tag{7}$$

where N_e is the electron number density. Here, l_0 again is a characteristic length of order the earth radius for earth matter. Thus, when one considers vacuum oscillation lengths comparable to the earth radius, the oscillation length and mixing can be very different in vacuum than in matter.

An interesting case would be the search for $\nu_\mu \to \nu_e$ and $\overline{\nu}_\mu \to \overline{\nu}_e$ oscillations that could be studied via long baseline experiments such as the proposed Minos experiment. The index of refraction effect eq. (7) has the opposite sign for $\overline{\nu}_e$ so that the matter effect may enhance one of the oscillations and suppress the other. A comparison of $\nu_\mu \to \nu_e$ and $\overline{\nu}_\mu \to \overline{\nu}_e$ would be of interest in searching for CP violation in the neutrino mixing in analogy with the CKM mixing of quarks. However, one needs to disentangle the matter effect from any possible CP violation [9].

The most interesting application of the matter effect was discovered by Mikhaeyev and Smirnov [10] and is referred to as the MSW effect. If the vacuum mixing angle is small and the lower mass state is mainly ν_e, then as a beam of ν_e passes from high density to low density, as ν_e emerging from the sun, the mixing angle will pass through $45°$ and a large transformation of ν_e will occur. Indeed, if the passage is adiabatic there could be almost a complete transformation. The MSW effect is a popular explanation of the solar neutrino problem, but more experiments are needed to tell if it is the answer. Shortly after reading the MS paper, I wrote a short unpublished paper [11] pointing out that the ν_e flux might be larger at night than during the day for some energies. If most of the neutrinos were to arrive at earth as ν_μ or ν_τ, then due to the effect of earth matter, the mixing angle would be larger in earth than in vacuum and some of ν_μ or ν_τ would convert back to ν_e. This day–night effect has been sought in the super-Kamiokande observations of solar neutrinos, but so far there are only limits which exclude certain values of Δm^2. It is quite possible that the effect is extremely small [12] for values of Δm^2 that fit the data very well.

The possibility of vacuum oscillations with an oscillation length comparable to earth–sun distance and a large mixing angle, was a much earlier proposed explanation [13]. The solar neutrino data suggest that there is a particularly large suppression of a monoenergetic neutrino from electron capture on ^{7}Be. For the case of vacuum oscillations, this suppression factor should vary during the year due to the change in the earth–sun distance. In calculating this variation in detail, the question arises whether this variation is smeared out because the neutrinos originate in different parts of the sun and so travel different distances. However, as I pointed out in my original paper, in this case the matter effect completely suppresses oscillations inside the sun and they only begin at the solar surface where ρ_e is small enough. It turns out that in any case the main smearing effect is the energy spread of the ^{7}Be neutrinos [14].

In conclusion, matter is transparent for neutrinos except for extreme densities and ultra-high energies. However, transparent media do have an index of refraction and the analog of birefringence could play an important role in the physics of neutrino oscillations.

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