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Defect correlation in liquid crystal: Experimental verification of cosmological Kibble mechanism

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Abstract. In this talk we present observation of correlated production of strength one defects and anti-defects formed in isotropic-nematic phase transition in NLC. We find the width σ of the distribution of *net* winding number, to be in good agreement with the value predicted by the Kibble mechanism for defect production.

Keywords. Topological defects; Kibble mechanism; liquid crystals; correlations.

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1. Kibble mechanism

Particle physics models of the early Universe bear several symmetry breaking phase transitions. *Topological defects* are supposed to have formed in such phase transitions.

Kibble [1], in 1976, proposed a mechanism of formation of topological defects. He postulated that:

1. At a phase transition the Universe is broken up into *domains* of broken symmetry phase, and the order parameter varies randomly from one domain to the other.
2. In between two domains, the order parameter interpolates following the shortest path on the vacuum manifold (order parameter space). This is called as the *geodesic rule*.

As an example we consider a phase transition associated with a spontaneous $U(1)$ symmetry breaking in $(2+1)$-dim. This allows for the existence of vortex solution where the phase of the order parameter varies by $\pm 2\pi$ around a closed loop, as explained in the talk by Srivastava. A typical field configuration in the system after a phase transition will look as in figure 1. Average order parameter in each domain is given by the arrows. The variation of the order parameter in the domain boundaries is obtained using the geodesic rule. This is shown within a dashed curve at one of the domain boundaries.

Using these prescriptions one can find junctions of domains in physical space, around which phase of the order parameter varies by 2π on a closed loop (denoted by the dotted circle), thereby leading to a vortex at the junction. Various other kinds of topological defects can be similarly argued to have formed via this mechanism.

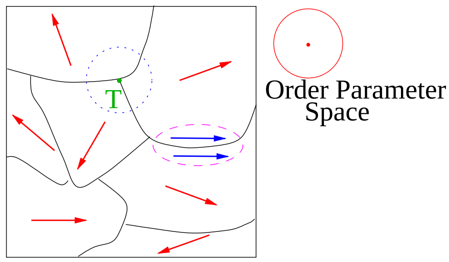
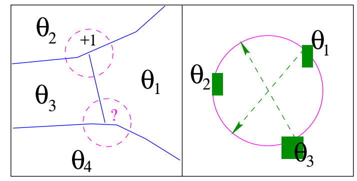


Figure 1.



(a) (b)

Figure 2. The configuration of order parameter in a 2-D system after a phase transition. The order parameter in each of the domains, denoted by solid boundaries, is given as a vector, whose magnitude and direction gives the magnitude and phase of the order parameter respectively.

2. Kibble mechanism in laboratory

The basic picture of formation of topological defects as proposed by Kibble for cosmology is based on topological ideas and is not sensitive to the various details of the models. Motivated by this fact, Zurek [2], in 1985, proposed to test Kibble mechanism in superfluid ^{4}He. Chuang *et al* [3], studied scaling of defects in nematic liquid crystal (NLC) in 1991.

However to verify Kibble mechanism there were two specific theoretical predictions to be checked:

1. Density of defects per domain
2. Strong correlation in defects and anti-defects.

In 1994, Bowick, Chandar, Schiff and Srivastava [4], for the first time experimentally verified the theoretical predictions of defect density. Later, similar studies were done in ^{4}He by Hendry *et al* [5].

In the present work we experimentally verify the second prediction, that the production of defect and anti-defect is highly correlated. Below we explain how the correlation arises.

3. Why one expects correlated production of defects and anti-defects?

In the case of $U(1)$ symmetry breaking the order parameter space is a circle (figure 2b). We take a junction of 3 domains, where a defect of winding $+1$ is formed (figure 2a). If we take a similar junction nearby, we can only have a -1 winding there, provided the angle in the fourth domain lie in an arc antipodal to the arc formed by phases in the two nearby domains.

However if the second junction is of different type (e.g. a junction of four or more domains), then one may get a $+1$ winding or a -1 winding there according to the field interpolation, but the resultant probability of forming a -1 defect in the vicinity of $+1$ defect is clearly enhanced. As a result in a given region on an average the probability of getting non-zero value for $\triangle N = n_+ - n_-$ gets suppressed. Here N is the total number of defects, n_+ and n_- are the total number of defects and anti-defects respectively.

4. Distinguishing random production from Kibble mechanism

In our above example, after a phase transition when topological defects have formed, we divide the whole sample space into boxes each containing on the average N defects. We then draw the distribution of $\triangle N$. If defects are formed without any correlation, then the distribution will have its peak at $\triangle N = 0$ and a standard deviation $\sigma = N^{1/2}$.

Now, suppose that defects formed via Kibble mechanism. We choose one box. Let its perimeter be L. $\triangle N$ inside the box equals the total winding on the perimeter. If the elementary domain size is ξ then there are L/ξ number of domains along the perimeter. According to Kibble mechanism, each domain has independent distribution of phase of scalar field. The order parameter space being a circle, the variation in phase in any two consecutive domains may be any value between 0 and π. So average angular variation is $\pi/2$. This thus becomes a random walk problem with steps $+1/4$ or $-1/4$ for $\triangle N$. The distribution will then have a peak at $\triangle N = 0$ with a standard deviation $\sigma = 1/4(L/\xi)^{1/2}$. The first prediction of Kibble mechanism says that total number of defects N, inside each box, is proportional to the total number of domains $(L/\xi)^2$, inside the box. For $U(1)$ case in $(2+1)$-dim. it can be shown that probability of a defect per domain is $1/4$. So standard deviation $\sigma \simeq 0.57N^{1/4}$ (or $0.71N^{1/4}$) if elementary domains are triangular (or square). So the width of distribution is more suppressed as a function of N in this case.

5. The experiment with nematic liquid crystal

In our experiment we take a droplet of NLC and study nematic-isotropic phase transition. The NLC consists of rod-like molecules. At high temperature these molecules are randomly oriented. Thus the order parameter which is given by the local alignment is zero. This is called *Isotropic phase*. Below a critical temperature the molecules become locally aligned and the order parameter is non-zero. This phase is called the *Nematic phase*. The orientation of the order parameter is given by a unit vector (with opposite directions identified) called as the *director*. This leads to the order parameter space $RP^2 \equiv (S^2/Z_2)$.

We observe strength one vortices formed in the I-N interface, for which the order parameter space is effectively a circle as we argue below. We first heat up the NLC droplet to its isotropic phase ($T_c \simeq 36°$C). As we let the drop to cool down slowly, we observe phase transition occurring uniformly over a thin layer in the drop. As a result an I-N interface is formed. At the interface the anchoring of the *director* forces it to lie on a cone with half angle $\simeq 64°$. This forces the order parameter space there to become a circle S^1.

Due to birefringence of NLC, when the sample is placed between a cross polarizer setup, then regions where the projection of the *director* on the plane containing the electric field $\vec{E}$ is either parallel or perpendicular to $\vec{E}$, polarization is maintained, and the region appears dark. Other regions depolarize light and appear bright. So around a defect of strength 's' there would be '4s' number of dark brushes. The sign of the winding can be found by rotating the cross polarizer setup and noticing the direction of rotation of the brushes about

Table 1. Comparison between theoretical predictions and experimental observation.

Theoretical predictions		Experimental observations
Random production	Kibble mechanism	
$\langle \triangle N \rangle = 0$	$\langle \triangle N \rangle = 0$	$\langle \triangle N \rangle = 0.09$
$\sigma \simeq 3.1$	$\sigma \simeq 1.02$ for triangular domains	$\sigma = 1.19 \pm 0.04$
	$\sigma \simeq 1.26$ for square domains	

the defect. Conventionally if brushes from a defect rotates in the same (opposite) direction of rotation of the cross-polarizer, it is a defect (anti-defect).

6. Results

We took a photograph containing several hundred defects and subdivided the picture into boxes each containing on the average $N \simeq 10$ defects. Plotting the distribution of $\triangle N$ we determined standard deviation of σ from this plot. The following table shows the results.

7. Conclusions

These results support the Kibble mechanism as the underlying mechanism of topological defect formation in NLC. The test can be improved upon by verifying the power law behavior of the variance $\sigma \simeq N^{1/4}$.

This is the first experimental verification of defect anti-defect correlation as far as we know. Also in systems where the domains are not visible (e.g. is phase transition of second order or via spinodal decomposition) experimentally measuring defect density is not possible. In such cases one can study the statistics of defect distribution as discussed above to understand the underlying mechanism of defect formation.

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