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# Topological defects in the left-right symmetric model and their relevance to cosmology

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**Abstract.** It is shown that the minimal left-right symmetric model admits cosmic string and domain wall solutions.

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## 1. Introduction

Consider the spontaneous symmetry breaking of a group $G$ down to a subgroup $H$ of $G$. Topological defects, arising according to the Kibble mechanism [1] when $G$ breaks down to $H$, are classified in terms of the homotopy groups of the vacuum manifold $G/H$. The relevant homotopy groups are $\Pi_i(G/H),\ i = 0,1,2$. If $\Pi_i(G/H)$ is nontrivial, topological defects can form. For $i = 0,1$ and $2$, the defects are domain walls, cosmic strings and monopoles respectively. Cosmic strings can explain large scale structure, anisotropies in cosmic microwave background radiation, and part of the baryon asymmetry of the Universe. Domain walls, on the other hand, if they exist are potentially problematic. They would dominate the energy density of the Universe and overclose it.

As a particle physics model, we consider one of the most attractive extensions of the electroweak model, based on the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ [2]. We show that the cosmic string solutions exist if the symmetry breaking $SU(2)_R \otimes U(1)_{B-L} \rightarrow U(1)_Y$ has occurred. These string defects may either be destabilized at the electroweak phase transition or may acquire additional condensates and continue to enjoy topological stability. The model also admits two kind of domain wall solutions which are stable only above the electroweak scale.

## 2. Cosmic strings

We begin with the phase in which only the first stage of symmetry breaking $SU(2)_R \otimes U(1)_{B-L} \rightarrow U(1)_Y$ has occurred. In the conventions of Mohapatra [2], the field signalling this breakdown is the $(1,0,2)$ field $\Delta_R$ which acquires the vacuum expectation value (vev) with the $(2,1)$ entry of the matrix being the only non-trivial component, $\langle \Delta_R \rangle_{21} = v_R$.

A cosmic string ansatz can be constructed by selecting a map $U^{\infty}$ from the circle $S^{\infty}$ at infinity into some broken $U(1)$ subgroup of the original group. Since $Y = T_R^3 + X$ (with $X = 2(B-L)$) is unbroken, we propose a cosmic string ansatz using the $U(1)$ generated by $\tilde{Y} = T_R^3 - X$. Furthermore, we select the internal parameter to be one-half times the spatial cylindrical angle $\theta$. Thus, $U^{\infty}(\theta) = \exp\{i(T_R^3 - X)\theta/2\}$. The $SU(2)$ acts on $\Delta_R$ by similarity transformation, so $\langle \Delta_R(\infty, \theta) \rangle_{21} = e^{i\theta} v_R$, so that the vev remains single valued; however,

$$U^{\infty}(2\pi) = \mathrm{e}^{-i\pi} \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \neq U^{\infty}(0)\ . \tag{1}$$

Hence we have identified a discrete stability group $Z_2$ which leaves the vev invariant but not the general matrix $\Delta_R$.

The stability criterion based on the $Z_2$ identified above does not survive the subsequent phase transition. The low energy vevs of the $(1,0,2)$ field $\Delta_L$ and the $(1/2,1/2,0)$ field $\phi$ are, respectively, $\langle \Delta_L \rangle_{21} = v_L$ and diag$(\kappa, -\tilde{\kappa})$ which are not invariant under the action of $U^{\infty}(2\pi)$. However, one may think of the above curve $U^{\infty}(\theta)$ as a projection to the subspace $SU(2)_R \otimes U(1)_{B-L}$ of the more general curve $\tilde{U}^{\infty}(\theta) = \exp\{i(T_R^3 + T_L^3 - X)\theta/2\}$. This leaves $\Delta_R(\infty, \theta)$ to be as above and leaves the $\phi$ vev invariant, but makes $\langle \Delta_L(\infty, \theta) \rangle_{21} = \mathrm{e}^{i\theta} v_L$. Thus the new vevs also possess a discrete stability group $Z_2'$, a simple generalization of the earlier $Z_2$. If such cosmic strings form, they should exist as relics at the present epoch.

## 3. Domain wall

At tree level the Lagrangian is symmetric under the exchange $\Delta_L \leftrightarrow \Delta_R$, reflecting the hypothesis of $L-R$ symmetry. If the vacuum values for these two Higgs fields are assumed to be as in the previous section, it can be shown [2] that their potential assumes the form

$$V(\Delta_L, \Delta_R) = -\mu^2(\Delta_L^2 + \Delta_R^2) + (\rho_1 + \rho_2)(\Delta_L^4 + \Delta_R^4) + \rho_3 \Delta_L^2 \Delta_R^2\ , \tag{2}$$

where the parameters are inherited from the original form of the potential [2]. If we define $\sigma(x) = \sqrt{\Delta_R^2 + \Delta_L^2}, \quad \xi(x) = \tan^{-1} \Delta_L/\Delta_R$, then the numerical solution for the domain wall configurations $\sigma(x)$ and $\xi(x)$ are shown in figure 1.

At the electroweak scale, the effective potential does not respect $L$-$R$ symmetry due to the nature of the $\phi$ self coupling. The $Z_2$ guaranteeing the topological stability of the walls now disappears.

In order to have the observed near-maximal parity violation at low energies, we must have $\kappa \ll v_R$. Also, to avoid fine tuning in the potential we must have $\tilde{\kappa} = 0$. But $v_L \ll \tilde{\kappa}$, so we shall set $v_L = 0$ (note that $\kappa, \tilde{\kappa}, v_L, v_R$ are vevs of $\phi, \tilde{\phi}, \Delta_L, \Delta_R$ fields respectively). So we are left with only two fields $\phi$ and $\Delta_R$. These fields admit a domain wall solution where the field $\phi$ develops a condensate in the core of the domain wall. Then the potential is simplified to

$$V(\phi, \Delta) = \lambda C^4 \phi^4 - \mu_\kappa^2 C^2 \phi^2 + \rho \Delta_R^4 - \mu^2 \Delta_R^2 + \alpha C^2 \phi^2 \Delta_R^2\ , \tag{3}$$

where $\lambda = \lambda_1 + \lambda_1'$, $\mu_\kappa^2 = \mu_{11}^2 + \mu_{22}^2$, $\rho = \rho_1 + \rho_2$, $\alpha = \alpha_{11} + \alpha_{22} + \beta_{11}$ and $C = \kappa/v_R$ (see [2]). We have obtained the domain wall profile numerically. We have assumed the

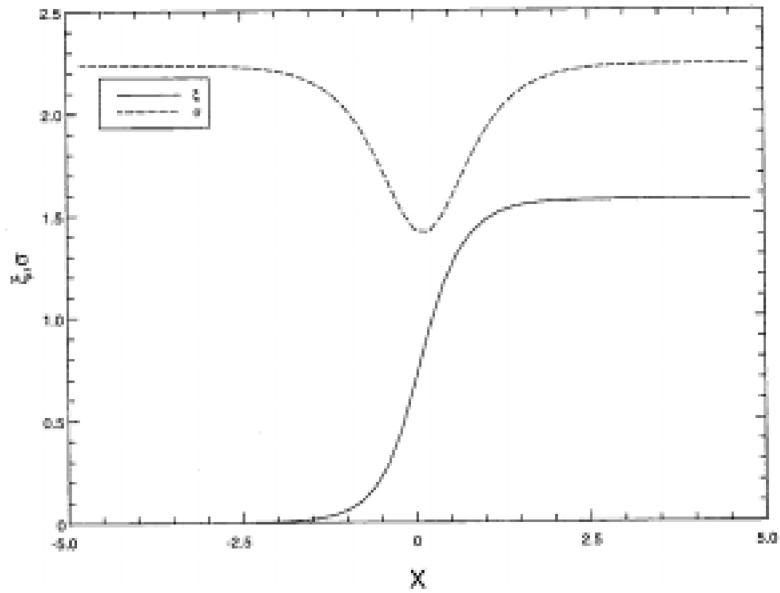


**Figure 1.** Domain wall solutions for $\rho_1 + \rho_2 = 0.1$, $\rho_3 = 0.9$, $\mu^2 = 1$ and $\beta = 0.7$.

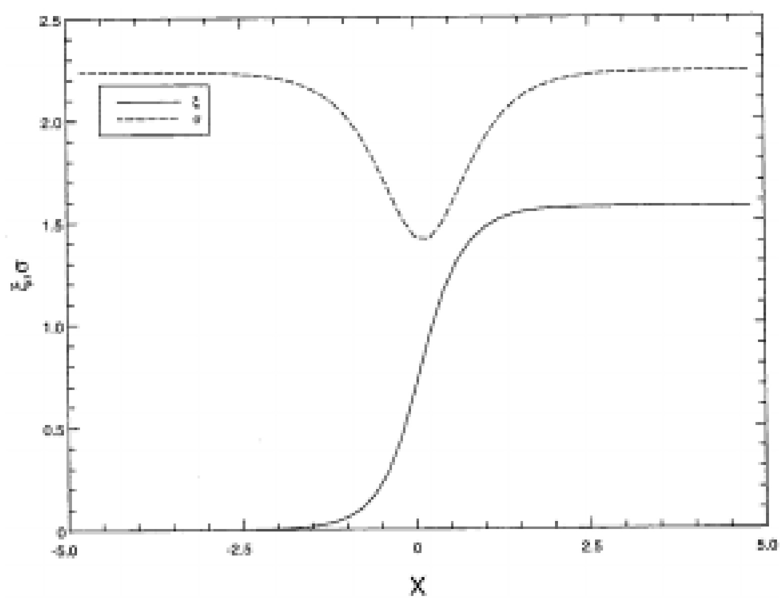


**Figure 2.** Domain wall solutions for $\rho = 0.5$, $\lambda = 0.01$, $\alpha = \mu_\kappa^2 = 0.4$, $v_R = 1.0$ and $C = 0.01$.

ansatz functions $R(x)$ and $f(x)$ for the nonzero components of $\Delta_R$ and $\phi$ respectively. Figure 2 shows our result.

It can be shown there exist unstable strings which are the boundary of these domain walls. The walls eventually shrink via surface tension, string intercommutation and nucleation of new string loops [3]. Thus they never dominate the energy density of the Universe, and can have interesting cosmological effects while they last.

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