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Dark matter halo cores in hierarchical clustering theories

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Abstract. In hierarchical clustering theories, smaller masses generally collapse earlier than larger masses and so are denser on the average. The core of a small mass halo could be dense enough to resist disruption and survive undigested, when it gets incorporated into a bigger object, and determine the halo structure in the inner regions. We examine in this talk the possible consequences of this idea in determining the structure of dark halo cores, by considering, both simple scaling arguments, and a novel fluid approach to self-similar collapse solutions for the dark matter phase space density.

Keywords. Cosmology; dark matter; large-scale structure of Universe; galaxies; formation; halos; clusters.

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1. Introduction

Much of the early work on the structure of dark matter halos of galaxies, concentrated on their density profiles in the outer regions, especially in the context of understanding the flat rotation curves of disk galaxies. The nature of the density profiles of dark matter halos, in their inner regions, is also important from several points of view. The structure of dark halo cores determines the efficiency of gravitational lensing by the galactic and cluster halos, the X-ray emissivity of clusters and the rotation curves of galaxies in the inner regions. These properties can be well probed by observations. So, if the core density profiles of dark halos, depend on features of structure formation models, like their initial power spectrum, one would have a useful observational handle on such features. It is therefore of interest to understand, ab initio, what determines the structure of dark matter halos and their cores.

Further, Navarro, Frenk and White (NFW) have proposed from their N-body simulations, that dark matter halos in hierarchical clustering scenarios develop a universal density profile, regardless of the scenario for structure formation or cosmology [1–3]. The NFW profile has an inner cuspy form with the density $\rho \propto r^{-1}$ and an outer envelope of the form $\rho \propto r^{-3}$. There does not appear to be any reason, apriori, why halo density profiles should prefer such a form. Recently, higher resolution simulations of galactic and cluster halo formation in a CDM model, by Moore *et al* [4,5], yielded a core density profile $\rho(r) \propto r^{-1.4}$, shallower than r^{-2}, but steeper than the r^{-1} form preferred by NFW. It is important to understand these results as well on general theoretical grounds. We summarise here some of our recent work, to understand the structure of dark halo cores [6].

2. Processes which could set the halo density profile

Let us consider an Einstein de-Sitter universe, with $\Omega = 1$. Also assume that the Fourier space power spectrum of density fluctuations is a power law, $P(k) = Ak^n$, where the spectral index n lies between the limits $-3 < n < 1$. In this case structure grows hierarchically with small scales going non-linear first and larger and larger mass scales going non-linear at progressively later times. What would decide the density profile of a dark matter halo in such a cosmological setting?

Firstly, when some mass scale decouples from the general Hubble expansion and collapses in an inhomogeneous fashion to form a dark matter halo, the changing gravitational potential and phase mixing will cause some amount of relaxation or 'virialisation' to occur. General constraints on the equilibrium properties of such a halo will be set by energy and mass conservation together with scaling laws which obtain in a hierarchical clustering scenario. Further, in the cosmological context, a collapsed mass is not isolated and will therefore continue to accrete surrounding material. Such a secondary infall onto the collapsed halo will alter/determine its structure in the outer regions.

We emphasize here a third process: When any mass scale collapses, in a hierarchical theory, it will already contain a dominant smaller mass dark halo which would have collapsed earlier, and is therefore denser on the average. It is possible that the core of such a smaller mass halo, is dense enough to resist disruption by tidal forces, and survive undigested, when it gets incorporated into the bigger object. A nested sequence of undigested cores in the center of the halo, which have survived the hierarchical inhomogeneous collapse to form larger and larger objects, could determine the inner density profile of the halo. We illustrate this idea schematically in figure 1.

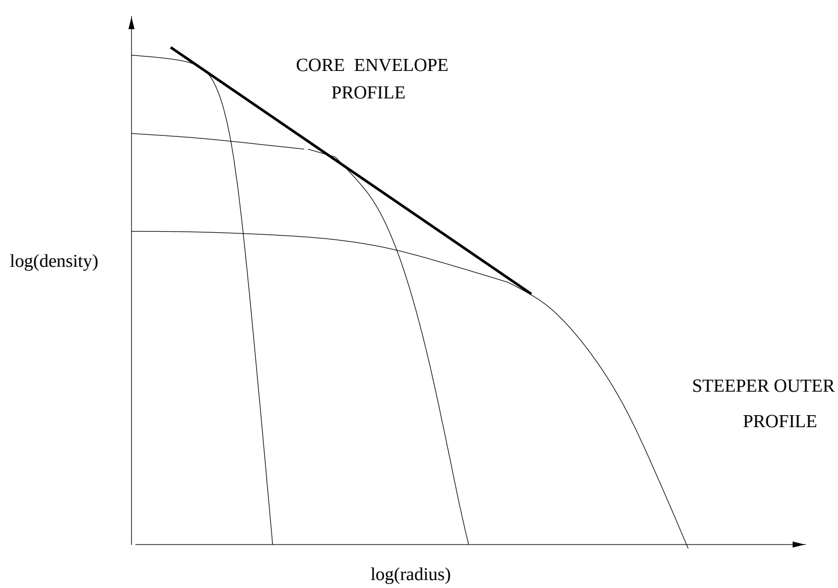

Figure 1. Schematic illustration of how the density profile of a large halo core could arise as an envelope of the density profiles of undigested cores of smaller mass halos.

Suppose, ignoring the detailed effect of undigested cores or secondary infall, a halo of mass M collapses to form a 'virialised' object with some characteristic density ρ_0 and core radius r_c. For $P(k) \propto k^n$, simple standard scaling arguments using linear theory (cf. [7–9]), predict that ρ_0 and r_c, scale with M as

$$\rho_0(M) \propto M^{-(n+3)/2}; \quad r_c(M) \propto M^{(n+5)/6}; \quad \rho_0 \propto r_c^{-(9+3n)/(5+n)}. \tag{1}$$

So, in the above sequential collapse to form larger and larger objects, the undigested core of each member of the sequence, typically contributes a density ρ_0 at a scale r_c, satisfying the relation $\rho_0 = c_1 r_c^{-(9+3n)/(5+n)}$, with some constant c_1. This suggests that the inner density profile of the bigger halo, which is the envelope of the profiles of the nested sequence of smaller mass cores, could have the form

$$\rho(r) \propto r^{-\alpha}, \qquad \alpha = \alpha_n = \frac{9+3n}{5+n}. \tag{2}$$

It is intriguing that the same form for the *density profile* (as against the correlation function) is also argued for by Peebles (ref. [7]; §26). In a paper which appeared during the course of our work, Syer and White [10] motivate the same density profile law as a fixed point of mergers, for the case when bigger halos form by purely mergers of smaller halos.

One can also state the above argument in terms of the velocity dispersion or the rotation velocity profiles. The typical velocity dispersion of a collapsed halo $\sigma \propto (M/r_c)^{1/2}$. Since the scaling argument gives $r_c \propto M^{(n+5)/6}$, we have $\sigma^2 \propto r_c^{(1-n)/(5+n)}$. So for any $n < 1$, smaller mass objects have a smaller velocity dispersion than larger mass objects. The survival of a nested sequence of cores during the inhomogeneous collapse to form bigger and bigger objects, then suggests that the velocity dispersion profile in the core regions will scale as $\sigma^2(r) \propto r^{(1-n)/(5+n)}$. For any $n < 1$, an alternate signature of undigested cores is then a velocity dispersion which increases with increasing radius in the above fashion. It is interesting to note in this context that, the cluster scale halo core in the Moore *et al* simulation [4], does indeed show such a velocity dispersion profile, with σ increasing with increasing r (Moore, private communication).

Although scaling laws suggest possible forms for the core density, and velocity dispersion profiles, dynamical constraints may not always allow these forms to obtain. To explore this dynamical issue further, we have looked [6] at a simple tractable model: the spherical, self similar collapse, of dark matter density perturbations, in a flat universe.

3. Self similar collapse and halo density profiles: A fluid approach

Consider the collapse of a single spherically symmetric density perturbation, in a flat background universe. Suppose the initial, average excess density contrast of an overdense region, is a power law in radius. Then there is no special scale in the problem either from initial conditions or cosmology. We expect to be able to describe the further evolution of such a density perturbation, through a self similar solution. Fillmore and Goldriech (FG) [12] and Bertschinger (B85) [11], looked at purely radial self similar collapse by solving for the self similar particle trajectory. We adopt a different approach here, examining directly the evolution of the distribution function of the dark matter.

3.1 *The self similar solution*

The evolution of dark matter phase space density $f(\mathbf{r},\mathbf{v},t)$ is governed by the Vlasov equation,

$$\frac{\partial f}{\partial t} + \mathbf{v}.\frac{\partial f}{\partial \mathbf{r}} + \mathbf{a}.\frac{\partial f}{\partial \mathbf{v}} = 0, \tag{3}$$

where $\mathbf{r}$ and $\mathbf{v} = \dot{\mathbf{r}}$ are the proper co-ordinate and velocity of the particles respectively. Also the acceleration $\mathbf{a} = \dot{\mathbf{v}} = -\nabla\Phi$, with

$$\nabla^2\Phi = 4\pi G\rho = 4\pi G\int f\mathrm{d}^3\mathbf{v}. \tag{4}$$

By direct substitution, it is easy to verify that these equations admit self similar solutions of the form

$$f(\mathbf{r},\mathbf{v},t) = k_2 k_1^{-3} t^{-q-2p} F\left(\frac{\mathbf{r}}{k_1 t^p}, \frac{\mathbf{v}}{k_1 t^q}\right); \qquad p = q+1 \tag{5}$$

where k_1, k_2 are constants which we will fix to convenient values below. We have used proper co-ordinates here since the final equilibrium halo is most simply described in these co-ordinates. (The same solution in co-moving co-ordinates is given in [13]).

Consider the evolution of a spherically symmetric density perturbation, in a flat universe whose scale factor $a(t) \propto t^{2/3}$. For self similar evolution, the density is given by

$$\rho(r,t) = \int f\mathrm{d}^3\mathbf{v} == k_2 t^{-2}\int F(y,\mathbf{w})\mathrm{d}^3\mathbf{w} \equiv k_2 t^{-2}\psi(y) \tag{6}$$

where we have defined scaled co-ordinates, $\mathbf{y} = \mathbf{r}/k_1 t^p$, $\mathbf{w} = \mathbf{v}/k_1 t^q$, and $r = |\mathbf{r}|$, $y = |\mathbf{y}|$. We have also used the relation $p = q+1$. For the flat universe, the background matter density evolves as $\rho_b(t) = 1/(6\pi G t^2)$. So the density contrast $\rho(r,t)/\rho_b(t) = \psi(y)$, where we take $k_2 = 1/(6\pi G)$.

3.2 *Linear and non-linear limits*

Let the initial excess density contrast averaged over a sphere of co-moving radius $x = r/a(t) \propto rt^{-2/3}$ be a power law $\bar{\delta}(x,t_i) \propto x^{-3\epsilon}$. Since ρ/ρ_b is a function of y alone, the $\bar{\delta}(x,t)$ will also be a function only of y. Note that, in the linear regime, it is the excess density contrast averaged over a *co-moving* sphere, which grows as the scale factor $a(t)$. So one can write for the linear evolution of the spherical perturbation

$$\bar{\delta}(r,t) = \bar{\delta}_0 x^{-3\epsilon} t^{2/3} \propto \bar{\delta}_0 r^{-3\epsilon} t^{2/3+2\epsilon} \propto \bar{\delta}_0 y^{-3\epsilon} t^{-3\epsilon p+2/3+2\epsilon}, \tag{7}$$

where we have substituted $r \propto yt^p$. This can be a function of y alone, for a range of t in the linear regime iff $-3\epsilon p + 2/3 + 2\epsilon = 0$, which gives

$$p = \frac{2+6\epsilon}{9\epsilon}. \tag{8}$$

We see that once the initial density profile is specified the exponents p, q of the self similar solution are completely determined. (For an initial $\bar{\delta}(x, t_i) \propto x^{-3\epsilon}$, the radius of the shell turning around at time t, $r_t(t) \propto t^p$. So a natural way of fixing the constant k_1 is by taking $k_1 t^p = r_t(t)$, and $y = r/r_t(t)$. We will do this in what follows.)

Consider now what happens in the non-linear limit. The zeroth moment of the Vlasov equation gives

$$\frac{\partial \rho}{\partial t} + \nabla_{\mathbf{r}}.(\rho \bar{\mathbf{v}}) = 0. \tag{9}$$

Here $\bar{\mathbf{v}} = \langle \mathbf{v} \rangle$ is the mean velocity. (Henceforth both $\langle\ \rangle$ or a bar over a variable denotes a normalised moment over f). In regions which have had a large amount of shell crossings, it seems plausible to demand that the halo particles have settled to nearly zero average infall velocity, that is $\bar{v_r} \equiv 0$. (They could of course still have velocity dispersions). From (9) , we then have $(\partial\rho/\partial t) = 0$, in the non-linear regime. In this regime therefore,

$$\rho(r, t) = H(r) = H(yt^p) = \frac{1}{6\pi G t^2}\psi(y). \tag{10}$$

This functional equation has only power law solution, because of the power law dependences on t. Substituting $H(r) = h_0 r^{-\alpha}$ into eq. (10) , and using $r \propto yt^p$, we get $y^{-\alpha} t^{-p\alpha} \propto t^{-2} D(y)$. This can only be satisfied for range of t in the non-linear regime provided $p\alpha = 2$. So, for an initial density profile with a power law slope 3ϵ, the power law slope of the density in the non-linear regime is given by,

$$\alpha = \frac{2}{p} = \frac{9\epsilon}{3\epsilon + 1}. \tag{11}$$

This result has been obtained by following the self similar particle trajectory, by B85 (for $\epsilon = 1$), and FG for $2/3 \leq \epsilon < 1$. We see that it can be simply obtained by just combining the self-similar solution f and the static core condition. (Obtaining the B85/FG result in this way has been independently noted earlier by Padmanabhan (private communication, unpublished notes 1994))

What should we choose for the value of ϵ? For a power law $P(k) \propto k^n$, the fractional density contrast averaged over a co-moving sphere of radius x, is distributed as a Gaussian, with a variance $\sigma(x) \propto x^{-(3+n)/2}$. This suggests a ‘typical’ spherically averaged initial density law for a halo collapsing around a randomly placed point of the form $\bar{\delta}(x, t_i) \propto x^{-(3+n)/2}$, or $3\epsilon = (3 + n)/2$. Suppose we use this value of ϵ for the initial density profile of a halo. Then the halo density in the static core regions will be $\rho(r, t) \propto r^{-\alpha}$, where, substituting $3\epsilon = (3 + n)/2$ in eq. (11)

$$\alpha = \alpha_n = \frac{9 + 3n}{5 + n}. \tag{12}$$

Remarkably, this is the same law we derived earlier for the core of a collapsed halo, assuming that the cores of sequence of sub-halos are left undigested, during the formation of the bigger halo. Note that for $n < 1$ the density law given by (12) is shallower than $1/r^2$.

FG also showed that a power law slope shallower than $1/r^2$, cannot obtain for purely radial collapse, And that while the above form for α should obtain for $2/3 \leq \epsilon < 1$, for $\epsilon < 2/3$, one goes to the limiting value $\alpha = 2$. However, this is only true for purely radial trajectories (cf. [14,15]). We see below, by considering the higher moments of the Vlasov equation, that $\alpha < 2$ can only obtain if the system has non-radial velocity dispersions.

3.3 *Jeans and energy equations*

Suppose we multiply the Vlasov equation by the components of $\mathbf{v}$ and integrate over all $\mathbf{v}$. Assume there is no mean rotation to the halo, that is $\bar{v}_\theta = 0$ and $\bar{v}_\phi = 0$. Then we get

$$\frac{\partial(\rho\bar{v}_r)}{\partial t} + \frac{\partial(\rho\bar{v}_r^2)}{\partial r} + \frac{\rho}{r}(2\bar{v}_r^2 - \bar{v}_\theta^2 - \bar{v}_\phi^2) + \frac{GM(r)\rho}{r^2} = 0. \tag{13}$$

$$\bar{v}_\theta^2 = \bar{v}_\phi^2. \tag{14}$$

Here $M(r)$ is the mass contained in a sphere of radius r.

Let us consider again a static core with $\bar{v}_r \equiv 0$. The Jeans equation gives two equations for the three unknown velocity dispersions, even for a static core. To see if one can close the system we can look at the second moments of the Vlasov equation (the energy equations). However these will involve the third moments, or the peculiar velocity skewness. Some form of closure hypothesis is needed in a fluid treatment of the Vlasov equation. For this we proceed as follows: One can firstly assume that initially the tangential velocities have zero skewness. Then in purely spherically symmetric evolution they would not develop any skewness, that is $\bar{v}_\theta^3 = \bar{v}_\phi^3 = \langle v_\theta v_\phi^2 \rangle = 0$ for all times. Also if the initial velocity ellipsoid had one of its principle axis pointing radially, we do not expect this axis to become misaligned in purely spherical evolution. This means we can assume $\langle v_r v_\theta^2 \rangle = \bar{v}_r \bar{v}_\theta^2$. Under these assumptions, and taking the static core condition $\bar{v}_r = 0$, we get, $(\partial(\rho\bar{v}_\theta^2)/\partial t) = 0$ or $\rho\bar{v}_\theta^2 = K(r)$ independent of t. For the self-similar solution we then have

$$\rho\bar{v}_\theta^2 = K(r) = K(yt^p) = k_2 k_1^2 t^{4q-2p} \int w_\theta^2 F(y, \mathbf{w}) \mathrm{d}^3\mathbf{w}. \tag{15}$$

Once again substituting a power law solution $K(r) = K_0 r^s$, to this functional equation, we get the constraint from matching power of t on both sides, $ps = 4q - 2p$. Using $p = q + 1$, we then get $s = 2 - 4/p = 2 - 2\alpha$, and so

$$\rho\bar{v}_\theta^2 = K_0 r^{2-2\alpha}. \tag{16}$$

Integrating the radial momentum equation using eqs (13), (14), (16) and using $\rho = h_0 r^{-\alpha}$, we have

$$\bar{v}_r^2 = r^{2-\alpha}\left[\frac{K_0}{(2-\alpha)h_0} - \frac{4\pi G h_0}{2(2-\alpha)(3-\alpha)}\right]$$
$$\equiv \frac{1}{(2-\alpha)}\left[\bar{v}_\theta^2(r) - \frac{GM(r)}{2r}\right]. \tag{17}$$

Several important points are to be noted from the above equation. A crucial one is that, when $\alpha < 2$, the RHS of eq. (17) can remain positive, provided one has a non zero tangential velocity dispersions. If one has a purely spherically symmetric collapse and zero tangential velocities, then the density law cannot become shallower than $\alpha = 2$ and maintain a static core with $\bar{v}_r = 0$. This agrees with FG. In fact for any $\alpha < 2$, one needs tangential velocity dispersions to be at least as large as $GM/2r$, comparable to the gravitational potential energy per unit mass. Further, one can see that to obtain static cores with $\alpha < 1$, the required tangential dispersions have to be necessarily larger than the radial velocity dispersions. Also note that for $\alpha < 2$, all the components of velocity dispersions decrease with decreasing radius, as suggested by the simple scaling arguments of the previous section.

Note that for a static core $\bar{v}_r^2$ should also be independent of t. However the energy equation for $\partial(\rho\bar{v}_r^2)/\partial t$, shows that a time independent radial velocity dispersion, can only obtain if the radial velocity skewness $\langle(v_r - \bar{v}_r)^3\rangle$ is also zero. In the core regions where large amounts of shell crossing has occurred, one can assume that a quasi 'equilibrium' state obtains, whereby all odd moments of the distribution function, over $(\mathbf{v} - \bar{\mathbf{v}})$, may be neglected. Such a treatment will correspond to considering a fluid like limit to the Vlasov equation.

However, the radial skewness will become important near the radius, where infalling matter meets the outermost re-expanding shell of matter. This region will appear like a shock front in the fluid limit. A possible treatment of the full problem in the fluid approach to the Vlasov equation then suggests itself. This is to take the radial skewness to be zero both inside and outside a 'shock or caustic' radius, whose location is to be determined as an eigenvalue, so as to match the inner core solution that we determine in this section with an outer spherical infall solution. One has to also match various quantities across this 'shock', using jump conditions, derived from the equations themselves. To do this requires numerical solution of the self consistent set of moment equations, to the scaled Vlasov equation. The details of such a treatment are given in our paper [6]. Here we summarise a few of the results.

4. Numerical solution of the moment equations

We write the scaled Vlasov equation in spherical co-ordinates and take moments. We define $V = \bar{w}_r$, $\Pi = \langle(w_r - \bar{w}_r)^2\rangle$ and $\Sigma = \bar{w}_\theta^2 = \bar{w}_\phi^2$. We also set the tangential velocity skewness to zero. As explained above, we take the radial skewness to be zero both inside and outside a 'shock or caustic' radius. The shock location, say $y = y_s$, is determined as an eigenvalue, to the complete problem, of matching the inner core-solution to an outer spherical infall solution, and by requiring the solutions to satisfy the inner boundary conditions

$$V = M = 0, \qquad y = 0. \tag{18}$$

We match the solutions across $y = y_s$ by specifying the jump conditions at $y = y_s$, derived from the moment equations.

Note that both radial and tangential velocity dispersions are likely to be generated during the inhomogeneous collapse to form the halo. So it is natural to take the initial, pre-collapse, velocity dispersions to be small. Radial velocity dispersions will be automatically generated, even if initially zero, when spherically collapsing shells start to cross expanding shells; that is where the radial skewness is important. On the other hand, in any spherically symmetric system, tangential velocities have to be necessarily introduced in an ad-hoc fashion. Since all the components of the velocity dispersion are expected to be generated together, the shock gives a natural location to introduce a tangential velocity dispersion. We have done this in most of the numerical examples. The solutions are therefore also described by a parameter $\Sigma(y_s) = \Sigma_2$. We discuss below a few typical examples of our numerical integration.

4.1 *Collapse onto an excess point mass, $\epsilon = 1$*

First consider the self-similar spherical infall onto a point mass, by adopting $\epsilon = 1$ and $\Sigma_2 = 0$. This problem was solved by B85 and FG by examining the self similar particle trajectory, and allows us to 'benchmark' the fluid approach. We find the eigenvalue $y_s = 0.4628$ for the above parameters. B85 solving the problem by looking at particle trajectories got the location of the outermost caustic as $y_s = 0.364$. This difference between our work and B85 could be because we have replaced a smooth transition region for the collisionless fluid, where velocity skewness is important, by a discontinuous shock. B85 found that the scaled density could be fitted asymptotically by a form $\psi(y) \approx 2.79 y^{-9/4}$ when they adopted a minimum $y = y_m \sim 0.02$ for the particle trajectory. We can integrate our equations and get converged solutions satisfying the boundary conditions upto $y_m \sim 2 \times 10^{-4}$. We find that $\psi(y) = \tilde{\psi} y^{-9/4} \approx 3.1 y^{-9/4}$ at y_m, while at $y \sim 2 \times 10^{-2}$ we find $\tilde{\psi} \approx 2.5$. These numbers bracket the asymptotic value of $\tilde{\psi} \sim 2.79$ obtained in B85. So there is fairly good agreement between our work and B85, given the differences in the value of y_m and the very different approaches.

4.2 $\epsilon < 2/3$ *and the importance of tangential dispersions*

We also consider solutions for initial density profiles, with $\bar{\delta}(r, t_i)$ shallower than r^{-2}, or $\epsilon < 2/3$. In this case, if the collapse were purely radial, FG showed that the final density profile approaches a $1/r^2$ form. We find, as expected, that the nature of the solutions, depends on the ratio of tangential to radial velocity dispersions.

In figure 2, we show the solution for the case $\epsilon = 0.4$, $\tilde{\Sigma}_2 = \Sigma_2 y_s^{2-\alpha} = 0.94$. For this solution, the value of $y_s = 0.4955$. We show both $\log(\Pi(y)$ (solid line) and $\log(\Sigma(y)$ (dashed line) in the same plot, so that they can be easily compared. From figure 2, one sees that tangential velocity dispersions for this solution is everywhere larger than the radial dispersions, by a factor ~ 1.3. ($(\Sigma/\Pi)^{1/2} \sim 1.3$). For $\epsilon = 0.4$, and a static core, we expect the scaled density to have the asymptotic behaviour $\psi(y) \propto y^{-\alpha}$, with $\alpha = 18/11$. This is plotted as a dashed line in the $\psi - y$ plot of figure 2. We see from comparing the solid and dashed lines, in this plot, that the density indeed has such an asymptotic behaviour. Also the velocity dispersions and rotation velocities increase with increasing radius, as the analytic theory of § 3 predicts. This case illustrates that it is possible to obtain solutions for $\epsilon < 2/3$, which have $\alpha = 9\epsilon/(1 + 3\epsilon) < 2$, provided the tangential velocity dispersions are large enough.

To illustrate the effect of decreasing tangential velocity dispersions, we show in figure 3, the properties of a solution with $\epsilon = 0.4$, but a smaller $\tilde{\Sigma}_2 = 0.65$. The location of the shock is at $y_s = 0.3797$. The core regions are nearly static but not completely so. For this case the radial velocity dispersions are everywhere larger than the tangential dispersions, by a factor ~ 1.15 as $y \to 0$. One sees a large difference between this solution (figure 3) and the one obtained for larger tangential velocity dispersion (figure 2). First we see that when radial dispersion dominates, the density profile is closer to the $\psi \propto y^{-2}$ form (dashed-dotted line) than the $\psi \propto y^{-\alpha}$ form (dashed line), although neither provides a good fit. Second the velocity dispersions are reasonably constant with radius as $y \to 0$ limit, instead of increasing with increasing radius. We have also looked at solutions

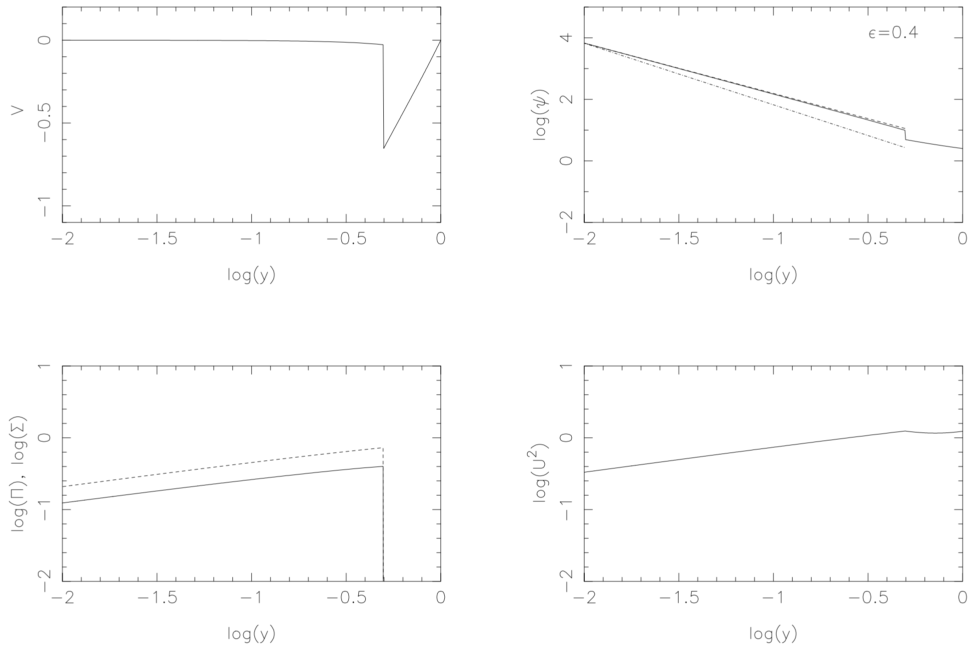

Figure 2. Self similar collapse solution for $\epsilon = 0.4$, $\tilde{\Sigma}_2 = 0.94$. The velocity V, scaled density ψ (solid line in the upper right plot), radial velocity dispersion Π (solid line in the lower left plot), tangential velocity dispersion Σ (dashed line in the lower left plot) and circular velocity squared U^2 are plotted against the scaled radius y. The pre shock spherical infall solution is also shown. In the ψ-y plot we also show for comparison the density laws $\psi \propto r^{-\alpha}$ (dashed line) with $\alpha = 9\epsilon/(1+3\epsilon)$ and $\psi \propto r^{-2}$ (dashed -dotted line).

with smaller values of ϵ, and also examples where the tangential velocity dispersions are introduced at the turn around radius ($y = 1$) rather than at the shock. These examples show very similar behaviour to the solutions discussed above (figures 2 and 3).

Our numerical solutions show the importance of tangential velocity dispersions, in deciding whether the self similar solution, with $\epsilon < 2/3$ retains a memory of the initial profile or whether the density profile tends to a universal $1/r^2$ form. For a large enough $\Sigma/\Pi > 1$, the core density profile is indeed close to the form $\rho \propto r^{-\alpha}$, with $\alpha = 9\epsilon/(1+3\epsilon)$. For $\Sigma/\Pi \sim 1$, some memory of the initial density profile is always retained. The core density profile is curved, steeper than $\rho \propto r^{-\alpha}$ form expected for static cores, but shallower than $1/r^2$. When $\Sigma/\Pi << 1$, the density profile goes over to the $1/r^2$ form derived by FG. Also for $\epsilon < 1/6$, one must necessarily have a tangential dispersion much larger than radial dispersion to get a static core region, with $\alpha < 1$. These differences from the static core results, arise because the boundary condition adopted in the numerical solution, correctly assumes only $\bar{v}_r(0) = 0$ at the origin, and not $\bar{v}_r(r) = 0$ for a range of radii near the origin (as would be required for a static core).

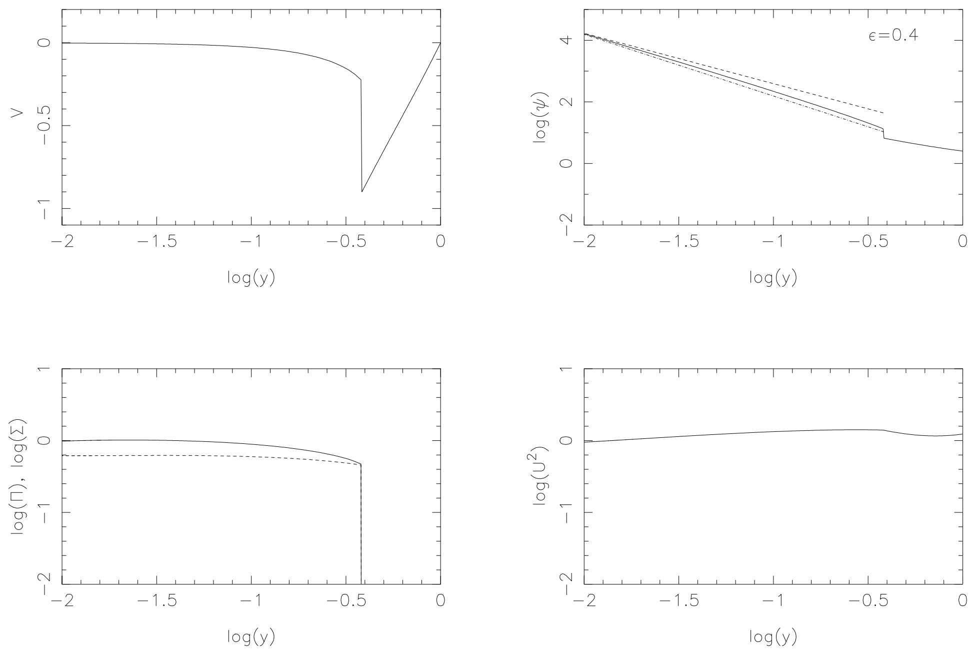

Figure 3. Self similar collapse solution for $\epsilon = 0.4$, $\tilde{\Sigma}_2 = 0.65$. The various quantities shown are same as in figure 2.

5. Discussion and conclusions

We have considered here the structure of the cores of dark halos in hierarchical clustering theories of galaxy formation. In such theories, it is very likely that cores of dark matter halos harbor undigested earlier generation material. Their density structure, in physical as well as phase space, will reflect the times and the cosmological densities when the core material was gathered.

In a flat universe with a power spectrum $P(k) \propto k^n$, a consequence of undigested cores, could be a cuspy core density profile, $\rho(r) \propto r^{-\alpha_n}$, with $\alpha_n = (9+3n)/(5+n)$. Or a velocity dispersion profile, which rises with increasing radius. In order to explore how and infact whether, this form will be realized dynamically, we focus on a simple tractable model: the spherical, self-similar collapse of dark matter density perturbations, in a flat universe. We have developed a novel fluid approach to study this problem.

Our results illustrate the general features which are likely to be important in determining the structure of halo cores. If newly collapsing material is constrained to mostly contribute to the density at larger and larger radii, then memory of initial conditions can be retained. The spherical self-similar collapse solutions, with $\alpha > 2$, or the solution with $\alpha < 2$ but a large enough tangential dispersion, illustrate this possibility. In the more general case, when newly collapsing material is able to occupy similar regions as the matter which collapsed earlier, the core density profile will only partially reflect a memory of the initial conditions. The solutions with $\alpha < 2$ and $\Sigma/\Pi \sim 1$ illustrates this feature. In either case we do not find any preference for a universal density profile. More details of our work, including also additional results from N-body simulations, will be given in ref. [6].

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