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Pancakes and filaments in cosmological gravitational clustering

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Abstract. We consider the geometrical properties of a distribution of matter evolving under gravitational clustering. Such a distribution can be studied using standard statistical indicators such as the correlation function as well as geometrical descriptors sensitive to 'connectedness' such as percolation analysis and Minkowski functionals. Applying these methods to N-body simulations and galaxy catalogues we find that the filling factor at the percolation threshold is usually very small reflecting the fact that the Universe consists of a network of filaments and pancakes, the latter being statistically more prominent.

Keywords. Cosmology; large scale structure; topology; shape.

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1. Introduction

The clustering of galaxies was well established already by the early 1970's mainly due to the pioneering work of Totsuji and Kihara [1] and Peebles [2] who showed that the two-point correlation function for galaxies had a distinctive power law form $\xi(r) \propto r^{-1.8}$ on scales less than 10 h^{-1} Mpc. However, the two point function does not provide a complete description of clustering. This is due to the fact that $\xi(r)$ and $P(k) = \langle|\delta_k|^2\rangle$ form a fourier transform pair. In the linear regime, assuming that density perturbations evolve from Gaussian initial conditions, $P(k)$ (and hence $\xi(r)$) provides a complete description of clustering since the phases θ_k in the fourier expansion $\delta\rho/\rho \propto \int \delta_k \exp{(ikx)}\mathrm{d}^3k$, $\delta_k = |\delta_k| \exp{(i\theta_k)}$ are uniformly random on $\{0, 2\pi\}$. However, as clustering advances into the nonlinear regime, the phases θ_k become correlated leading ultimately to the formation of filaments, pancakes and voids – which are the hallmarks of the evolved large scale structure of the Universe we see today. Since the phases θ_k are themselves not represented in the statistic ξ, the latter provides us with an incomplete though robust measure of clustering.

It therefore becomes important to complement traditional measures of clustering, such as ξ with statistical measures which are more sensitive to the geometry of a distribution and the 'connectedness' of matter over large scales. Historically the first measure attempting to quantify the morphology of large scale structure was percolation analysis adapted to cosmology in the early 1980's by Zeldovich [3] and Shandarin [4]. This was soon followed by the genus characteristic and minimal spanning trees (see Sahni and Coles [5] for a review and references). More recently a powerful means of studying the geometrical properties of large scale structure has emerged in the form of Minkowski functionals (MF) which include both percolation and genus curves as a subclass.

2. Percolation analysis

Percolation analysis attempts to quantify the connectedness of a distribution by examining its filling factor at different density thresholds. For instance when applied to a smoothed density distribution (constructed from N-body simulations or galaxy surveys) a good indication of percolation is provided by examining the volume fraction in the largest cluster – equivalently its filling factor FF_∞ – as a function of the total filling factor. (The filling factor of a distribution at a given threshold is the volume fraction in regions above that threshold, $FF = \int_\delta^\infty P(\delta)\mathrm{d}\delta$). In the case of the largest cluster, FF_∞ is defined as the ratio of the volume in the largest cluster to the total volume occupied by all clusters lying above a specified threshold. At very high values of the threshold 'δ' both FF and FF_∞ are small, since only a small number of very dense clusters lie above a high threshold and the scatter in their sizes is usually small. However as δ is reduced the total FF increases as does FF_∞ indicating that many more clusters lie above the new threshold and that some of them have merged leading to an increase in the size of the largest cluster. On reducing the value of δ further we find that a critical value FF_c is reached at which FF_∞ increases dramatically, indicating the onset of percolation in the largest cluster which at this point, spans the entire survey region. The value of FF_c as well as the full percolation curve shown in [6] provide us with very useful information about the geometrical properties of the distribution such as its large scale 'connectedness' etc.

Examining percolation in gravitational N-body simulations we find that FF_c usually decreases with time, as the distribution evolves under gravitational instability. For CDM-type models evolving from Gausian initial conditions, the value of FF_c today can be as small as 0.05, which is much smaller than the value $FF_G \simeq 0.16$ for the original Gaussian random field. This result indicates that the evolved distribution is significantly non-Gaussian and that at the threshold of percolation, its most dense features occupy a relatively small volume, suggesting that the largest elements in the distribution are more likely to be oblate or prolate. A distribution whose overdense regions percolate more easily than random (i.e. $FF_c < FF_G$) is called network like, in order to emphasise its 'stringy' features. (Occasionally one finds that $FF_c > FF_G$, in which case overdense regions percolate at significantly lower density thresholds than gaussian. This difficulty in percolation is associated with a clumpy mass distribution gastronomically referred to as a 'meatball' topology or perhaps more appropriately, a 'vada-sambar' topology, in which an isolated overdense region is a 'vada' submerged in a percolating underdense region – 'sambar'.)

Distributions evolving under gravitational instability also show another interesting property: when a similar analysis is carried out for underdense regions (voids) one finds that instead of being small, the critical filling factor is now large so that $FF_v > FF_G$ for voids. Voids therefore possess a 'bubble-like' topology in which isolated voids (bubbles, because they are underdense) are surrounded by a percolating overdense phase. For moderate density thresholds both overdense and underdense regions percolate and this phase is referred to as being 'sponge-like'. In figure 1 we show the evolution of the percolation curve for N-body simulations with power law initial spectra. We find that percolation transition is

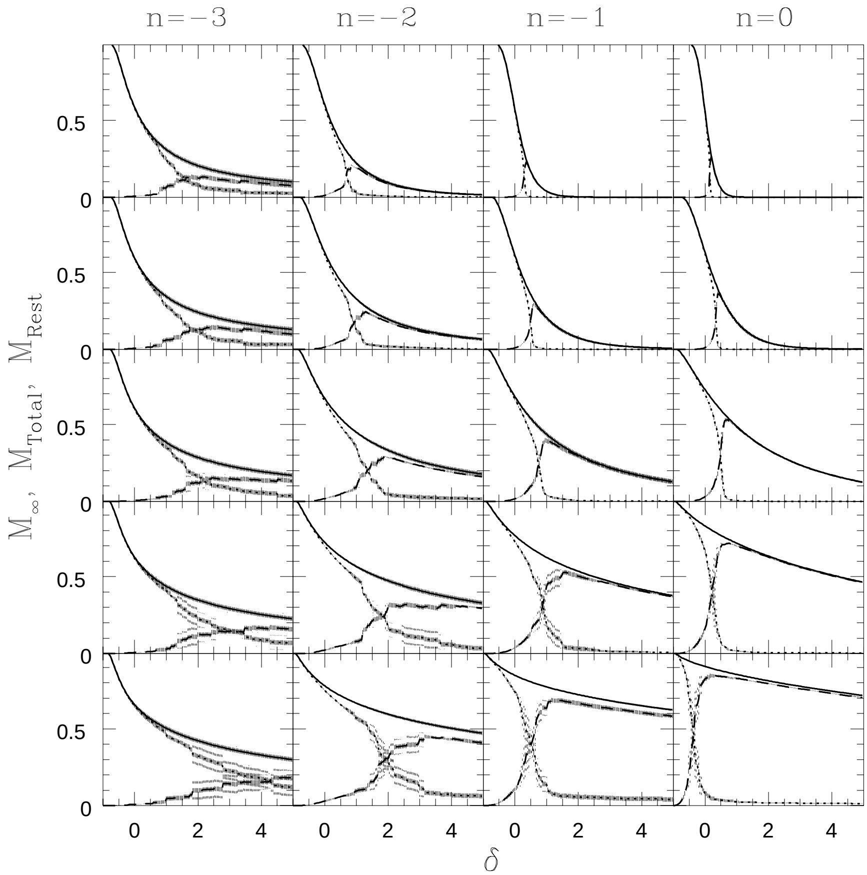

Figure 1. Percolation curves are shown for an N-body simulation evolving under gravitational instability from Gaussian initial conditions with a scale free initial spectrum $P(k) = k^n, n = -3, -2, -1, 0$. The cosmological epoch increases from top to bottom and the primordial spectrum increases from left to right. The dotted line corresponds to M_∞, the mass fraction in the largest cluster; $M_{\rm Rest}$ corresponds to the mass fraction in all clusters with the exception of the largest; $M_{\rm Total}$ is the mass fraction in *all* clusters above a given density threshold, $M_{\rm Total} = M_\infty + M_{\rm Rest}$. The percolation transition is characterised by a *sharp increase* in M_∞. The value of $M_{\rm Rest}$ is sharply peaked just before the onset of percolation indicating that the number of clusters is maximum at this threshold. For more details see Sathyaprakash, Sahni and Shandarin [12].

marked by a sharp increase in M_∞, the mass fraction in the largest cluster above a given threshold. Just before percolation the value of $M_{\rm Rest}$ peaks, indicating that most of the matter is in clusters other than the largest. Since clusters are very numerous at this threshold the latter provides a natural choice of δ at which to study cluster properties, a point to which we shall return in a later section when we discuss shapes.

3. Genus curve

The above analysis of the percolation curve is both supported and complemented by the analysis of another geometrical indicator which is sensitive to the connectedness of a distribution – the genus curve. The genus curve is constructed by studying the *topology* of isodensity surfaces above/below a given density threshold. The topology of a compact two dimensional surface can be characterised by its 'genus' (equivalently, the Euler characteristic) defined as $4\pi G(\nu) = \int_{S_\nu} K \mathrm{d}A$ where K is the Gaussian curvature and $\nu = \delta/\sigma_\delta$ the threshold [7]. At very high density thresholds clusters are topologically simply connected, whereas at lower thresholds neighboring clusters begin to merge leading to a complicated 'sponge-like' topology with a large genus. (Visually, the genus is simply proportional to the number of handles on a surface.) Again at very low thresholds voids will be simply connected with a low genus. The symmetry between overdense and underdense regions characteristic of a Gaussian random field is reflected in the remarkably simple (and symmetric) expression for its genus

$$G(\nu) = A(1 - \nu^2)\exp(-\nu^2/2). \tag{1}$$

Distributions evolving under gravitational clustering have a genus curve which gradually evolves away from the symmetric form (1) [6].

4. Shapefinder statistics

A small filling factor at percolation does not differentiate whether overdense regions are likely to be filamentary or pancake-like. Nor is the answer to this question provided by current catalogues of the redshift distribution of galaxies most of which are quasi-two-dimensional. Until recently the shapes of overdense and underdense regions were probed by fitting these regions by ellipsoids and then diagonalising the moment of inertia tensor. Although this method does yield some insight into the morphology of structures forming under gravitational instability, it cannot be said to provide a definitive characteristic of 'shape'. Consider for instance the simple example of a thin torus: clearly this object is filamentary in nature, in the sense that it is quasi-one-dimensional. Yet since it lies in two dimensionsal plane fitting it by an ellipsoid will make it seem 'oblate' rather than prolate! Similarly the curved surface of a cup is locally a curved pancake which can easily be mistaken to be spherical if one were to fit the whole object by an ellipsoid. In all these cases the filling factor of the object is small (thin torus, empty cup), yet the statistic (ellipsoid) artificially fills the empty regions (transforming the torus into a disc and the cup into a sphere) giving the object greater volume and thereby ignoring its true morphology.

In an attempt to define a 'shape quantifier' which did not suffer from these drawbacks, Sahni, Sathyaprakash and Shandarin [8] introduced the Shapefinder statistic which was constructed from the Minkowski functionals and hence did not rely on fitting by ellipsoids to determine shape. The four Minkwoski functionals charactering a compact object are [9]: (i) its volume (V), (ii) surface area S, (iii) its integrated mean curvature C and (iv) its genus G. Out of these four quantities the Shapefinder trio $\{L, W, T\}$ is constructed: $L = C/4\pi$, $W = S/C$, $T = 3V/S$. (L, W, T, can be thought to characterise the length, width and thickness of a body.) One can also define a dimensionless pair of Shapefinders

$\{P, F\}$ where $P = W - T/W + T$ and $F = L - W/L + W$, $0 \le P, F \le 1$. $\{P, F\}$ = $\{0, 0\}$ for a sphere, $\{0, 1\}$ for a filament, $\{1, 0\}$ for a pancake and $\{1, 1\}$ for a ribbon.

In two dimensions the two Minkowski functionals are the area S of a connected region, its perimeter L and its genus G (or number of holes). When combined these two dimensionful numbers can give a single *dimensionless* number characterising shape

$$\mathcal{F} = \frac{L^2 - 4\pi S}{L^2 + 4\pi S} \tag{2}$$

where $0 \le \mathcal{F} \le 1$; $\mathcal{F} = 0$ for a disc and $\mathcal{F} = 1$ for a filament.

Both two and three dimensional shape-statistics have been applied to N-body simulations of structure formation and to galaxy catalogues [10–12]. The results appear to favour filaments over pancakes, with the latter also being statistically significant. N-body simulations of structure formation indicate that as gravitational instability advances the filling factor decreases as more matter gets concentrated in filaments and pancakes [13,10]. Analysis of three dimensional catalogues, although very preliminary, appears to suport this viewpoint as shown in figure 2 [14]. The two dimensional shapefinder statistic can also be

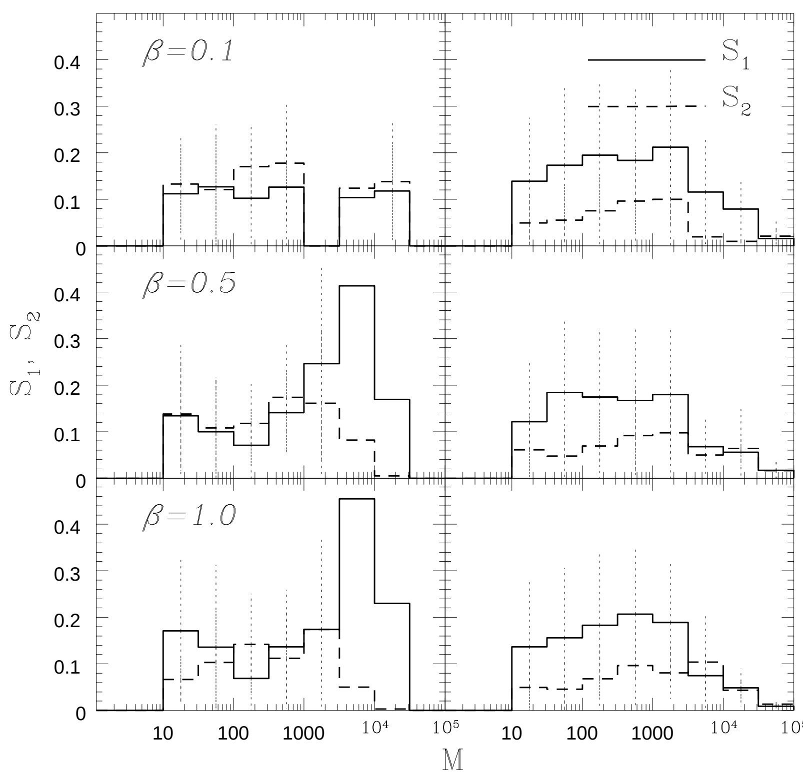

Figure 2. The shape of clusters at the percolation threshold is shown for the IRAS 1.2 Jy catalogue (left) and the randomized IRAS catalogue (right). S_1 represents filamentarity and S_2 planarity. For more details see Sathyaprakash *et al* [14].

used to study the shapes of hot and cold spots in the cosmic microwave background and the shapes of gravitationally lensed quasars and galaxies.

5. Conclusions

We have analysed the connectedness of large scale structure using percolation and genus curves, as well as statistics which are sensitive to shape. Our results unambiguously demonstrate that as gravitational clustering advances, the departure from gaussian initial conditions becomes more pronounced. This is reflected in a significantly lower/higher filling factor at percolation for overdense/underdense regions so that the former have a network-like topology whereas the latter are more 'bubble-like'. Applying shape statistics to probe overdense regions in N-body simulations we find that they are more prone to being filamentary and planar, and that the extent of filamentarity progressively increases as the system evolves under gravitational clustering. Preliminary analysis of three dimensional (IRAS) and quasi-two dimensional galaxy catalogues (LCRS) confirms this analysis supporting the hypothesis that the large scale structure of the Universe evolved from gravitational instability.

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