PRAMANA
— journal of
physics
© Indian Academy of Sciences
Vol. 53, No. 6
December 1999
pp. 989–993

Multi-fractal analysis of the galaxy distribution in the Las Campanas redshift survey

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Abstract. We have carried out a multi-fractal analysis of the distribution of galaxies in the three Northern slices of the Las Campanas redshift survey. In this analysis we have studied the scaling properties of the distribution of galaxies on length scales from 20 $h^{-1}\mathrm{Mpc}$ to 200 $h^{-1}\mathrm{Mpc}$. Our main results are: (1) The distribution of galaxies exhibits a multi-fractal scaling behaviour over the scales 20 $h^{-1}\mathrm{Mpc}$ to 80 $h^{-1}\mathrm{Mpc}$, and, (2) the distribution is consistent with homogeneity on the scales 80 $h^{-1}\mathrm{Mpc}$ to 200 $h^{-1}\mathrm{Mpc}$. We conclude that our results are consistent with the Universe being homogeneous at large scales and the transition to homogeneity occurs somewhere in the range 80 $h^{-1}\mathrm{Mpc}$ to 100 $h^{-1}\mathrm{Mpc}$.

Keywords. Cosmology; large scale structures.

PACS No. 98.65.Dx

1. Introduction

The isotropy of the cosmic microwave background radiation (CMBR) strongly suggests that the Universe seems to be homogeneous on large scales. The standard model of cosmology is built on this foundation. From the point of view of mathematical simplicity, the Einstein's equations take a simple form if we make the assumption of homogeneity and isotropy of space and the distribution of matter over large scales. There has, however, been a lot of debate on this issue.

Coleman and Pietronero [1] applied the fractal analysis to galaxy distributions and concluded that it exhibits a self-similar behaviour up to arbitrarily large scales. Their claim that the fractal behaviour extends out to arbitrarily large scales implies that the Universe is not homogeneous on any scale and hence it is meaningless to talk about the mean density of the Universe. These conclusions are in contradiction with the cosmological principle and the entire framework of cosmology, as we understand today, will have to be revised if these conclusions are true.

On the other hand, several others [5,4] have applied the fractal analysis to arrive at conclusions that are more in keeping with the standard cosmological model. They conclude that while the distribution of galaxies does exhibit self similarity and scaling behaviour, the scaling behaviour is valid only over a range of length scales and the galaxy distribution is homogeneous on very large scales. Various other observations including the angular distribution of radio sources and the X-ray background testify to the Universe being homogeneous on large scales [6,7].

Recent analysis of the ESO slice project [8] also indicates that the Universe is homogeneous over large scales. The fractal analysis of volume limited subsamples of the SSRS2 [9] studies the spatial behaviour of the conditional density at scales up to 40 h^{-1} Mpc. Their analysis is unable to conclusively determine whether the distribution of galaxies is fractal or homogeneous and it is consistent with both the scenarios. A similar analysis carried out for the APM-Stromlo survey [10] seems to indicate that the distribution of galaxies exhibits a fractal behaviour with a dimension of $D = 2.1 \pm 0.1$ on scales up to 40 h^{-1} Mpc. In a more recent paper [11] the fractal analysis has been applied to volume limited subsamples of the Las Campanas redshift survey. This uses the conditional density to probe scales up to 200 h^{-1} Mpc. They find evidence for a fractal behaviour with dimension $D \simeq 2$ on scales up to 20–40 h^{-1} Mpc. They also conclude that there is a tendency to homogenization on larger scales (50–100 h^{-1} Mpc) where the fractal dimension has a value $D \simeq 3$, but the scatter in the results is too large to conclusively establish homogeneity and rule out a fractal Universe on large scales.

Here we study the scaling properties of the galaxy distribution in the Las Campanas redshift survey (LCRS) [12]. This is the deepest redshift survey available at present. Here we apply the multi-fractal analysis [5,4] which is based on a generalization of the concept of a mono-fractal to address the following questions [14]:

(a) Does the Universe show transition to homogeneities?
(b) If so, what is the length scale at which this transition occurs?

We first discuss the basic features of the Las Campanas redshift survey (LCRS) which are relevant to us. We then introduce the concepts of mono-fractals, multi-fractals and the generalized dimension spectrum. For a good discussion on fractals the reader may refer to [2,3]. After this we discuss the intricacies and constraints in the analysis of the survey and the kind of workarounds which are necessary to circumvent these problems. From our analysis the multi-fractal nature of LCRS is then discussed. For a discussion on the applications of fractal concepts to galaxy distributions, the reader may refer to [13]. Lastly we summarize the conclusions.

2. Las Campanas redshift survey

The Las Campanas redshift survey consists of data of angular positions and red-shifts (among other details) of about 24000 galaxies. The survey goes to a depth of about $z = 0.2$ or equivalently to a recession velocity of about 600 km/s. These 24000 galaxies are spread over 6 slices. We treat the galaxies as points. The distribution of points is studied using multi-fractal characterization. In particular, we are interested in investigating as to whether or not, the distribution of points tends to homogeneity.

3. Fractal dimension

Consider a distribution of points in a box. Divide each side of the box into half. Thus the number of smaller boxes will now be 2^d, where d is the dimension of space in which the particles are embedded. We then count the number of non empty boxes. All the 2^d boxes need not necessarily have points in them. We then further divide each of these boxes into halves. This procedure is repeated and at every stage we count the number of non-empty boxes, $N(r)$, where r is the linear size of the boxes. If $N(r)$ scales as a power- law in r in the limit $r \longrightarrow 0$ we call the exponent as the box-counting dimension $D_{\rm box}$

$$D_{\rm box} = -\lim_{r\longrightarrow 0} \frac{{\rm d}\log N(r)}{{\rm d}\log r}. \quad (1)$$

A natural question which arises at this stage is the justification of calling this parameter as 'dimension'. To understand this, consider a smooth curve in, say, 3 dimensions. Adopting the above procedure, we enclose it in a $3D$ box, divide the box into smaller and smaller (and hence, more and more) boxes and count the number of non-empty boxes at every stage. Clearly only the infinitesimally small boxes along the curve will contribute to this number. Hence, in the limit of the size of the boxes tending to zero, the number of non-empty boxes will scale as $N(r) \propto r^1$, thus giving the box-counting dimension to be 1 which is the same as our usual understanding of dimension of a smooth curve. We can, however, envisage a situation, in which, there is a distribution of points, for which, the parameter which we call dimension, turns out to be a fraction. We call such distributions as fractals.

There is, however a problem in using the above definition in practice. In a physically relevant situation, we only have a countable set of points. In such a case, when we take the limit $r \longrightarrow 0$, once the size of the boxes become smaller than the inter-particle separation, the number of non-empty boxes do not change as we reduce the box-size. In such a case, the box-counting dimension turns out to be zero in the limit $r \longrightarrow 0$. Hence, in practice, using the definition in this strict sense is of little value. Operationally we look for a reasonable range of r in which $N(r)$ scales as a power-law in r and call the exponent as the box-counting dimension.

Box-counting dimension is only one of the definitions of fractal dimension. Another measure of the fractal dimension is the correlation dimension. Consider again a distribution of points. Choose a point at random and draw boxes (centered at this randomly chosen point) of increasing size. Count the number of points inside the boxes. We denote the number of points inside a box of linear size r and centered at the ith point by $n_i(r)$. For every value of r, $n_i(r)$ is averaged over all the centers. We denote this average by $\mathcal{N}(r)$.

$$\mathcal{N}(r) = \langle n_i(r)\rangle_i. \quad (2)$$

If $\mathcal{N}(r)$ scales as r^{D_c} we define D_c as the correlation dimension. From the above definitions we can now generalize the concept of dimension. We count the number of points $n_i(r)$ by the procedure sketched above for correlation dimension. The generalized dimension D_q is defined as,

$$D_q = \frac{1}{q-1}\frac{{\rm d}\log(\langle n_i^{q-1}(r)\rangle_i)}{{\rm d}\log(r)}. \quad (3)$$

Clearly, $q = 1$ corresponds to the box-counting dimension and $q = 2$ to correlation dimension. (Since the right hand side of this is ill-defined for $q = 1$ we need to take the limit as $q \longrightarrow 1$) for box-counting dimension. If for a distribution of points, D_q is independent of q, the distribution is a mono-fractal, while otherwise it is a multi-fractal.

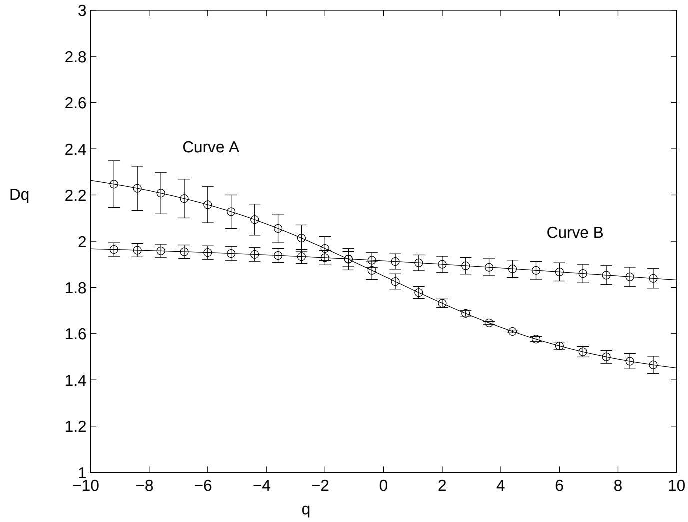

Figure 1. The spectrum of generalized dimension is shown for a subsample of the slice with declination $\delta = -12°$. Curve A refers to small scales (<80 h^{-1} Mpc) and Curve B to large scales (>120h^{-1} Mpc).

If the value of D_q is constant with q and further, if this constant value is also equal to the ambient dimension, the distribution is homogeneous. In the analysis of LCRS, we essentially study the nature of D_q for different ranges of length-scales.

4. Constraints and their workarounds

The survey consists of galaxies in 6 conical slices of declinations $\delta = -3°$, $-6°$, $-12°$, $-39°$, $-42°$ and $-45°$. Of these, the first three are in the north galactic cap while the last three are in the south galactic cap and we have restricted our analysis to the three slices in the north galactic cap. The vertex angle of each cone is $\theta = 90° - \delta$. There are several constraints arising from the geometry and the observational strategy of the survey. To begin with, the slices are very thin ($1.5°$) as compared to a range of about $80°$ in RA. Hence, if we use it for a $3D$ analysis, we cannot study the large scale behaviour. (The largest scales that can be contained within the slice is severely restricted by the slice thickness. Hence, we collapse the slice into a $2D$ conical surface, open the surface and make it flat, analyze the distribution of points (galaxies) on the $2D$ and compute the generalized dimension spectrum, D_q.

Since the circles we draw should be contained within the survey region, the circles cannot be centered on the galaxies near the boundary of the survey. The larger the scales we analyze, the larger the circles and hence, larger are the galaxies excluded from choice.

5. Conclusions

We have analyzed the survey in two ranges in length scales, one below 80 h^{-1} Mpc and the other above 120 h^{-1} Mpc. The averaged quantity $\langle n_i^{q-1}(r)\rangle_i$ shows a scaling behaviour with r. However, the value of D_q remains constant with q over scales larger than 120 h^{-1} Mpc while it is a function of q for scales smaller than 80 h^{-1} Mpc. The behaviour of D_q is shown in figure 1. Further, the q-independent value of D_q for scales larger than 120 h^{-1} Mpc is close to 2, the ambient dimension of the slice. This strongly suggests the following conclusions:

1. That the Universe seems to attain homogeneity over scales bigger than 120 h^{-1} Mpc.
2. That the transition to homogeneity occurs at a scale of around 100 h^{-1}Mpc.

References

[1] P H Coleman and L Pietronero, *Phys. Rep.* **213**, 311 (1992)
[2] K Falconer, in *Fractal geometry*: *mathematical foundations and applications* (John Wiley, 1990)
[3] J Feder in *Fractals* (Plenum Press, 1989)
[4] S Borgani, *Phys. Rep.* **251**, 1 (1995)
[5] V J Martinez and B J T Jones, *Mon. Not. R. Astron. Soc.* (1990)
[6] K S Wu Kelvin, Ofer Lahav and Martin J. Rees, preprint, astro-ph/9804062 (1998)
[7] P J E Peebles, preprint, astro-ph 9806201 (1998)
[8] Guzzo, *New Astronomy* **2**, 517 (1998)
[9] A Cappi, C Benoist, L N Da Costa and S Maurogordato, astro-ph/9804085 (1998)
[10] F Labini Sylos and M Montuori, astro-ph/9711134 (1997)
[11] L Amendola and E Palladino, *Astrophys. J.* In press, astro-ph/9901420 (1999)
[12] S A Shectman, S D Landy, A Oemler, D L Tucker, H Lin, R P Kirshner and P L Schechter, *Astrophys. J.* **470**, 172 (1996)
[13] V J Martinez, in *Applying fractals in astronomy*, edited by A Heck and J M Perdang (Springer-Verlag, 1991) p. 135
[14] S Bharadwaj, A K Gupta and T R Seshadri, *Astron. Astrophys.* (in press) astro-ph/9903252 (1999)