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Large scale structure

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Abstract. We briefly discuss some aspects of the problem of forming large scale structures in the Universe. The basic picture that initially small perturbations generated by inflation grow by the process of gravitational instability to give the observed structures is largely consistent with the observations. The growth of the perturbations depends crucially on the contents of the Universe, and we discuss a few variants of the cold dark matter model. Many of these models are consistent with present observations. Future observations hold the possibility of deciding amongst these models.

Keywords. Cosmology; theory; large scale structure.

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1. Introduction

Our present understanding of the Universe is based on two important assumptions

1. the Universe is homogeneous and isotropic on large scales
2. gravitation governs the dynamics of the Universe on large scales.

The time-evolution of such a Universe can be described by just one function of time – the scale factor $a(t)$ which tells us about the expansion or contraction of the Universe (these being the only two possibilities which preserve assumption (1). The evolution of $a(t)$ is governed by the laws of gravitation eq. (4), and the observation that all the distant galaxies are moving away from us leads us to the conclusion that the Universe is expanding [1]. A large variety of observational evidence supports the picture of an expanding Universe that is very uniform on large scales [2], most important being the cosmic microwave background radiation (CMBR) [3] which is a nearly isotropic black body radiation of temperature $2.726\mathrm{K}$. This radiation is believed to be cosmological in origin, and the high degree of isotropy of the CMBR is an indication that the Universe is indeed very uniform on large scales. In addition, the temperature of the radiation falls as $T_{\mathrm{CMBR}}(t) \propto a^{-1}(t)$ as the Universe expands, indicating that the Universe was hotter in the past when $a(t)$ was smaller. This model (the hot big bang model) has been extremely successful in explaining many features of the observed Universe, of which the accurate predictions of the abundances of the light nuclei and the number of neutrino families are of particular importance. This model is currently accepted as the *standard model* of cosmolgy.

The observed Universe is not uniform and exhibits structures on a variety of scales, starting from galaxies $\sim 50\,\mathrm{Kpc}$, clusters of galaxies $\sim 1\,\mathrm{Mpc}$, voids and superclusters $\sim 20\,\mathrm{Mpc}$ and larger. There is some evidence that the Universe tends to homogeneity on scales $> 100\,\mathrm{Mpc}$ [2,4]. The problem at hand is to understand the formation of these structures within the framework of the standard model of cosmology.

2. Brief introduction to observations

Before going on to the theory of structure formation, we very briefly touch upon a few of the observational aspects of the problem. The most direct method of observing the large scale structures (LSS) in the Universe is through observations of galaxies. The first step in this process is to identify and determine the angular positions of all the galaxies in a certain part of the sky with fluxes within a specified range. The next task is to determine the distances to the galaxies. Direct determination of the distances to galaxies is very difficult and one uses the fact that the radiation from distant galaxies gets stretched to the red due to the expansion of the Universe to determine the distances. When the redshift $z = \Delta\lambda/\lambda_e$ is small it can be interpreted as a Doppler shift and it is related to r, the distance to the emitting galaxy,

$$cz = v = H_0 r, \tag{1}$$

where v is the recession velocity of the galaxy and $H_0 = 100h\,\mathrm{km/s/Mpc}$; $h \sim 0.65$ is the present value of the Hubble parameter [5]. These observations of angular positions and recession velocities ($\simeq$ distance) constitute a *redshift survey*. Figure 1 shows the distribution of galaxies in the Las Campanas redshift survey (LCRS) [6], which is one of the largest surveys currently available.

The next step is to identify and quantify the structures in the distribution of galaxies. The deviations from a uniform distribution (fluctuations) are usually quantified using

$$\Delta_g(\mathbf{x}) = \frac{N_g(\mathbf{x}) - \bar{N}}{\bar{N}}, \tag{2}$$

where $N_g(\mathbf{x})$ is the number density of galaxies at the position $\mathbf{x}$ in the survey and $\bar{N}$ is the mean number density of galaxies. The Fourier transform of $\Delta_g(\mathbf{x})$

$$\delta_g(\mathbf{k}) = \int \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{x}} \Delta_g(\mathbf{x}) \mathrm{d}^3\mathbf{x} \tag{3}$$

is used to define the power spectrum $P_g(k) = \mid \delta_g(\mathbf{k}) \mid^2$ which is very commonly used to characterize the statistical properies of the LSS [7,8]. Figure 2 shows the power spectrum for the LCRS. It also shows the power spectrum for the APM survey [10] which has observations of only the angular positions of the galaxies and does not have any information about the distances.

The quantity $k^3 P(k)$ gives a rough estimate of the fluctuations in the distribution of galaxies at the length-scale k^{-1}.

Using galaxies as tracers to map the LSS has the problem that the relation between the distribution of galaxies and the distribution of matter (which is believed to be mostly

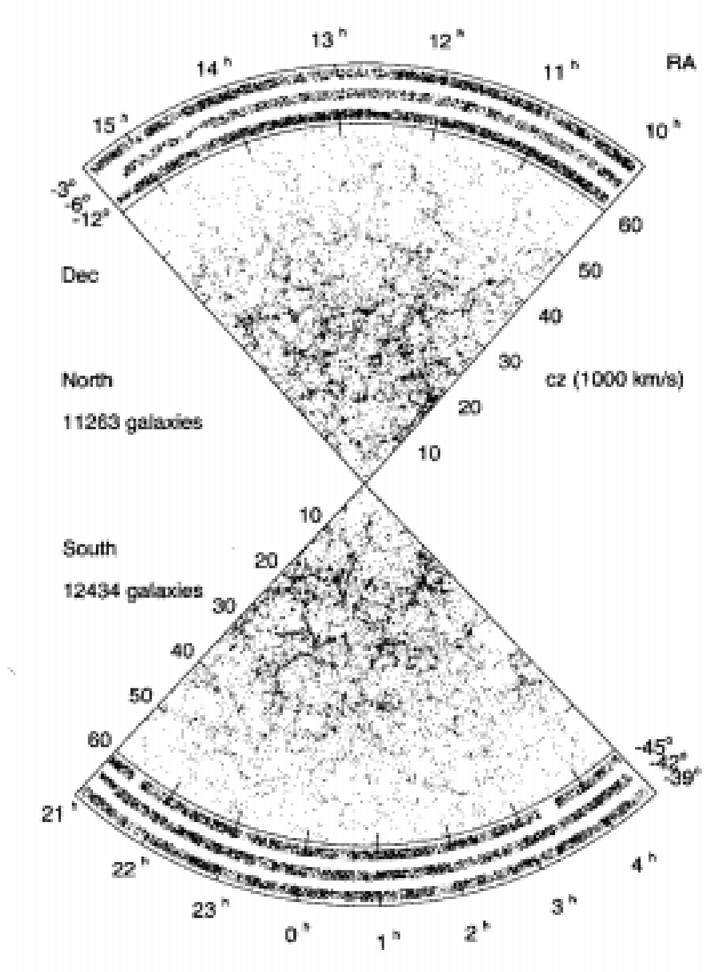

Figure 1. This figure shows the distribution of galaxies in the Las Campanas redshift survey.

invisible i.e. dark matter) is not known. There are reasons to believe that the two distributions might be different and the simplest (not the most realistic)way this may be taken into account is through a linear bias parameter b which relates the fluctuations in the matter distribution $\Delta(\mathbf{x})$ to the fluctuations in the distribution of galaxies i.e. $\Delta(\mathbf{x}) = b\,\Delta_g(\mathbf{x})$. The power spectrum for the matter distribution is then related to the galaxy power spectrum $P_g(k)$ through the relation $P(k) = b^2 P_g(k)$ (e.g. [11]).

Another very important effect arises because the inhomogeneities in the matter distribution effects the motion of the galaxies. As a consequence, the velocity of any galaxy has two parts $\mathbf{v} = \mathbf{v_H} + \mathbf{v_P}$, where $\mathbf{v_H}$ is the part which arises due to the expansion of the Universe eq. (1) and the peculiar velocity $\mathbf{v_P}$ which is due to the inhomogeneities in the matter distribution. This affects the redshifts and hence the distance determination to the

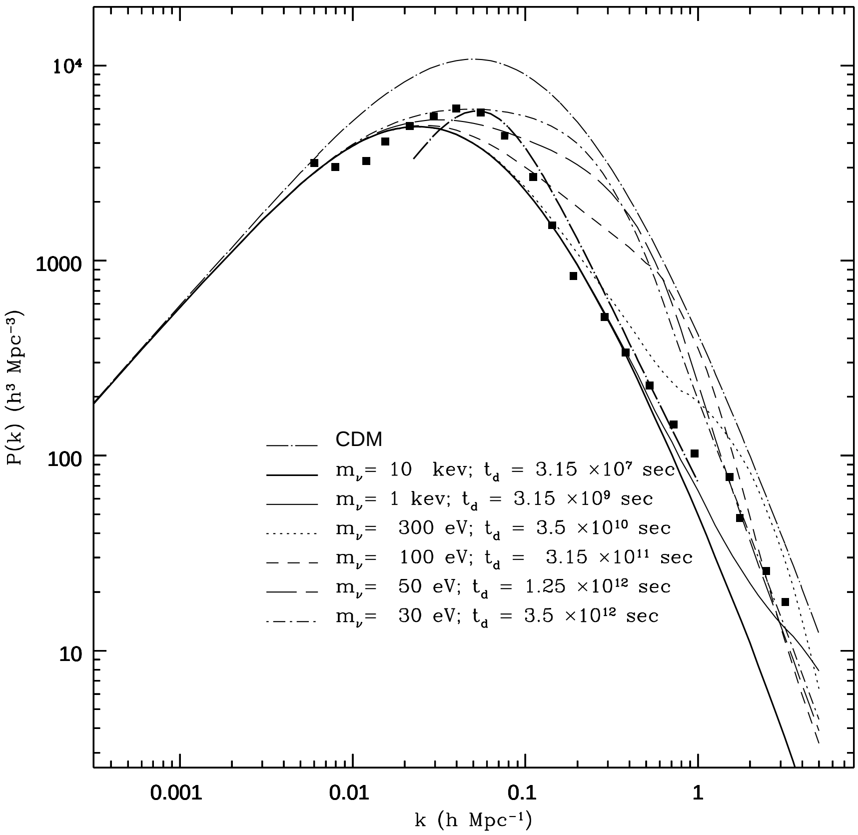

Figure 2. The APM power spectrum (filled squares) and the best fit to the LCRS power spectrum (dashed-dotted thick curve). We assume $b = 1.2$ for both of these. We also use $h = 0.5$. The power spectra for several decaying neutrino models all obtained by varying m and t_d keeping $m_\nu^2 t_d$ a constant and the CDM power spectrum are also shown.

galaxies. The final effect is that the galaxy power spectrum determined from redshift surveys turns out to be anisotropic in its $\mathbf{k}$, whereas one expects the actual matter and galaxy power spectra to be isotropic. This effect has to be taken into account when analyzing redshift surveys, and it can be utilized to extract some very important cosmological information [8,12].

3. Theory

It is currently believed that the observed LSS was formed from some initially small fluctuations $(\Delta(\mathbf{x}, t_i) \ll 1)$ which grew by the process of gravitational instability to produce the structures which are observed now.

3.1 *Initial fluctuations*

A number of possible mechanisms have been proposed to explain how the initial fluctuations were generated, and inflation has been most successful amongst them. Most models of inflation make three generic predictions [13]

1. the initial density fluctuation $\Delta(\mathbf{x}, t_i)$ is a Gaussian random field i.e. the phase $\phi(\mathbf{k}, t_i)$ in $\delta(\mathbf{k}, t_i) = \sqrt{P(\mathbf{k}, t_i)}\mathrm{e}^{i\phi(\mathbf{k}, t_i)}$ is random, with no correlation between the phases at different values of $\mathbf{k}$.
2. The initial power spectrum is of the form $P(k, t_i) = Ak$.
3. The Universe is spatially flat.

3.2 *Time evolution of the fluctuations*

The dynamics of the Universe is governed by the gravitational field produced by the differnet constituents of the Universe. For a uniform Universe this gives us the equation governing the evolution of the scale factor

$$\frac{\mathrm{d}}{\mathrm{d}\eta} a(\eta) = \sqrt{\frac{8\pi G}{3}\rho(\eta) a^4(\eta)} = H_0 \sqrt{\omega(\eta)}\ . \tag{4}$$

where $\eta(t) = \int_0^t (1/a(t'))\,\mathrm{d}t'$ is the conformal time. and $\rho(\eta)$ is the total energy density of the Universe. In general this will be the sum of the contributions from the different constituents of the Universe i.e. $\rho(\eta) = \sum_a \rho_a(\eta)$. We next introduce the critical density $\rho_c(\eta)$ which has the value $\rho_{c0} = 3H_0^2/8\pi G$ at present. It is often convenient to express the present value of the density ρ_{a0} of any particular component a in terms of the critical density using the density parameter $\Omega_{a0} = \rho_{a0}/\rho_{c0}$.

The normalization of the scale factor is chosen so that its present value is 1. The quantity $\omega(\eta) = \rho(\eta) a^4(\eta)/\rho_{c0}$ used in (4) is introduced as a convenient way of parametrizing the time evolution of the density and it has been chosen so that it has the value 1 at present. The behaviour of $\omega_a(\eta)$ will be different for each constituent of the Universe and some are listed below:

a. Relativistic particles – the CMBR, massless neutrinos or neutrinos with low mass $(m_\nu \ll T_{\mathrm{CMBR}})$ are in this group and we have $\omega_r(\eta) = \Omega_{r0}$. This component is sometimes referred to as radiation.

b. Pressureless dust – baryons (not coupled to CMBR), cold dark matter particles (CDM) and non-relativistic massive neutrinos $(m_\nu \gg T_{\mathrm{CMBR}})$ are in this group and $\omega_m(\eta) = a(\eta)\Omega_{m0}$. This component is sometimes referred to as matter.

c. Spatial curvature – the effect of this geometrical quantity can be expressed in terms of a density which can be either negative, zero or positive and $\omega_K(\eta) = a^2(\eta)\Omega_{K0}$.

d. Cosmological constant – this is the energy density of vacuum and $\omega_\Lambda(\eta) = a^4(\eta)\Omega_{\Lambda 0}$.

It follows that in the early stages of the Universe ($a(\eta) \ll 1$) the relativistic particles are the most dominant component and this is referred to as the radiation dominated era. As the Universe expands $a(\eta)$ increases and there comes an epoch when $\omega_r = \omega_m$. At this epoch the density of the relativistic particles and the density of the pressureless dust becomes equal. This is referred to as the epoch of matter radiation equality and the value of the conformal time at this epoch is denoted by η_{eq}. After this the pressureless dust dominates the Universe and this is referred to as the matter dominated era. In the absence of spatial curvature and cosmological constant the Universe continues to remain matter dominated. If there is a positive Ω_{K0} then a spatial curvature dominated era follows the matter dominated era, and if $\Omega_{\Lambda 0} > 0$ then the final state of the Universe is a cosmological constant dominated phase. Figure 3 shows the behaviour of $\omega(\eta)$ in a situation without curvarure and cosmological constant.

We next briefly discuss the dynamics of the evolution of fluctuations in a uniform, expanding Universe. The fluctuations are treated as perturbations to the uniform background solutions and the equations governing the evolution of the perturbations are non-linear and they cannot be solved in general. In the situation when these perrturbations are small ($\Delta(\mathbf{x}, \eta) \ll 1$) it is possible to linearize the equations governing $\Delta(\mathbf{x}, \eta)$ and analytically study the evolution of the perturbations. It should be noted that the perturbations are present only in the radiation and matter densities, the curvature and cosmological

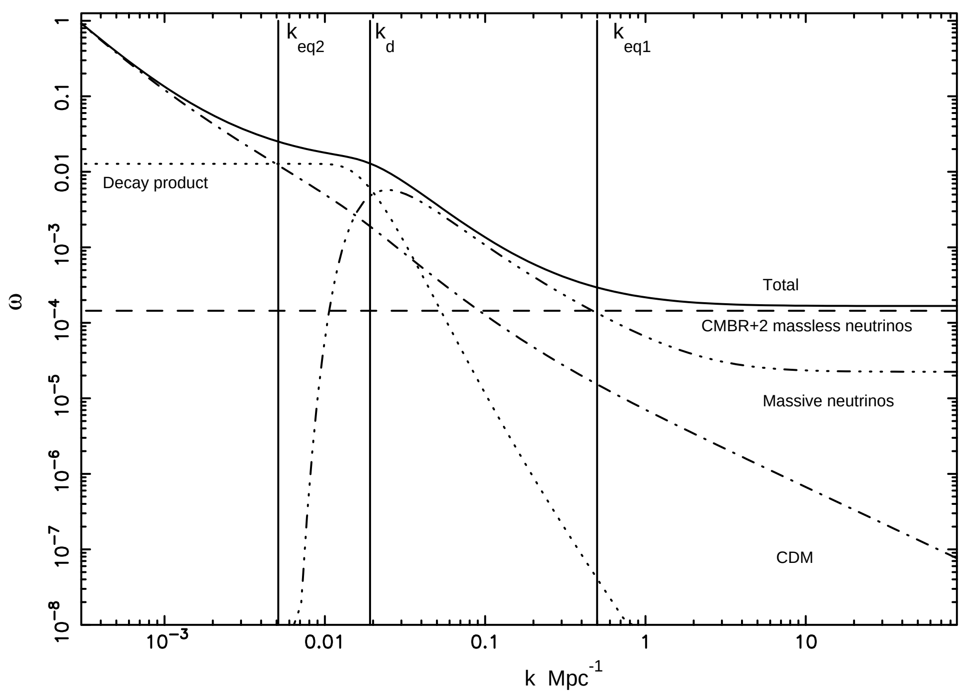

Figure 3. The contribution to $\omega(\eta)$ from the various components is shown as a function of the mode $k = \pi/(c\eta)$ which enters the horizon at the epoch η.

constant are unaffected by the perturbation. It should also be noted that the initial perturbations in the radiation density, matter density and the gravitational field can be related using the fact that inflation generates adiabatic perturbations. We list below the main results of the linear analysis of the matter density perturbations [14]:

1. In the radiation dominated era perturbations $\delta(\mathbf{k},\eta)$ with $k < 1/c\eta$ grow as the Universe expands. The perturbations stop growing once $k > 1/c\eta$. The length-scale $c\eta$ defines the size of the horizon and this keeps on increasing as the Universe expands. Perturbations grow as long as they are larger than the horizon and they stop growing once they enter the horizon.
2. In the matter dominated era perturbations on all length-scales grow in the same way.

The net effect is that the shape of the power spectrum which is initially generated by inflation gets modified as it evolves through the radiation dominated era and the final linear power spectrum can be expressed as

$$P(k) = AkT(k) \tag{5}$$

where $T(k)$ is the transfer function. The shape of the transfer function depends on what are the constituents of the Universe.

We first consider a Universe with only two kinds of constituents 1. relativistic particles and 2. pressureless dust. The radiation make a constant contribution $\omega_r(\eta) = \Omega_{r0}$ while the contribution from matter $\omega_m(\eta) = a(\eta)\Omega_{m0}$ increases as the Universe expands, and the Universe proceeds from a radiation dominated era to a matter dominated era at the epoch

$$\eta_{\rm eq} = 2(\sqrt{2}-1)\frac{\sqrt{\Omega_{r0}}}{\Omega_{m0}H_0}\,. \tag{6}$$

In the radiation dominated era the Jeans length is of the order of the horizon ($\sim c\eta$), and perturbations in all components grow when they are on scales larger than the horizon (i.e. they satisfy $kc\eta \ll 1$). The growth stops if the modes enter the horizon (i.e. $k \sim \pi/(c\eta)$) in the radiation dominated era. Once the Universe gets matter dominated, matter perturbations on all scales grow in the same way $(\delta(k,\eta) \propto \eta^2)$. Using $k_{\rm eq} = \pi/(c\eta_{\rm eq})$ to denote the mode which enters the horizon at the epoch of matter-radiation equality, we can say that all modes with $k < k_{\rm eq}$ entered the horizon in the matter dominated era and they all grow by the same factor during the course of the evolution. The modes with $k > k_{\rm eq}$ enter the horizon in the radiation dominated era and as a consequence they experience a period of stagnation when they do not grow. The amplitude of these perturbations is suppressed by the factor $(k_{\rm eq}/k)^2$ relative to the modes which enter in the matter dominated era. This fact can be used to crudely model the transfer function using the simple form $T(k) = 1$ for $k < k_{\rm eq}$ and $T(k) = (k_{\rm eq}/k)^4$ for $k \geq k_{\rm eq}$, and it depends on just one quantity—the mode which enters the horizon at the epoch of matter radiation equality

$$k_{\rm eq} = \frac{\pi}{2(\sqrt{2}-1)}\frac{\Omega_{m0}}{\sqrt{\Omega_{r0}}}\frac{H_0}{c}\,. \tag{7}$$

Figure 4 shows the sCDM transfer function calculated by numerically integrating the linear perturbation equations. The numerically calculated transfer function can be fitted by using a function of the form [15]

$$T(k) = \left[1 + (ak + (bk)^{3/2} + (ck)^2)^\nu\right]^{-2/\nu}, \tag{8}$$

where $a = (6.4/\Gamma)h^{-1}\,\mathrm{Mpc}$, $b = (3/\Gamma)h^{-1}\,\mathrm{Mpc}$, $c = (1.7/\Gamma)h^{-1}\,\mathrm{Mpc}$, and $\Gamma = 5\ k_{\mathrm{eq}}h^{-1}\,\mathrm{Mpc}$ is the dimensionless shape parameter. The shape parameter Γ can be expressed directly in terms of the densities as

$$\Gamma = \Omega_{m0}h \times \left(\frac{\Omega_{r0}h^2}{4.18 \times 10^{-5}}\right)^{-1/2}. \tag{9}$$

It should be noted that here, and in the rest of the paper, we ignore the dynamical effect of the baryon density Ω_{B0}.

3.3 $h = 0.5$ *sCDM model and variants*

In the standard CDM model Ω_{r0} $(= 4.18 \times 10^{-5}\mathrm{h}^{-2})$ has contributions from the CMBR photons and 3 massless neutrino species, and the matter, which is largely made up of CDM particles has $\Omega_{\mathrm{m0}} = \Omega_{\mathrm{CDM0}} = 1$. This gives us $k_{\mathrm{eq}} = 0.2\mathrm{h}^2\mathrm{Mpc}^{-1}$ and $\Gamma = 0.5$. The predictions of the sCDM model can be compared with the observed power spectrum. The normalization of the sCDM power spectrum is fixed using the large-angular scale

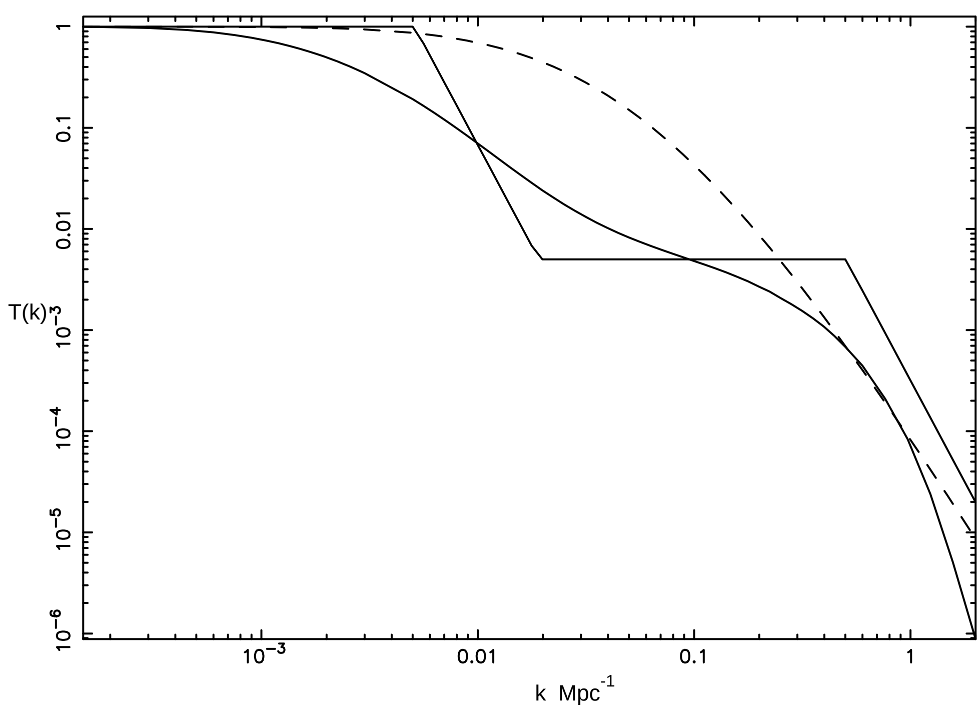

Figure 4. The dashed curve shows the transfer function for the $h = 0.5$ CDM model. The solid curves show the transfer function for a decaying neutrino model with $h = 0.5$, $m_\nu = 200\,\mathrm{eV}$ and $t_d = 10^{13}\,\mathrm{s}$.

anisotropies in the CMBR observed by COBE-DMR [16,17]. The relative biasing between the theoretically predicted matter power spectrum and the observed galaxy power spectrum is unknown, hence it is only possible to compare the shapes of the two. A comparison of the shapes shows that the sCDM power spectrum does not match the observed power spectrum [18]. This is also seen in figure 2. While the standard CDM model predicts a shape parameter $\Gamma = 0.5$, observations indicate a CDM-like power spectrum with $0.22 \leq \Gamma \leq 0.29$ [9,10] and [11].

The r.m.s. mass fluctuation in randomly placed spheres of radius $8\mathrm{h}^{-1}\,\mathrm{Mpc}$ (denoted by σ_8) provides a sensitive probe of the matter power spectrum at scales around $k = 0.2\mathrm{h}\,\mathrm{Mpc}^{-1}$, and various studies show that the observed abundance of rich clusters of galaxies at the present epoch is consistent with $\sigma_8 \sim 0.5$–0.8 [19], whereas the standard CDM model normalized to the four-year COBE data predicts $\sigma_8 = 1.22$ [17].

All of this evidence indicates that the matter power spectrum predicted by the sCDM model has too much power at small scales and several variants of the standard CDM model have been proposed to overcome this shortcoming. We next briefly discuss two variants of the CDM model which have been successful in overcoming the shortcomings of the sCDM model. In both these models the epoch of matter-radiation equality is delayed in comparison with sCDM and as a consequence Γ is smaller. The delayed onset of the matter dominated era causes a reduction in power at small scales which experience a longer period of stagnation and this solves the problems faced by the sCDM model.

There are essentially two ways to decrease Γ: (1) by decreasing the amount of matter which can clump at small scales (i.e., decreasing Ω_{m0}) or (2) by increasing the radiation content of the Universe (i.e., increasing Ω_{r0}).

In the $\Lambda\mathrm{CDM}$ model [20] the cosmological constant makes a non-zero contribution to the energy density and $\Omega_{\Lambda 0} = 0.7$ and $\Omega_{m0} = 0.3$ are the values most favoured by current observations [21]. There two main features of this model are (a) Γ decreases because of the lower value of Ω_{m0} (b) The normalization of the power spectrum is different from sCDM because of the integrated Sachs–Wolfe effect.

In the decaying neutrino model ($\tau\mathrm{CDM}$) in addition to the CDM and 2 massless neutrinos there is a massive neutrino which decays to produce relativistic particles. For this decay to be effective in reducing Γ the neutrino mass m_ν and the life-time t_d have to be such that the Universe passes through a phase where the massive neutrino behaves like pressureless dust and causes a matter dominated era before it decays. Once the neutrinos decay the relativistic decay products dominate the Universe until it passes on to the final CDM dominated era. The evolution of the different components is shown in detail in figure 2. It should be noted that the relativistic decay products cannot be photons. This model decreases Γ by increasing Ω_{r0} [22] and it is possible to have models with the neutrino mass in the range $50\,\mathrm{eV} \leq m_\nu \leq 10\,\mathrm{keV}$ provided the lifetime t_d is adjusted so as to keep the quantity $m_\nu^2(\mathrm{keV})t_d(\mathrm{yr})$ constant at a value around 100. Decaying neutrino models with masses in the range $30\,\mathrm{eV} \leq m_\nu < 50\,\mathrm{eV}$ are ruled out. Figure 2 shows the power spectrum for some values of m_ν and t_d.

4. Summary and discussion

We have very briefly discussed the formation of large scale structures in a scenario where these arise from some initially small perturbations generated by inflation. Variants of the cold dark matter model succesfully reproduce the observed large angular scale anisotropies in the CMBR and the clustering of galaxies on large scales. Both of these phenomena involves large scales where the linear theory of density perturbations which is well understood is valid. Future LSS and CMBR observations are expected to be able to discriminate between the different models of structure formation and possibly single out the correct one [23].

The linear theory of density perturbations cannot be applied at small scales (< 10 Mpc) where the density fluctuations are very large. Most studies at small scales are based on N-body simulations where the evolution of perturbations is studied numerically on a computer. These studies have gone a long way towards giving a better understanding of the formation of galaxies and clusters of galaxies and bias which is very crucial for interpreting observations of galaxy clustering at large scales.

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