Topology of the Universe: Background and recent observational approaches

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Abstract. Is the Universe (a spatial section thereof) finite or infinite? Knowing the global geometry of a Friedmann–Lemaître (FL) universe requires knowing both its curvature and its topology. A flat or hyperbolic ('open') FL universe is *not* necessarily infinite in volume.

Multiply connected flat and hyperbolic models are, in general, as consistent with present observations on scales of 1–20 h^{-1}Gpc as are the corresponding simply connected flat and hyperbolic models. The methods of detecting multiply connected models (MCM's) are presently in their pioneering phase of development and the optimal observationally realistic strategy is probably yet to be calculated. Constraints against MCM's on ∼1–4 h^{-1}Gpc scales have been claimed, but relate more to inconsistent assumptions on perturbation statistics rather than just to topology. Candidate 3-manifolds based on hypothesised multiply imaged objects are being offered for observational refutation.

The theoretical and observational sides of this rapidly developing subject have yet to make any serious contact, but the prospects of a significant detection in the coming decade may well propel the two together.

1. Cosmic topology

This workshop is on observational cosmology: how observations confront cosmological theory. Unfortunately, one of the fundamental aspects of Friedmann–Lemaître models of the Universe is weak in theoretical predictions. General relativity says nothing about how big the Universe should be. It describes curvature, which divides up constant curvature 3-manifolds ('spaces') into three classes corresponding to the three possible signs of curvature.

For example, a canonical flat multiply connected model is the hypertorus, T^3, which can be thought of as a cube whose opposite faces are identified. This is a flat 3-manifold without any edges or boundaries, but finite in volume. A T^3 universe may be as small as 1 h^{-1}Gpc or as big as the horizon for the same values of Ω_0, λ_0, Ω_b, σ_8 and H_0 [1]. Evolution in the luminosity functions of galaxies and quasars, the star formation rate history of the Universe, and similar observational quantities do not distinguish between the different

models. They do not constrain the size of the Universe. Although the 'curvature radius' and H_0 have strong effects on the size of the *observable* Universe, i.e. on the horizon radius, they only have weak effects on the size of the Universe itself.

How can the theory (that spatial sections are 3-manifolds) be confronted with observations? In short, by photons travelling many times across the Universe so that multiple topological images are seen of single objects. In a multiply connected universe, objects (or regions of CMB plasma) would be seen several times in different directions and (in general) at different redshifts. This would be something like gravitational lensing, except that the whole Universe would be the lens and the angular and radial distance differences in multiply imaged objects would be, in general, big fractions of π and of the horizon radius respectively, as opposed to arcsecond and sub-parsec differences in the case of gravitational lensing.

2. Recommended reading

Recent reviews of the different observational strategies include [2,3] (the latter also includes a brief historical and mathematical background).

A fuller review including theoretical aspects of cosmic topology and pre-1993 observational work is that of [4], but due to exponential growth in the subject, the number of published articles on the subject has roughly doubled since then.

Proceedings of the 1997 Cleveland and 1998 Paris workshops on cosmic topology are available as [5] and [6] respectively.

Mathematical tools, particularly including a 'census' of a few thousand small compact hyperbolic 3-manifolds are available at [7].

3. A survival kit for the observer: jargon

The minimum concepts and jargon that the workshop participant or reader should retain from the above literature are probably:

  (i) *'Compact'* essentially means finite in spatial volume.
 (ii) To avoid confusion, the word 'open' is dropped in favour of *'hyperbolic', 'negatively curved', '$\Omega_0 < 1$'* or *'$k < 0$'*; and 'closed' is dropped in favour of *'elliptic', 'spherical', 'positively curved', '$\Omega_0 > 1$'* or *'$k > 0$'* (otherwise, compact hyperbolic models would be referred to as closed open models ...).
(iii) *'Geodesic'* generally means a geodesic in 3-space, but at times is used to mean a geodesic in $3 + 1$ space-time.
(iv) The entire (*comoving* spatial section of the) Universe can be represented as a polyhedron embedded in H^3, R^3 or S^3 (for $k < 0, = 0, > 0$ respectively) of which faces are identified with one another in some way — this is the *'fundamental polyhedron'* or *'Dirichlet domain'*.
 (v) By pasting together copies of the fundamental domain, an space H^3, R^3 or S^3 (respectively) can be constructed which corresponds, for the observer, to the apparent space in which objects at high redshift are located under the hypothesis of trivial topology [9] — this is termed the *'universal covering space'*, $\widetilde{M}$.

(vi) In the covering space, the isometries mapping multiple *'topological images'* (or *'topological clones'*) to one another form a group, Γ, whose elements are linear combinations of a set of *'generators'*.

(vii) The 3-manifold can formally be written as $M = \widetilde{M}/\Gamma$.

(viii) For convenience, one often swaps thinking and calculating between the fundamental polyhedron and the covering space.

4. An example of a candidate 3-manifold

In the commonly studied case of the rectilinear toroidal models, multiple topological images of an object form a rectilinear grid in comoving space. Among a small selection of the brightest known galaxy clusters, three form a right angle of equal arm lengths to within 2–3° and 1% accuracy respectively [10].

Are the Coma cluster, RX J1347.5–1145 and CL 09104+4109 three images of a single cluster or is the right angle just a coincidence?

A list of arguments for and against this T^2 candidate is provided in the discussion section of [11], and a comparison with COBE data is presented in [12].

5. Projects

This is a workshop. The following are ideas suggested for projects.

5.1 *Theory*

(i) What should the topology of the Universe be? Can a theory of quantum gravity or of quantum cosmology make any serious predictions about what the topology of the Universe should be at $t \sim t_0$?

(ii) A group Γ relates the covering space $\widetilde{M}$ of a multiply connected universe to the fundamental polyhedron $M = \widetilde{m}/\Gamma$. The standard model of particle physics relates different particles to one another by a group, e.g. $\mathrm{SU}(2)_L \times \mathrm{U}(1)_{YW} \times \mathrm{SU}(3)^C$. Could the Universe be considered a particle at the quantum epoch and the spatial transformations of Γ be related to the gauge bosons?

5.2 *Observation*

Methods:

(iii) The classical magnitude-redshift relation yielded only weak constraints on the curvature parameters (Ω_0, λ_0) until an empirical way of improving supernovae of type Ia as standard candles was devised. The results are impressive, even though theoretical understanding of the method of sharpening the standard candle is weak [13,14]. Could some sort of similar 'trick' improve the presently published methods to the point of extracting a significant topological detection?

(iv) Realistic simulations including all the observational difficulties could be used to opti-mise the cosmic crystallography [15–17] and local isometry search methods [18,17].

(v) Realistic simulations and analysis should also be used to find the best way to apply the matched circles principle [19–21,12].

Candidates:

(vi) Generate specific candidates.

(vii) Observationally refute these in order to understand systematic errors.

New catalogues:

(viii) radio: GMRT — $5 \lesssim z \lesssim 10$? proto-clusters

(ix) mm/sub-mm: MMA/LSA — $5 \lesssim z \lesssim 10$? galaxies

(x) cm: MAP, Planck —$z \approx 1100$ or . . . (integrated Sachs–Wolfe effect) $z \ll 1100$ CMB (plasma); $z \sim 1$–3 clusters (SZ effect)

(xi) optical: SDSS, VLT — $z \sim 1$–3? quasars, galaxies

(xii) Xray: XMM — $z \sim 1$–2 clusters, quasars

Local (10 kpc $-$ 100 Mpc):

(xiii) Understand the galaxy (or the local unit of large scale structure) well enough to say what its topological image must have looked like at $z \sim 2$–5 and from an 'arbitrary' angle. This would be a 'safe' theme for a thesis project, since the theoretical and/or observational work done in understanding the galaxy would be valid independently of its use in identifying or refuting high redshift topological images of the galaxy.

References

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