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The breaking of O(6) symmetry in ^{118}Xe and ^{120}Xe

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Abstract. The spectra of the isotopes of xenon are analysed from the point of view of O(6) symmetry breaking. It is pointed out that the excitation energies of the states 0_2^+ and 0_3^+ can be used in detecting breaking of the symmetry. The nature of symmetry breaking in ^{118}Xe and ^{120}Xe is indicated.

Keywords. IBM; O(6); dynamical symmetry breaking.

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1. Introduction

The isotopes of xenon and barium were long known to correspond to the O(6) dynamical symmetry of IBM [1]. However, several recent investigations, both theoretical and experimental, indicate that the issue is far from settled [2–5]. The experimental study in [2] clearly questions the conventional O(6) dynamical description of ^{134}Ba. The strong fragmentation of the low energy octupole strength observed in ^{196,198}Pt and considered to be a signature of O(6), does not occur in ^{134}Ba. In [4] the study of the evolution of nuclear structure in the xenon–barium region has been taken up. It has been reported [4] that the barium isotopes correspond to dynamical symmetry intermediate between U(5) and SU(3) while the xenon isotopes fit more a U(5)–O(6) description. In this paper we discuss the breaking of O(6) symmetry in Xe isotopes. It is found that the neutron deficient isotopes ^{118,120}Xe deviate from an O(6) description. It is pointed out that the relative energies of the O^+ states $(0_1^+, 0_2^+, 0_3^+)$ may serve to determine the nature of the symmetry of the nucleus. This criterion, together with $B(E2)$ values may be used to suggest the symmetries of the ^{118}Xe and ^{120}Xe isotopes.

2. Fitting of the energy spectra

The experimental data [9] of the isotopes $^{118-130}$Xe were fitted to the O(6) expression [1]:

Table 1. The experimental and fitted values of energies in keV for the xenon isotopes $^{118-130}$Xe.

Levels	^{118}Xe		^{120}Xe		^{122}Xe		^{124}Xe		^{126}Xe		^{128}Xe		^{130}Xe	
	Expt.	Fit	Expt.	Fit	Expt.	Fit	Expt.	Fit	Expt.	Fit	Expt.	Fit	Expt.	Fit
0_1^+	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2_1^+	337.32	272.96	322.40	268.92	331.18	277.42	354.02	301.72	388.02	302.84	442.91	331.76	536.08	368.58
4_1^+	810.27	735.70	795.80	722.60	828.30	737.30	879.17	799.80	941.97	800.50	1033.15	865.00	1204.61	960.10
0_2^+	830.36	940.50	908.40	938.52	1149.30	1012.14	1268.73	1112.04	1313.66	1128.42	1582.97	1300.68	1793.54	1449.30
2_2^+	928.10	586.46	875.80	581.76	843.08	614.80	846.88	662.40	879.88	678.98	969.47	765.32	1122.15	851.88
2_3^+	1228.31	1526.96	1274.10	1520.28	1494.88	1626.94	1628.38	1784.44	1678.55	1807.04	1999.64	2066.00	2150.21	2301.78
3_1^+	1366.18	1068.42	1271.50	1059.24	1214.04	1117.14	1248.07	1221.24	1317.66	1232.58	1429.56	1386.12	1632.52	1542.66
6_1^+	1396.81	1388.22	1397.10	1361.04	1466.57	1379.64	1548.71	1494.24	1635.04	1492.98	1737.04	1599.72	1944.09	1681.80
4_2^+	1441.16	1153.70	1401.00	1139.72	1402.50	1187.14	1438.30	1294.04	1488.49	1302.02	1603.41	1443.08	1808.18	1604.50
2_4^+	1640.34	1719.96	–	–	1716.30	1722.42	1978.50	1985.17	2064.00	2064.44	2272.85	2228.76	2296.10	2340.78
0_3^+	1721.20	1447.00	1623.20	1623.60	–	–	1689.90	1683.45	1760.52	1761.60	1877.32	1897.00	2017.09	1972.20
2_5^+	1838.23	2033.46	–	–	2065.40	2059.80	–	–	–	–	2663.00	2933.12	–	–
4_3^+	1701.63	1676.20	1712.60	1661.12	–	–	–	–	1902.80	1928.92	2023.00	2165.68	2081.96	2410.00
5_1^+	1922.12	1782.80	1816.70	1761.72	1774.38	1836.94	1837.40	2002.84	1903.66	2015.72	1966.55	2236.88	2362.08	2487.30
6_2^+	1997.00	1910.72	1985.50	1882.44	2056.47	1941.94	2144.46	2112.04	1867.16	2119.86	2280.90	2322.32	–	–
8_1^+	2073.40	2230.52	2098.90	2184.24	2217.30	2204.44	2331.40	2385.04	–	–	2512.54	2535.92	2696.88	2811.96
7_1^+	2559.80	2686.96	2460.40	2648.96	2459.00	2739.20	–	–	–	–	–	–	–	–

$$E = E_0 - \frac{A}{4}\sigma(\sigma+4) + B\tau(\tau+3) + CJ(J+1), \qquad (1)$$

where $E_0 = (A/4)N_{\rm B}(N_{\rm B}+4), N_B$ being boson number and A, B, C are parameters. The quality of the fit can be judged from the factor $Q/E_{2_1^+}$ where

$$Q = \frac{1}{N}\sqrt{\sum_i (E^i_{\rm fit} - E^i_{\rm exp})^2}\ ,$$

where N = number of excitation energies fitted and $E_{2_1^+}$ is the energy of the lowest 2^+ state. The experimental and fitted values of energies for the xenon isotopes $^{118-130}$Xe are given in table 1. The values of A, B, C, Q and $Q/E_{2_1^+}$ are given in table 2. It appears that the best fit occurs for ^{124}Xe and the quality deteriorates as the neutron number decreases. In addition to the energy fits, the $B(E2)$ values give valuable information regarding the nature of dynamical symmetry of a nucleus. Although some experimental information regarding the $B(E2)$ values of the Xe isotopes are available, this is not enough to distinguish between various symmetries. For example, with a suitable choice of parameters, the energies of the levels with $\sigma = N_{\rm B}$ in O(6) and $N_d = \tau$ in U(5) can be made identical [6,7]. The predicted ratios of $B(E2)$ values of observed transitions are also the same. Therefore, more experimental information on the states with $\sigma < N_{\rm B}$ are needed before one can decide without ambiguity if a nucleus obeys O(6) dynamical symmetry.

In the isotopes of Xe some experimental data on the energies for bands $\sigma < N_{\rm B}$ are available. Of these, the energies of the 0^+ states are significant because the lowest two correspond to $\sigma = N_{\rm B}$ while the third 0^+ state has $\sigma = N_{\rm B} - 2$. The assignment of the quantum numbers and the energies as predicted by eq. (1) are as follows:

State	σ	τ	L	E
0_1^+	$N_{\rm B}$	0	0	0
0_2^+	$N_{\rm B}$	3	0	$18B$
0_3^+	$N_{\rm B} - 2$	0	0	$A(N_{\rm B}+1)$

Thus

$$\frac{E_{0_3^+}}{E_{0_2^+}} = \frac{A}{18B}(N_{\rm B}+1). \qquad (2)$$

Table 2. The parameters A, B, C, Q (eq. (1)) and $Q/E_{2_1^+}$ for Xe isotopes.

Nucleus	$E_{2_1^+}$	A	B	C	Q	$Q/E_{2_1^+}$
^{130}Xe	536.08	328.7	80.55	7.73	55.51	0.103
^{128}Xe	442.91	271.0	72.26	7.12	37.22	0.084
^{126}Xe	388.63	220.2	62.62	8.68	36.28	0.093
^{124}Xe	354.02	187.0	61.78	9.10	29.22	0.083
^{122}Xe	331.18	144.5	56.23	8.75	36.32	0.110
^{120}Xe	322.40	147.6	52.14	10.06	43.97	0.136
^{118}Xe	337.32	144.7	52.25	10.66	47.89	0.142

Table 3. Variation of $E_{0_3^+}/E_{0_2^+}$ as a function of boson number $N_{\rm B}$ for xenon isotopes.

Nucleus	$N_{\rm B}$	$E_{0_2^+}$	$E_{0_3^+}$	$E_{0_3^+}/E_{0_2^+}$	$(E_{0_3^+}/E_{0_2^+})/(A/18B)$
^{130}Xe	5	1793.54	2017.09	1.125	4.96
^{128}Xe	6	1582.94	1877.32	1.186	5.69
^{126}Xe	7	1313.66	1760.52	1.340	6.86
^{124}Xe	8	1268.73	1689.90	1.332	7.92
^{122}Xe	9	1149.30	–	–	–
^{120}Xe	10	908.40	1623.20	1.787	11.36
^{118}Xe	9	830.36	1721.20	2.073	13.47

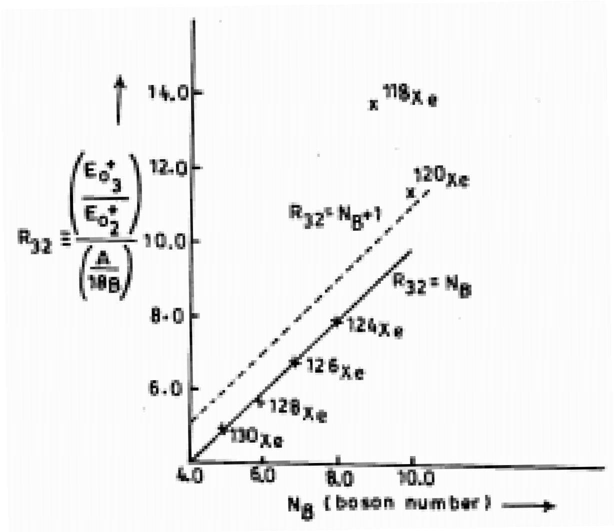

Figure 1. A plot of $(E_{0_3^+}/E_{0_2^+})/(A/18B)$ as a function of boson number.

The experimental values of the ratio $E_{0_3^+}/E_{0_2^+}$ for various isotopes of xenon are shown in table 3.

In figure 1 the ratio $(E_{0_3^+}/E_{0_2^+})/(A/18B)$ is plotted against $N_{\rm B}$. The dotted line shows the expected behaviour.

3. Discussion

From a study of table 3 and figure 1 the following facts are noted:

1. For the isotopes $^{124-130}$Xe the ratio $(E_{0_3^+}/E_{0_2^+})/(A/18B) \cong N_{\rm B}$ and not $(N_{\rm B}+1)$ as is expected from (2). This indicates a systematic lowering of the $\sigma = N_{\rm B} - 2$ band which may be signature of γ-softness.

2. For the isotopes ^{120}Xe and ^{118}Xe there is a departure from this trend. In particular, for ^{118}Xe this ratio is much larger than expected. This indicates that these isotopes, especially ^{118}Xe do not fit the O(6) behaviour.

If one assumes subshell closure at $N = 64$, the boson numbers corresponding to the isotopes change. But this makes the situation even worse. The boson numbers for ^{130}Xe and ^{128}Xe remain unaltered but those for ^{124}Xe, ^{122}Xe, ^{120}Xe and ^{118}Xe decrease drastically making the agreement between $E_{0_3^+}/E_{0_2^+}$ versus N_{B} plot worse.

For comparison we calculate the ratio $E_{0_3^+}/E_{0_2^+}$ in the other two dynamical symmetries: U(5) and SU(3). In U(5) the $0_1^+, 0_2^+$ and 0_3^+ states correspond to $(n_d = v = n_\Delta = 0)$, $(n_d = 2,\ v = n_d - 2 = 0,\ n_\Delta = 0)$ and $(n_d = 3,\ v = 3,\ n_\Delta = 0)$ respectively. Assuming that the energies are given by

$$E = E_0 + \varepsilon n_d + \alpha n_d(n_d + 4) + 2\beta v(v + 3) + 2\gamma L(L + 1), \tag{3}$$

$$\frac{E_{0_3^+}}{E_{0_2^+}} = \frac{\frac{3}{2} + \frac{21}{2}\left(\frac{\alpha}{\varepsilon}\right) + 18\left(\frac{\beta}{\varepsilon}\right)}{1 + 12\left(\frac{\alpha}{\varepsilon}\right)}. \tag{4}$$

Since α and $\beta << \varepsilon$ this ratio is $\sim$ 1.3. In the SU(3) limit the energies are given by

$$E = E_0 + 2\gamma L(L + 1) + \frac{2}{3}\delta(\lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu). \tag{5}$$

The quantum numbers (λ, μ) associated with 0_1^+, 0_2^+ and 0_3^+ are $(2N_{\mathrm{B}}, 0)$, $(2N_{\mathrm{B}} - 4, 2)$ and $(2N_{\mathrm{B}} - 8, 4)$ respectively.

Therefore

$$\frac{E_{0_3^+}}{E_{0_2^+}} = \frac{2 - \frac{3}{N_{\mathrm{B}}}}{1 - \frac{1}{2N_{\mathrm{B}}}}. \tag{6}$$

For ^{120}Xe $(N_B = 10)$ this ratio turns out to be 1.73 which is also the experimental value, but for ^{118}Xe $(N_{\mathrm{B}} = 9)$, the predicted value is 1.71 as compared to the experimental value 2.07.

In addition to energy, the $B(E2)$ values also give significant clues regarding the dynamical symmetry of a nucleus. Defining

$$R_1 = \frac{B(E2;\ 4_1 \to 2_1)}{B(E2;\ 2_1 \to 0_1)}, \tag{6a}$$

$$R_2 = \frac{B(E2;\ 6_1 \to 4_1)}{B(E2;\ 2_1 \to 0_1)}, \tag{6b}$$

$$R_3 = \frac{B(E2;\ 8_1 \to 6_1)}{B(E2;\ 2_1 \to 0_1)}, \tag{6c}$$

Table 4. The $B(E2)$ ratios R_1, R_2, R_3 for ^{118}Xe and ^{120}Xe.

Nucleus	Ratio	U(5)	SU(3)	O(6)	Expt.
^{118}Xe	R_1	1.78	1.39	1.37	1.11
	R_2	2.38	1.46	1.50	0.88
	R_3	2.67	1.41	1.49	0.49
^{120}Xe	R_1	1.8	1.40	1.38	1.77
	R_2	2.5	1.48	1.52	1.85
	R_3	2.8	1.45	1.55	$>$1.28

we summarise in table 4 the experimental and predicted values of R_1, R_2, R_3 for the nuclei ^{118}Xe, ^{120}Xe for various symmetries.

It is evident from the data that in case of ^{118}Xe the predicted values of the ratios R_1, R_2, R_3 in the limit U(5) are much higher than the experimental values. The predicted SU(3) and O(6) values are quite close to each other though they are also somewhat higher than the experimental values. Also the energy ratio $E_{0_3^+}/E_{0_2^+} = 2.07$ is close to the SU(3) predicted value 1.71. It is possible that this nucleus has a symmetry intermediate between SU(3) and O(6).

In the case of ^{120}Xe, the experimental $B(E2)$ values are closer to but somewhat less than U(5) values. The energy ratio $E_{0_3^+}/E_{0_2^+} = 1.79$ fits exactly the value predicted by SU(3). Although the ratio $(E_{0_3^+}/E_{0_2^+})/(A/18B)(= 11.43)$ is also close to the O(6) predicted value $(= 11)$, there is departure from the systematic downward trend of the ratio as compared to the O(6) value (see figure 1). The nucleus ^{120}Xe may possibly be described by a hamiltonian which is intermediate between SU(3) and U(5) dynamical symmetry.

4. Conclusion

We have analysed the spectra of Xe isotopes for signatures of O(6) symmetry breaking. It is pointed out that the energy ratio $E_{0_3^+}/E_{0_2^+}$ can be an useful indicator for this purpose. Two main conclusions follow from the analysis:

1. The O(6) symmetry breaks down for the neutron deficient isotopes ^{120}Xe and ^{118}Xe. In the absence of experimental $B(E2)$ values for transitions between bands of different σ, the nature of symmetry breaking can only be speculated upon from $B(E2)$ data within the $\sigma = N_{\mathrm{B}}$ band.
2. It is confirmed that the isotopes ^{130}Xe, ^{128}Xe, ^{126}Xe and ^{124}Xe obey O(6) symmetry as pointed out by Casten and von Brentano [1]. But the surprising new element is that the boson numbers to be associated with these isotopes are found to be one less than expected. The effect of this changed boson number on the $B(E2)$ values is rather small.

The last point calls for a re-examination of the conventional counting of boson numbers in nuclei. It raises the interesting possibility of defining an effective boson number which differs from the conventional boson number.

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