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An alternative approach to determine the spot-size of a multi-mode laser beam and its application to diode laser beams

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Abstract. An alternative approach is suggested to determine the spot-size of a multi-mode laser beam. It has been shown by simulations that the suggested approach can give the beam quality factor and characteristic radius with less than 5% error. Unlike the power content method, the proposed method is applicable to the beams even with diameter one tenth of the CCD size. The new approach has been applied to a multi-mode diode laser output and it is shown that the $ABCD$ matrix analysis can be used for beam propagation, with the measured parameters of the laser.

Keywords. Multi mode beam; M^2 parameter; diode laser.

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1. Introduction

Since the development of efficient high-power laser diodes and new solid state laser materials, the end-pumped solid state lasers have generated a considerable interest. The influence of pump and laser mode size on overlap efficiency is an important factor for efficient operation of these lasers. Optimization of pump beam size and its divergence is necessary to enhance the overlap efficiency of the system [1]. For optimization of the pump beam size and its divergence, a proper optical system to propagate the pump beam from the exit of the diode laser to the laser crystal is required. The laser diode output is generally an arbitrary multi-mode beam. If the beam quality factor M^2 and initial generalized complex radius of the pump beam is known then a simple paraxial $ABCD$ matrix approach can be applied for the optimization of pump beam parameters [2,3]. For estimation of these parameters of a laser beam, determining the spot size of the laser beam, in the near- or far-field region is a fundamental problem. Several techniques have been suggested for experimental measurement of laser profiles [4,5]. Theoretically, the second moment method is the most suitable method while experimentally a small amount of noise present in the beam profile can lead to highly erroneous results. Therefore, the power content methods are recommended by the ISO commission. The error can still be higher even with these methods because of the critical noise and offset processing. We report here a new approach for determining the spot size. We propose that the width of an arbitrary multi-mode laser beam should be measured by least square fitting of a Gaussian profile. With the help of computer simulations we have shown that the proposed method is relatively immune to noise and offset present in the recorded profiles. The value of Rayleigh range computed with this method is much closer to the real value than the same computed by power content method. Therefore, the paraxial $ABCD$ method for beam propagation can be applied more accurately when the beam parameters are computed with this method. We have also computed a reasonable scale factor to reduce the M^2 value computed by this method to the M^2 value of second moment method. We applied this method to find out the M^2 value and the complex radius of curvature of a multi-mode diode laser. The computed values were used to simulate the propagation of such a beam through a linear optical system. Simulations were done with simple paraxial $ABCD$ technique and the results obtained match excellently with the experimental results.

2. Theoretical

For the analysis of any arbitrary, irregular, multi-mode beam, the waist position of the beam must be known. Stable resonators have a well-defined waist position and size of their lowest order TEM_{00} mode is also well known while in case of unstable resonators these parameters are not known. In unstable resonators, position of waist can be redefined by using a spherical lens at the output and collimating the beam. Consider an arbitrary, irregular, multi-mode collimated beam coming from any kind of laser. Assume that at z distance away from its waist position $2W(z)$ is the diameter of that beam along one of the axis. If $I(x, y)$ is its normalized intensity distribution at that plane then as per second moment method the value of $W(z)$ will be given as [6]

$$W(z)^2 = 4 \times \int\int (x - x_0)^2 \, I(x, y) \, \mathrm{d}x \, \mathrm{d}y, \tag{1}$$

where x_0 is the beam center. If we make use of the fact that in Cartesian coordinate system any collimated optical beam can always be described as a linear combination of Hermite–Gaussian modes [7], then eq. (1) can be rewritten as

$$W(z)^2 = \frac{4 \times \sum_n S_n \int\int (x - x_0)^2 \, I_n(x, y) \, \mathrm{d}x \, \mathrm{d}y}{\sum_n S_n} = \frac{\sum_n S_n \, \omega_n(z)^2}{\sum_n S_n}, \tag{2}$$

where S_n, $\omega_n(z)$ and $I_n(x, y)$ are respectively the weight factor, radius and normalized intensity profile of the nth order Hermite–Gaussian beam. Equation (2) gives the relation between the irregular multi-mode beam spot size and the spot sizes of Hermite–Gaussian modes participating in the formation of the beam. The spot size of nth order Hermite–Gaussian beam is always proportional to its characteristic spot size $\omega_0(z)$ at a particular plane z [7]

$$\omega_n(z) = a_n \, \omega_0(z), \tag{3}$$

where $\omega_0(z)$ is $1/e^2$ radius of lowest order Gaussian spot and a_n is the proportionality constant for the nth mode. When the spot size is measured by second moment method the value of $a_n = \sqrt{2n+1}$ [8]. With the help of (2) and (3) the relation between spot size $W(z)$ and characteristic spot size or the Gaussian beam spot size $\omega_0(z)$, participating in the formation of the beam can be given as

$$W(z)^2 = M^2\,\omega_0(z)^2, \tag{4}$$

where M^2 is the beam quality factor and is given as

$$M^2 = \frac{\sum_n S_n\, a_n^2}{\sum_n S_n}. \tag{5}$$

It shows that the factor M^2 is a constant for a particular beam and depends upon the orders and respective weight factors of Hermite–Gaussian modes participating in the beam formation. A circularly symmetric beam can also be described as a linear combination of Laguerre–Gaussian modes. The M^2 factor for circularly symmetric beams will be given as

$$M^2 = \frac{\sum_l \sum_m S_{lm}\, a_{lm}^2}{\sum_l \sum_m S_{lm}}, \tag{6}$$

where S_{lm} and a_{lm} are respectively the weight factor and proportionality constant for lth order Laguerre–Gaussian beam with mth azimuthal index. When the radius is measured with second-moment method, then for a Laguerre–Gaussian beam the value $a_{lm} = 2l + m + 1$ [9]. When these multi-mode beams pass through any linear optical system such that there is no diffraction losses, the content of different orders of Hermite–Gaussian modes and their relative weight factors always remain constant. Therefore, the value of M remains invariant. Since M is a constant for all type of beams and the relation between Gaussian beam radius $\omega_0(z)$ and its waist radius ω_0 is well known, therefore, variation of beam radius $W(z)$ with z can be written as

$$W(z) = M\,\omega_0\sqrt{1 + \left(\frac{z\,\lambda}{\pi\,\omega_0^2}\right)^2}, \tag{7}$$

where λ is the wavelength of the beam in the medium. It is clear from (7) that the multi-mode beam spot size at any plane z can be obtained once the M factor and waist size of the lowest order Hermite–Gaussian beam is known. Let Θ be the far-field divergence angle of the beam, the value of this angle can be obtained from (7) and is given as

$$\Theta = \frac{W(z)}{z} = \frac{M\,\lambda}{\pi\,\omega_0}, \quad z \to \infty. \tag{8}$$

Equation 8 can be rewritten as

$$\Theta\, W_0 = \Theta\, M\,\omega_0 = \frac{M^2\,\lambda}{\pi}, \tag{9}$$

where W_0 is the waist size of the multi-mode beam and is related to the Gaussian beam waist ω_0 as given in (4). Equation (9) leads to the fact that in the absence of any diffraction losses, product of the size of the waist of a beam and its far-field divergence angle remains invariant while passing through any linear optical system. The M factor for a multi-mode beam can be obtained with the help of eq. (9) once its far-field divergence angle Θ and waist size W_0 are known. Similarly from the measured M factor value, the Gaussian beam waist radius ω_0 can be obtained with the help of eq. (4). The Rayleigh range for such a multi-mode beam is given as

$$z_{\mathrm{R}} = \frac{\pi\,\omega_0^2}{\lambda} = \frac{\pi\,W_0^2}{M^2\,\lambda}. \tag{10}$$

The generalized complex radius of curvature at a distance z from the waist is given as

$$q = i\frac{\pi\,\omega_0^2}{\lambda} + z. \tag{11}$$

Equation (11) shows that the generalized complex radius of curvature for a multi-mode beam is nothing but the complex radius of curvature for the lowest order Hermite–Gaussian beam participating in the formation of this beam. Therefore, with the knowledge of generalized complex radius of curvature, the matrix method can be used [3, 5] to calculate the propagation of such a beam through any linear optical system characterized by an $ABCD$ matrix. This can be said in another way, when the wavelength λ of the multi-mode beam is replaced with an effective wavelength $\lambda_e = M^2\,\lambda$, then the paraxial propagation laws valid for a Gaussian beam becomes valid for the multi-mode beam.

3. Simulation

Three types of multi-mode profiles were generated with $\omega_0 = 10\,\mu$m, aperture length $L = 800\,\mu$m, and $\lambda = 1.0\,\mu$m. This corresponds to Rayleigh range $z_{\mathrm{R}} = 314.159\,\mu$m.

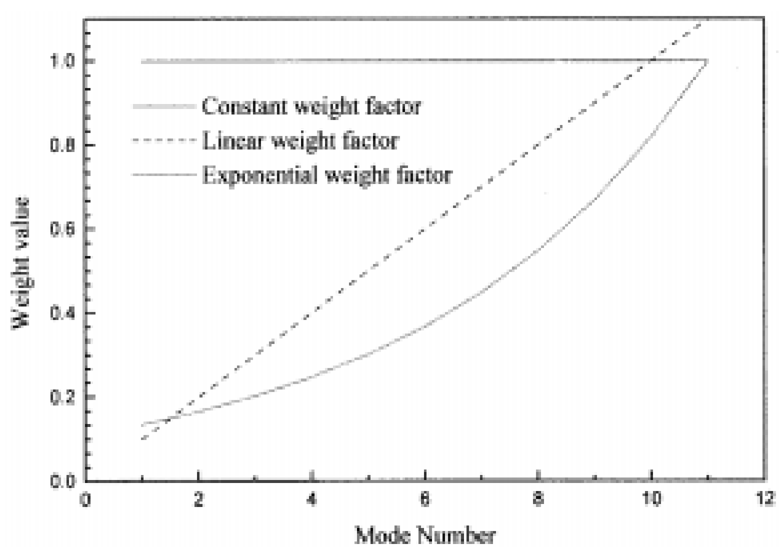


Figure 1. Weight factor profiles used for generating three different types of multi-mode profiles.

All the profiles were one-dimensional profiles with 1025 data points. In the generation of these multi-mode profiles, first eleven Hermite–Gaussian profiles with varying weight factor were incoherently added. In the first case, all the modes have equal weight factor, in the second case the weight factor increases linearly with mode number, while in the third case

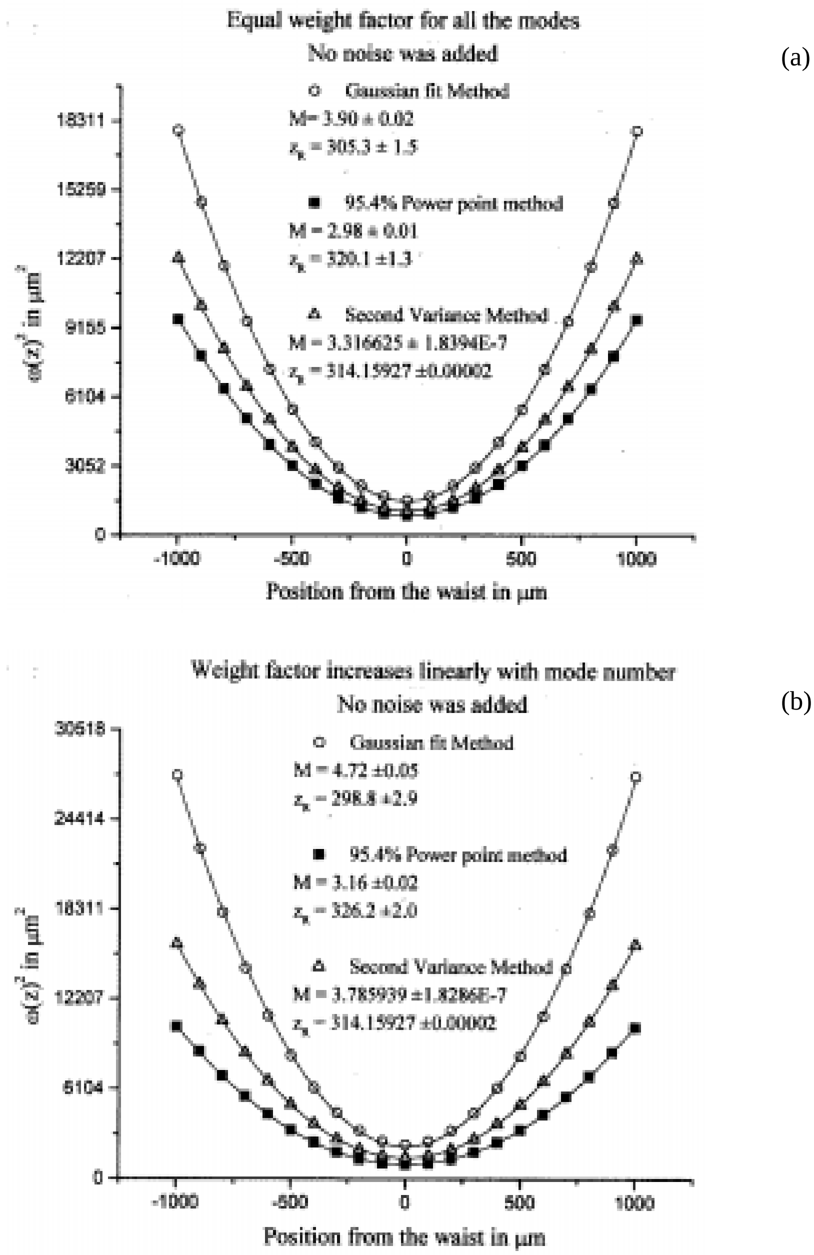
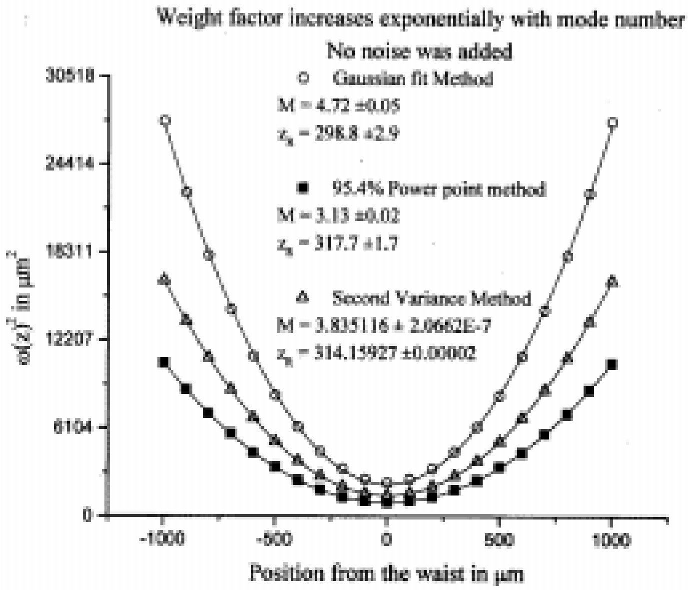


Figure 2a, b.

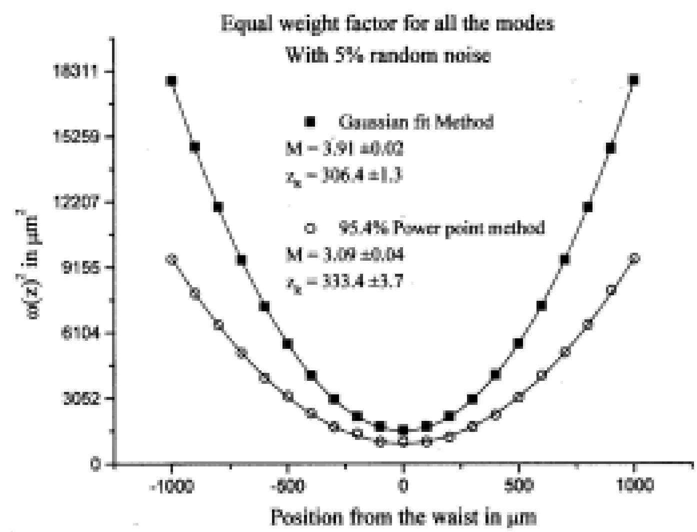


(c)

Figure 2a–c. Variation of multi-mode beam radius with distance from the waist plane. The radius was measured with three different methods namely; second moment method, 95.4% power content method and Gaussian fit method. No noise or offset was added to the generated profiles. The profiles were generated by incoherently adding first eleven Hermite–Gaussian profiles with (**a**) equal weight factor for all the modes, (**b**) weight factor increasing linearly with mode number, (**c**) weight factor increasing exponentially with mode number. Solid lines correspond to the least square fit of eq. (7).

the weight factor increases exponentially with mode number. All the weight factor profiles are shown in figure 1. The computed M factor as per eq. (5), for all these three cases will respectively be $M_1 = 3.3166$, $M_2 = 3.7859$, and $M_3 = 3.8351$. In all these cases, the beam profiles were generated at 21 different positions from the beam waist. First the profiles were generated without any noise or offset, and then the radii of these profiles were estimated with second moment method, power content method and Gaussian profile fit method. The results are shown in figures 2a, 2b and 2c. The values of M^2 and Rayleigh range are obtained by least square fit of (7). It can be seen that the values of Rayleigh range $z_{\rm R}$ and M factor obtained from the second-moment method curves are perfectly matching with the computed values. The whole simulations were repeated again, first by adding random noise of 5% of peak value to all the profiles and then by further adding an offset of 0.5% of peak value to these profiles. The results are respectively shown in figures 3a, 3b, 3c and 4a, 4b, 4c. The results of second moment method with random noise and offset were highly erroneous, therefore these results are not included in figures 3 and 4. It can be seen that the values of M^2 and Rayleigh range obtained by Gaussian fit method are quite consistent in all the three conditions. The values obtained by power content method becomes erroneous with the presence of small offset. It is clear from the central part of the figure 4a, b, c that when the size of the beam is smaller than one tenth of the CCD size, the power content method cannot be used, although the proposed method is applicable even with smaller beam sizes. With the Gaussian fit method the value of Rayleigh range obtained in all the nine profiles is within 4.5% of the real value, while with power content method the error is much larger. The scale factor for reducing M_{G} and W_{G} of Gaussian fit method to M and W of second-moment method is $S = M/M_{\mathrm{G}} = W/W_{\mathrm{G}} = 0.798 \pm 0.027$. The value is computed from the ratios of the widths of 63 profiles estimated with

(a)



(b)

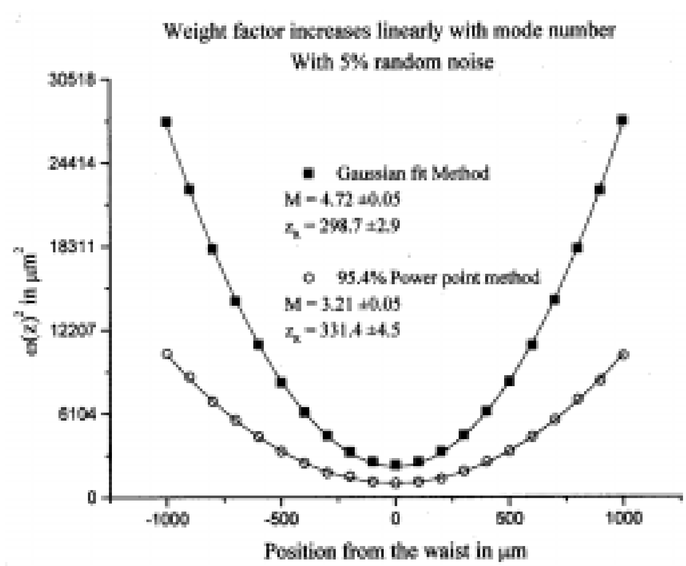


Figure 3a, b.

(c)

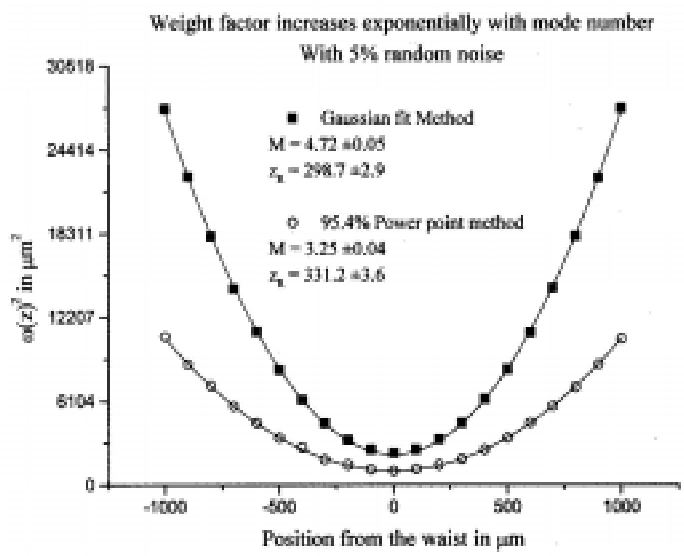


Figure 3a–c. Variation of multi-mode beam radius with distance from the waist plane. The radius was measured with two different methods namely; 95.4% power content method and Gaussian fit method. The profiles were generated by incoherently adding first eleven Hermite–Gaussian profiles with (**a**) equal weight factor for all the modes, (**b**) weight factor increasing linearly with mode number and (**c**) weight factor increasing exponentially with mode number. Random noise of the order of 5% of peak value was added to the generated profiles. Solid lines correspond to the least square fit of eq. (7).

(a)

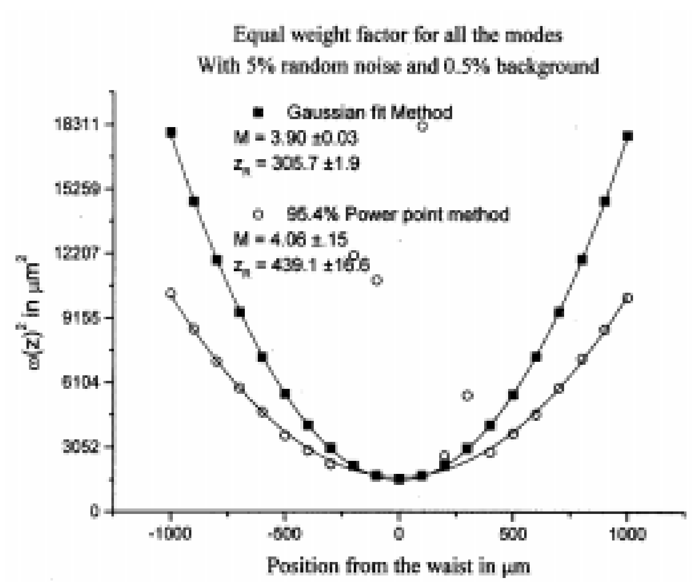
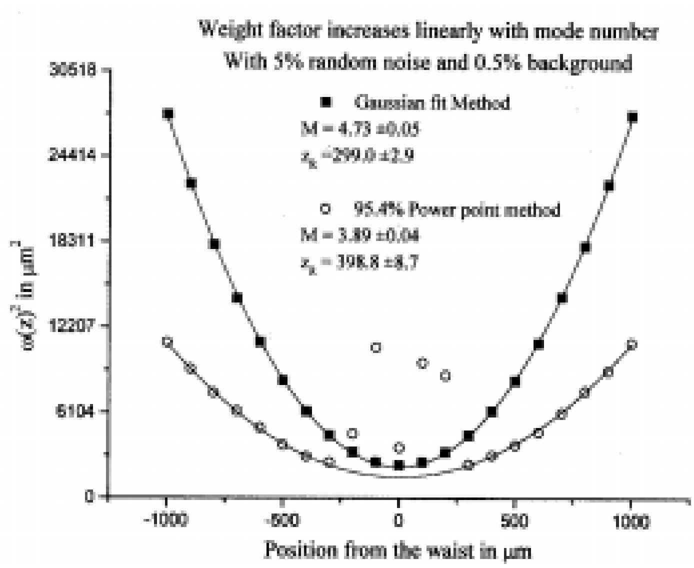


Figure 4a.

(b)



(c)

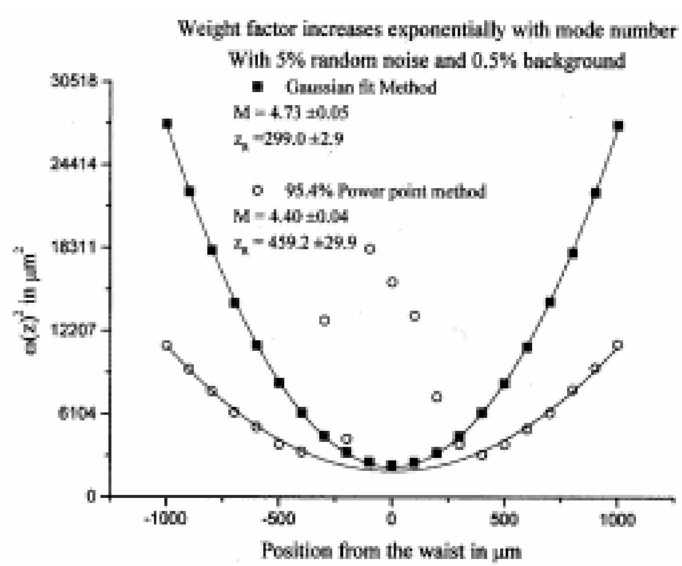


Figure 4a–c. Variation of multi-mode beam radius with distance from the waist plane. The radius was measured with two different methods namely; 95.4% power content method and Gaussian fit method. The profiles were generated by incoherently adding first eleven Hermite–Gaussian profiles with (**a**) equal weight factor for all the modes, (**b**) weight factor increasing linearly with mode number and (**c**) weight factor increasing exponentially with mode number. Random noise of the order of 5% of peak value and an offset of 0.5% of peak value were added to the generated profiles. Solid lines correspond to the least square fit of eq. (7). In case of 95.4% power content the bad data points near the waist are not taken into account for least square fitting.

Gaussian fit method and second moment method in different conditions. Within the prescribed error the value of the factor can be used for any experimental measurement by the proposed Gaussian fit method. These results also show that with the proposed Gaussian fit method the beam quality factor and Rayleigh range or characteristic beam radius ω_0 of a multi-mode beam can be computed within an error of less than 5%.

4. Experimental

In our diode pumped Nd:YVO_4 laser, we used a FTI make 1.0 W diode with fast axis collimator. The $1/e^2$ divergence angles are $\theta_x = 8°$ and $\theta_y = 0.9°$. A collimating lens f_1 of focal length 8.0 mm is placed at the output of the diode. The distance of f_1 from exit plane was optimized in such a way that the diameter of pump beam remains smaller than the 1.0 cm aperture in front of the focusing lens. This aperture was present due to the lens holder. The optimized distance of f_1 from the exit plane was about 7.7 mm. A focussing lens f_2 of 25.0 mm focal length was placed 247 mm away from the collimating lens. The laser crystal is generally placed at the focused plane of the beam. For experimental determination of M^2 values, the focused beam was imaged with the help of a 50.0 mm focal length lens on a CCD. The image magnification factor was 11.2±0.1 and the CCD was an SBIG ST6 camera with a 16 bit dynamic range. The error in the magnification factor was estimated from the error in the measured distances and the error in focal length of the lens used. The pixel sizes are 23 μm and 27 μm in horizontal and vertical directions respectively. Experimental setup for recording the focus beam images is shown in figure 5. Images of spot size were recorded at several planes before and after the focused spot size. The camera was oriented in such a way that the axes of the laser beams were almost parallel to the CCD arrays. For reducing the random noise present, all the rows were added to get the horizontal profile and all the columns were added to get the vertical profile. The radii of these spot sizes along both the axes were measured with Gaussian fit method. The measured values of the radii were reduced to actual values by dividing them with the magnification factor. A typical Gaussian fit to the diode laser output is shown in figure 6. Measured variation of spot size with propagation distance is shown in figure 7. The values of M^2 factor and W_0 were obtained for both the axes by least square fitting of eq. (7) to the experimental data. Values of M^2 and W_0 corresponding to second moment method were obtained from these measured values by applying the scale factor computed by simulation. The reduced M^2 values along both the axes are $M_x^2 = 38.8 \pm 1.2$, $M_y^2 = 1.9 \pm 0.5$ and reduced W_0 values along both the axes are

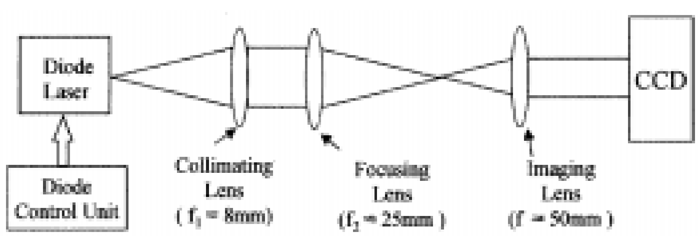


Figure 5. Experimental setup for recording the diode laser beam profiles.

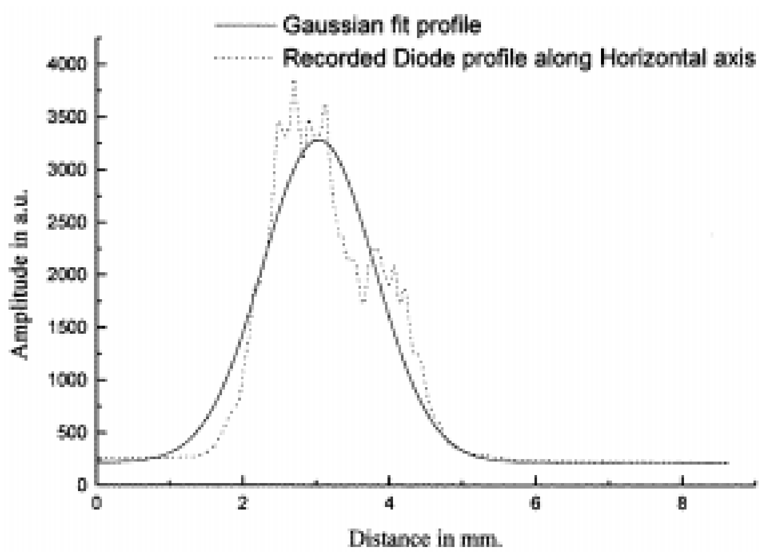


Figure 6. A typical magnified diode-laser profile along horizontal axis with a Gaussian fit.

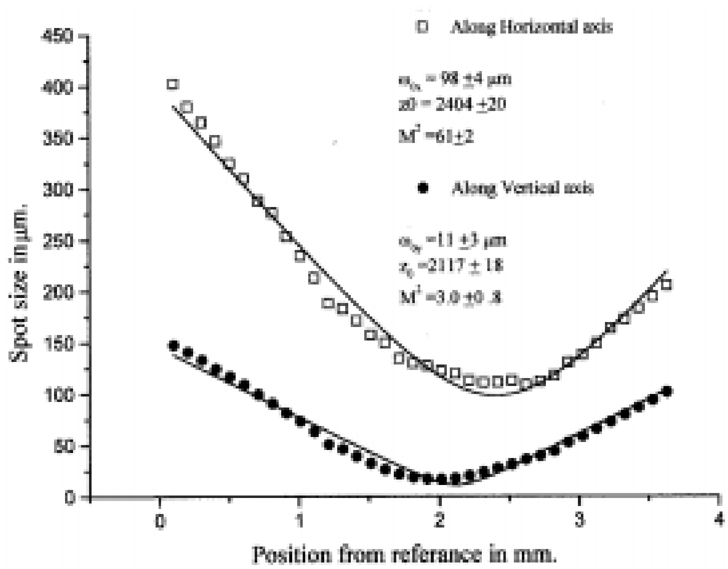


Figure 7. Measured variation of spot size with distance from a reference plane. The entrance plane of the imaging lens was the reference plane. Solid line corresponds to least square fit of eq. (7). The values of M^2 and W_0 are for Gaussian fit method. To obtain corresponding second-moment method values the appropriate scale factor has to be used.

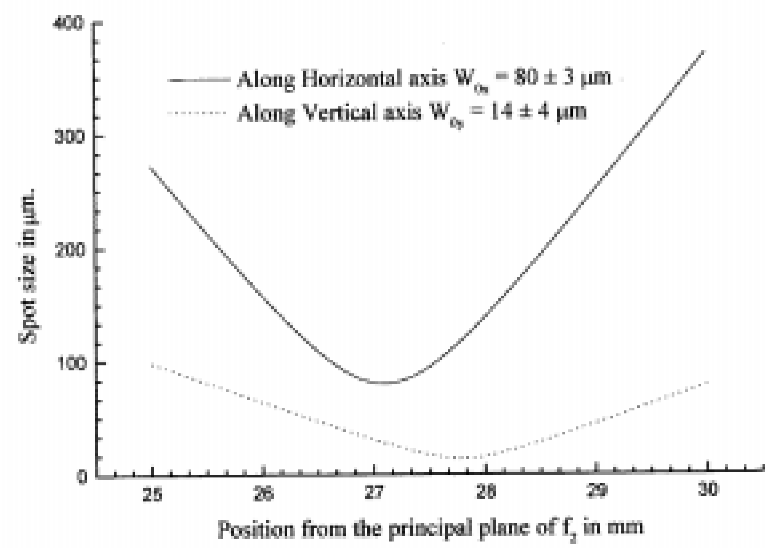


Figure 8. Variation of the computed waist size of the diode-laser beam with distance from the principal plane of the focusing lens.

$W'_{0x} = 78 \pm 3\,\mu$m and $W'_{0y} = 8.8 \pm 2.4\,\mu$m. From the reduced M^2 values and the given far-field divergence angles, the diode laser waists at the exit plane, along both the axes were computed with the help of eq. (9). The corresponding initial complex radii of curvature were computed with the help of eqs (10) and (11). The values so obtained are $q_{x0} = iz_{Rx} = i0.5$ mm and $q_{y0} = iz_{Ry} = i2.0$ mm. Once the initial complex radius of curvature is known, the value of final complex radius of curvature can easily be obtained with the help of $ABCD$ matrix, corresponding to the optical system from the initial plane to the final plane. The value of final spot size can be obtained from the measured complex radius of curvature. A computer program based on Mathematica software was developed for $ABCD$ analysis. We applied the $ABCD$ matrix analysis for computing the pump beam spot size at various planes after the principal plane of the focusing lens. Figure 8 shows the computed variation of pump-beam spot sizes with the distance from the principal plane of the focusing lens. The computed waist sizes along both the axes are $W'_{0x} = 80 \pm 3\,\mu$m and $W'_{0y} = 14 \pm 4$ μm. The errors in these computed values are calculated from the measured errors in M^2 values. It can be seen that within experimental errors, the computed values of W_0 matches excellently with the experimentally measured values.

5. Conclusion

We have demonstrated with computer simulations that, with our proposed Gaussian fit method, the beam quality factor and characteristic waist of a multi-mode beam can be measured with an error of less than 5%. Unlike the power content method, the proposed method is applicable to the beams even with diameter one tenth of the CCD size. We applied this method to one of the most complex output beam of a diode laser and have shown that the measured values of beam quality factor and waist size are reliable and can be used for $ABCD$ analysis.

References

[1] Y F Chen, T S Liao, C F Kao, T M Huang, K H Lin, and S C Wang, *IEEE J. Quantum Electron.* **32**, 2010 (1996)
[2] P A Belanger, *Opt. Lett.* **16**, 196 (1991)
[3] A E Siegman and Steven W Townsend, *IEEE J. Quantum Electron.* **29**, 1212 (1996)
[4] ISO-Norm, document ISO/TC 172/SC 9 WGI (1991-02-19), *Test method for width, divergence and radiation characteristic factor of laser beams* (1991)
[5] N Reng and B Eppich, *Opt. Quantum Electron.* **24**, S973 (1992)
[6] A E Siegman, New developments in laser resonators, *SPIE* **1224**, 2 (1990)
[7] A E Siegman, *Lasers* (University Science Book, Mill Valley, CA, 1986)
[8] W H Carter, *Appl. Opt.* **19**, 1027 (1980)
[9] R L Phillips and L C Andrews, *Appl. Opt.* **22**, 643 (1983)