Beauty-conserving strangeness-changing rare semileptonic decays of $B_s$ meson

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Abstract. The beauty-conserving strangeness-changing decays of $B_s$ meson are examined. In the charm sector, charm-conserving strangeness changing ($\Delta c = 0, \Delta s \neq 0$) decays are Cabibbo suppressed and are governed by the CKM element $V_{us}$ which is much smaller than the CKM diagonal element $V_{cs}$, so may be of little interest. On the other hand, in the $b$-sector, beauty-conserving strangeness changing ($\Delta b = 0, \Delta s \neq 0$) decays are CKM allowed as the CKM matrix element $V_{us}$ governing such decays is much larger than $V_{bc}$ or $V_{ub}$, which govern respectively the $b \rightarrow c$ or $b \rightarrow u$ transitions. The phase space available, however, is too small for the decays considered here. The numerical estimates for the decay widths of two such modes of $B_s$ meson are presented.

Keywords. Semileptonic decays; $B_s$ meson.

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1. Introduction

Because semileptonic decays are relatively simple and experimentally accessible, they are the primary tool for obtaining quark couplings to $W$ mesons and also for studying the effect of nonperturbative strong interaction through form factors which are difficult to calculate. It is expected that the decays of the charmed and heavier hadrons will be simpler as the strong interaction effects will be smaller and their study would help in the understanding of decay processes, in general. Indeed many semileptonic decays of charmed and $b$ hadrons have been used in extracting [1] the CKM matrix elements with the help of heavy-quark effective theory (HQET) and other tests on the standard model. Usually [2], the dominant (CKM allowed) modes of decays are the ones in which there is change of heavy flavor. As a matter of fact in heavy hadron decays it is assumed that the decays take place due to the decay of the heavy quark, with the accompanying light quark(s) acting as spectator. Such decays of hadrons involve large momentum transfer when the daughter hadron contains only the lighter quarks than the parent one.

Apart from this usual behaviour, there is a possibility of special class of decays in which the heavy quark remains as such and only the light quark undergoes a weak transition,
giving us the heavy flavor conserving and strangeness changing rare decay modes. In the charm sector, the charm changing ($\Delta c \neq 0$) decays are Cabibbo allowed whereas the charm conserving but strangeness changing ($\Delta c = 0$, $\Delta s \neq 0$) decays are Cabibbo suppressed. The CKM matrix element $V_{cs}$, involved in the former case, is much larger than the CKM matrix element $V_{us}$ for the latter case. So, the charm-conserving but strangeness changing decays are of little interest. In the $b$-hadron decays, however, the CKM matrix element ($V_{ub}$ or $V_{us}$) is larger in $b$-conserving strangeness changing ($\Delta b = 0$, $\Delta s \neq 0$) decay mode than the CKM matrix element ($V_{bc}$ or $V_{ub}$) in $b$ to $c$ or $b$ to $u$ transitions i.e. $\Delta b \neq 0$ transitions. So, from the point of view of the CKM elements, the $b$-conserving strangeness changing decays will be CKM allowed. It is the phase space in $b$-conserving decays which is too small as compared with that in $b$-changing modes. In some such decay modes, however, the rate for such decays may have a significant fraction. Since in the decays under consideration, only the light quark participates in weak interactions, it is possible that the information we obtain about the decay of light quark(s) may help us in the understanding of $K$-meson decays, hyperon decays and hadron structure.

As the heavy flavor is conserved in such decays, $q^2$ involved is very small, i.e. $q^2 \approx 0$. So, we just need the form factor value at $q^2 = 0$ and no extrapolation of the form factors, for $q^2$ dependence, is required here. This $q^2$, however, may not be small at light quark scale. By matching the predictions with experimental values, we may be able to get the values of the form factors at small $q^2$ and thus may be able to distinguish between various models which predict form factor values at $q^2 = 0$.

In this paper, we have calculated the branching ratios for two such decay modes of $B_s$ meson. The branching ratios come out to be very small but can be comparable to those in heavy flavor changing rare decay modes.

2. Preliminaries

The amplitude for semileptonic decay $M \to Xl\bar{\nu}$ of a meson $M$ into a meson $X$ is given by [3]

$$A(MQq \to Xq'\bar{q}l\bar{\nu}) = -i \frac{G_F}{\sqrt{2}} V_{q'Q} L_\mu H^\mu,$$

where the leptonic current can be written in terms of Dirac spinors $u_l$ and $v_\nu$:

$$L_\mu = \bar{u}_l \gamma_\mu (1 - \gamma_5) v_\nu.$$  \hspace{1cm} (2)

The hadronic current $H^\mu$ is related to the matrix element of the current operator $J_\mu$ as

$$H^\mu = \langle X | q' \gamma^\mu (1 - \gamma_5) Q | M \rangle.$$  \hspace{1cm} (3)

The hadronic current is to be constructed from the available four vectors, which are momenta and spin-polarization vectors. The coefficients of the vector and axial vector thus formed, the form factors, are functions of the Lorentz invariant $q^2$. Thus $H^\mu$ is written in terms of form factors which enable us to isolate the effects of strong interactions on the amplitude. Here, $V_{q'Q}$ is the appropriate CKM matrix element.

For the case of $B_s \to Bl\bar{\nu}$ decay, where $B_s$ and $B$ are pseudoscalar mesons with momenta $p$ and $p'$ and mass $M$ and $m_B$ respectively, there are only two independent four
Semileptonic decays of $B_s$ meson vectors, the sum $p + p'$ and the difference $q = p - p'$ and so two independent form factors. Here, the hadronic current $H^\mu$ will not have any axial vector contribution and in the limit of vanishing lepton mass is described [4] by only one form factor $F_1(q^2)$ when $l = e$ or $\mu$ and

$$\langle B(p')|V^\mu|B_s(p)\rangle = F_1(q^2)(p + p')^\mu.$$  \hfill (4)

For the process $B_s \rightarrow B^*l\nu$, each term in the current must be linear in the polarization vector $e$ of the vector meson.

Again in the limit $m_l \rightarrow 0$, the decay $B_s \rightarrow B^*l\nu$ is essentially described by three form factors: namely, $A_1(q^2)$, $V(q^2)$ and $A_2(q^2)$,

$$\langle B^*(p', e)|V^\mu - A^\mu|B_s(p)\rangle = \frac{2ie^{\mu\nu\alpha\beta}}{M + m_{B^*}}\epsilon_\nu p'_\alpha p_\beta V(q^2) - (M + m_{B^*})\epsilon^{*\mu}A_1(q^2)$$

$$+ \frac{e^*q}{M + m_{B^*}}(p + p')^\mu A_2(q^2).$$ \hfill (5)

The form factors $A_1(q^2)$ and $A_2(q^2)$ can be associated with the exchange of a particle with quantum numbers $J^P = 1^+$ whereas $V(q^2)$ is associated with $J^P = 1^-$.  

3. Results and conclusion

The differential decay rate for the semileptonic decay $B_s \rightarrow Bl\nu$ is given by [5]

$$\frac{d\Gamma(B_s \rightarrow Bl\nu)}{dq^2} = \frac{G_F^2}{192\pi^3}|V_{us}|^2 \frac{\lambda^3(M^2, m_{B_s}^2, q^2)}{M^3}|F_1^{B_s \rightarrow B}(q^2)|^2,$$ \hfill (6)

because $p_B = (\lambda(M^2, m_{B_s}^2, q^2))/(2M)$ where $\lambda(x, y, z) = (x^2 + y^2 + z^2 - 2xy - 2xz - 2yz)^{1/2}$.

Since $q^2$ is close to zero, the form factor will have nearly a constant value.

By integrating the (6) with respect to $q^2$ over the range 0 to $(M - m_B)^2 = q_{\text{max}}^2$, we will get the total decay rate for the decay $B_s \rightarrow Bl\nu$,

$$\Gamma(B_s \rightarrow B^+\bar{e}\nu) = \frac{G_F^2}{192\pi^3}\frac{|V_{us}|^2}{M^3}|F_1(0)|^2 I_1,$$ \hfill (7)

where

$$I_1 = \int_0^{(M - m_B)^2} \lambda^3(M^2, m_{B_s}^2, q^2)dq^2.$$ \hfill (8)

With $I_1 = 3.0124 \times 10^{-3}$ GeV$^4$ we get

$$\Gamma(B_s \rightarrow Be\bar{\nu}) = 2.1 \times 10^{-20}|F_1(0)|^2 \text{ GeV}.$$  

Using $\tau_{B_s} = 1.54 \times 10^{-12}$ s [6], the branching ratio turns out to be:

$$B(B_s \rightarrow Be\bar{\nu}) = 0.492 \times 10^{-7}|F_1(0)|^2.$$
The decay $B_s \rightarrow B^* \ell \nu$ is complicated as compared to $B_s \rightarrow B \ell \nu$ because the latter involves only one form factor whereas the former involves three form factors. The things get simpler, however, if we consider the limit of vanishing lepton mass i.e. $q^2 \rightarrow 0$. In this limit, the terms proportional to $A_0(q^2)$ and $A_3(q^2)$ in (5) do not contribute to the total amplitude and hence to the decay rate. The decay is determined by only one form factor $A_0$ at $q^2 = 0$:

$$\frac{d\Gamma(B_s \rightarrow B^* \ell \nu)}{dq^2} = \frac{G_F^2}{192\pi^3} |V_{us}|^2 \lambda^2 \left( M^2, m_{B^*}^2, q^2 \right) |A_0| (q^2)^2.$$ (9)

Again, the form factor is expected to be constant because of the smallness of $q^2$.

By integrating (9) with respect to $q^2$, we can get the total decay rate,

$$\Gamma(B_s \rightarrow B^* \ell \nu) = \frac{G_F^2}{192\pi^3} \frac{|V_{us}|^2}{M^3} |A_0|^2 I_2,$$ (10)

where

$$I_2 = \int_0^{(M-m_{B^*})^2} \lambda^2 \left( M^2, m_{B^*}^2, q^2 \right) dq^2,$$ (11)

and with $I_2 = 8.067 \times 10^{-5} \text{GeV}^4$ we get

$$\Gamma(B_s \rightarrow B^* \ell \nu) = 5.76 \times 10^{-22} |A_0(0)|^2 \text{ GeV},$$

and the branching ratio turns out to be,

$$B(B_s \rightarrow B^* \ell \nu) = 0.135 \times 10^{-8} |A_0(0)|^2.$$

As is clear, the branching ratios come out to be very small for both the cases. Their measurement may be difficult but not necessarily hopeless. Some hope of observing such decays are hadronic machines including the BTeV and LHC-B experiments. It is estimated that at CERN LHC-B, approximately $2 \times 10^{11}$ [8] $B_s$ mesons per year will be produced. Using this information, we can easily calculate the number of expected events for these two decays at LHC to be

$$N(B_s \rightarrow B \ell \nu) = B(B_s \rightarrow B \ell \nu) \times (2 \times 10^{11}) \approx 10 \times 10^3,$$ (12)

$$N(B_s \rightarrow B^* \ell \nu) = B(B_s \rightarrow B^* \ell \nu) \times (2 \times 10^{11}) \approx 3 \times 10^2.$$ (13)

From this estimation it follows that at future LHC collider it may be possible to detect these decays. To conclude, the calculations involve no parameter and when compared with data can give us form factor value at $q^2 = 0$ which may be employed to test various models.

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References