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Origin of mutual exclusiveness in Bohr's complementarity principle

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Abstract. According to Bohr's complementarity principle, two distinct types of complementarity exist – one of complementary variables and other in the so-called wave-particle complementarity experiments. Some authors have claimed that mutual exclusiveness (ME) in both the cases arise due to uncertainty principle and have analysed the second type in terms of Fourier space analysis and consequent putative "momentum kick" distribution. Some others, on the other hand, have identified the collapse hypothesis as the actual quantum mechanical principle responsible for ME in the interferometry experiments. In this paper the momentum space analysis is thoroughly examined vis-a-vis the general quantum mechanical description in terms of the changes in the wave function. It is argued that such alternative explanations are not in full conformity with the strict quantum mechanical description.

Keywords. Bohr's complementarity principle; wave-particle duality; uncertainty relation; mutual exclusiveness; quantum mechanics.

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1. Introduction

Recently we have discussed [1, 2] that two intuitive formulations of the notion of complementarity by Bohr may be placed into two distinct classes on the basis of the origin of mutual exclusiveness. While in the case of complementary variables such as position and momentum (class I) it is the quantum mechanical uncertainty relation which is at work, the collapse hypothesis ensures this exclusiveness in the so-called wave-particle complementarity experiments (class II) of properties (such as interference fringes in the far-field intensity distribution and Welcher weg information) [3].

It is important to note that here we refer to complementarity between a pair of properties, rather than a pair of variables as in class I. Usually it is difficult to associate specific operators corresponding to these properties [1, 2]. The information on these properties are of course obtained from the measurement of some variables but the complementarity exists not between these variables but between some properties defined from the measurement of the variables. For example in the double slit experiment with a beam of electrons one measures the density distribution of electrons on the screen. Interference is the existence of bright and dark fringes, which is a property of the density distribution. The possibility of assigning suitable pair of operators was explored in [2] and we have noted that the commutation relation of such suggested pair of operators does not lead to any uncertainty type relation.

In contrast to what we have just stated, many authors have analysed the class II type of complementarity to show that ME really follows from the uncertainty principle. While admitting that the simultaneous observation of wave and particle behaviour is prohibited usually by the position-momentum uncertainty relation, Scully *et al* [4] advanced a novel experiment to allow investigation of other mechanisms that enforce complementarity. It is argued that in their specific example the incompatibility between Welcher weg knowledge and the appearance of interference cannot be traced to any uncertainty relation. But in a relatively recent letter Storey *et al* [5] have concluded from a Fourier space analysis, with special reference to Scully *et al*'s work, that the principle of complementarity is a consequence of the uncertainty principle. If this is true, then our claim that ME in class II type of complementarity follows from the collapse hypothesis must be wrong. Our main purpose in this paper is to critically examine whether uncertainty principle has any role in ME in these cases.

Apart from the examples of Scully *et al* and Schulman [6] there are obvious instances where the uncertainty argument cannot be dragged. For example in the double slit experiment if we put a 100% efficient detector close to slit 1 and record on the screen only those events for which the detector does not click, the result does not show any interference. In this case the disappearence of interference takes place without any exchange of energy or momentum between the detector and the particle and collapse of wave function offers the only explanation.

In the following sections we analyse the reasons which lead to the conclusion that in all so-called wave-particle complementarity experiments collapse of wave function offers the only consistent quantum mechanical explanation for the mutual exclusiveness.

2. Wave-particle complementarity in double slit experiment

In the typical example of recoiling double slit arrangement, the essential part of Bohr's argument [7] is that in order to observe the interference pattern, the slits must be localised with an accuracy of the order of the slit width and the corresponding uncertainty in the slit momentum is always much greater than the difference between the recoil momenta in the scattering from the two slits. Consequently the Welcher weg information is not available when the photons do produce interference pattern.

Interference is basically a property of the characteristic wave function. Any explanation of the disappearance of the interference due to a measurement without clarifying how the state vector is altered by the measurement, seems incomplete. The quantum mechanical uncertainty relation that follows from non-commutation of canonically conjugate variables [8] nowhere uses the concept of simultaneous approximate measurement of non-commuting observables. But this is the essence of Heisenberg's formulation [9] of the gedanken experiment used in the standard explanations. Margenau [8] from a critical analysis has pointed out that the non-statistical notion of uncertainty principle in a simultaneous measurement of canonically conjugate dynamical variables pertaining to a single particle is not amenable to any meaningful interpretation within the framework of quantum mechanics (QM). In particular Heisenberg's uncertainty principle uses a peculiar mixture of classical and quantum concepts to evince the breakdown of classical laws and in most cases offers a semiclassical explanation for some quantum phenomena. When the semiclassical arguments are pushed to the logical extreme, contradictions emerge. This can be resolved only if we follow pure quantum mechanical description in terms of the state function.

The mental picture emerging from the analysis in terms of the Heisenberg uncertainty principle is that each individual microparticle produces its own pattern and if we have the which-slit information, "shifted interference patterns add together, washing out the fringes" [5]. This implies the unacceptable conclusion that superposition is not lost even after the exact which-slit detection. The gradual build-up of the interference pattern in actual experiments [10, 11] with a series of incoming single micro-entity one at a time confirms that each registration of the micro-entity on the screen conforms to one of the two patterns (either a continuous distribution or interference fringes) depending on whether the experimental set-up is providing the which-slit information or not. In the shifted fringe interpretation even after an exact which-slit detection the only change in the original ψ-function is believed to be a change in the wavelength due to momentum transfer and the superposed structure (i.e. $\psi = \psi_1 + \psi_2$) remains unaltered. The analogy appears to be with the double slit experiment using white light. The fallacy of this argument becomes apparent because in this interpretation the central bright band is always present and there is no washing out of the entire fringe system. There is no doubt that Heisenberg's form of the uncertainty principle played an important role in the initial stages of the development of QM. Even now it offers in many cases a quick glimpse of some peculiar quantum results. The use of semiclassical notions is its real strength because one can get some picture of what may be happening to the individual particles. But it is well to remember that these pictures are not part of the wave function description of QM. Since in this paper we are interested in the quantum mechanical principle which enforce ME in the complementarity, we shall ignore semiclassical explanations using Heisenberg's form of uncertainty relation.

The whole situation is clarified if we trace the changes in the state vector at different stages. We consider the electron interference experiment performed with one electron at a time. The electron is represented by a packet with dispersion in momentum negligible compared to the mean momentum. After emerging from the double slit the state vector is written as $\psi = \psi_1 + \psi_2$, where ψ_i is the packet emerging at the slit $i (i = 1, 2)$. If just outside the slit 1 a narrow beam of photons of wave number vector $\vec{q}$ is projected parallel to the slit, then before the interaction between the electron and the photon, the state vector for the joint system may be written as

$$(\psi_1 + \psi_2)|\vec{q}\rangle$$

where $|\vec{q}\rangle$ is the ket for the initial photon beam. Due to interaction the photon has a finite probability of being scattered by the electron and the state vector becomes:

$$a\psi_1'|\vec{q}\,'\rangle + b\psi_1|\vec{q}\rangle + c\psi_2|\vec{q}\rangle,$$

where $|\vec{q}\,'\rangle$ is the ket for the scattered photon and ψ_1' is the modified packet near slit 1 and a, b and c are suitable amplitude factors. For a precise which-slit detection it is necessary that $|\vec{q}\,'\rangle$ and $|\vec{q}\rangle$ are orthogonal packets. If we consider the totality of all electron spots on the photographic plate we get a fringe pattern from $b\psi_1 + c\psi_2$ part of the wave vector while the total position density is given by $|a\psi_1'|^2 + |b\psi_1 + c\psi_2|^2$. In general this pattern is fainter than that due to $\psi_1 + \psi_2$ and in the special case $b = 0$ (photon scattering probability = 1) the pattern gets washed out. If the scattered photon is detected say on a photographic plate, we get the which-slit information and immediately the state vector of the electron collapses to $\psi_1'|\vec{q}'\rangle$. The totality of all electrons for which a scattered photon is detected will not give any interference fringe. It is important to note that fringes may be washed out even if there is no collapse. In the above example if we let $b = 0$, the fringes will be washed out even if the scattered photon is not recorded on a photographic plate. The entanglement with orthogonal detector states (for microscopic detectors) represents potentiality from which one can either get the which-slit information by using microreader on the detector states or retrieve the interference fringes with the help of quantum eraser for some special type of micro detectors [4,12–14] — the two extreme actualities. But there is no conflict with the complementarity principle. The latter only asserts that one cannot have precise which-path information and interference fringes together and does not deny that interference effect may be washed out without any which-path information.

3. Critique of the Auckland group analysis

In quantum mechanics momentum transfer has a clear meaning only in special cases where both initial and final states are eigenfunctions of momentum and a conservation of momentum condition is imposed through Fermi golden rule. In other cases, particularly for wave packets, it is difficult to attach unequivocal meaning to this term. However, it is instructive to explore the possibility of momentum transfer description for a Welcher weg detector system interacting with the interfering state vector. Storey *et al* [5] have made just such an attempt. In most simple terms the Fourier space analysis by Storey *et al* can be properly transcribed by taking the interfering packet at $t = 0$ as $\psi_1 + \psi_2$ and $\psi_1|D_1\rangle + \psi_2|D_2\rangle$ without and with detector state entanglement, where ψ_i is the packet localised near the slit $i (i = 1, 2)$ and $|D_1\rangle$ and $|D_2\rangle$ are the corresponding detector states. If ξ be the variable measured then the detector state may be expressed as a linear superposition:

$$|D\rangle = \Sigma_\xi O_\xi |\xi\rangle,$$

where O_ξ's represent the probability amplitudes of the basis states $|\xi\rangle$'s. Now for a particular outcome ξ for a measurement on the detector the interfering packet collapses to $\psi_f = \psi_1 O_\xi^1 + \psi_2 O_\xi^2$. The interference effect in the far field region is given by:

$$I_\xi = |\psi_f|^2 = |\psi_1|^2 |O_\xi^1|^2 + |\psi_2|^2 |O_\xi^2|^2 + [\psi_1^* \psi_2 O_\xi^{1*} O_\xi^2 + \text{C.C.}].$$

The interference is given by the last term in the bracket. Here only those cases where $|\xi\rangle$ state is detected are selected. Summing over all detector states we get the resultant intensity :

$$I = |\psi_1|^2 + |\psi_2|^2 + [\psi_1^* \psi_2 \Sigma_\xi O_\xi^{1*} O_\xi^2 + \text{C.C.}].$$

The visibility function is defined by Storey *et al* as $\mathcal{V}$ where, $\mathcal{V} = |\Sigma_\xi O_\xi^{1*} O_\xi^2| = |\langle D_1|D_2\rangle|$ — the overlap between the two detector states. In the case of microdetectors non-orthogonal detector states are possible and one can have partial which-path knowledge and partial interference pattern and other possibilities also emerge in the case of coherent detector states [15].

If $\mathcal{V} = 0$, interference disappears i.e. $|D_1\rangle$ and $|D_2\rangle$ are now orthogonal states. Thus partial interference pattern appears due to non-orthogonality of detector states. Up to this the analysis of Storey *et al* is perfectly general and gives a quantum mechanical treatment of the problem of partial detection. But hereafter they introduce an assumption which restricts the generality of the conclusions. They consider O_ξ^1 as a function of x_1 and make a Fourier transform $\mathcal{O}_\xi^1(p)$ and then assume that $\mathcal{O}_\xi(p)$ satisfy the property $\mathcal{O}_\xi(p) = 0$, for $|p| \geq p_m$, i.e. $\mathcal{O}_\xi(p)$ is a compact function with a cut off at $\pm p_m$. With this assumption it can be proved that $\mathcal{V}$ satisfies the inequality:

$$1 \geq \mathcal{V} \geq 1 - p_m d/\hbar \tag{1}$$

where d is the separation between the slits. Here, p_m is interpreted as momentum transfer to the interfering 'particles'. For a precise which-path detection, $\mathcal{V} = 0$ and we get $p_m \geq \hbar/d$. This has the form of an uncertainty relation. Hence, Storey *et al* conclude that when the detection is precise there is a lower limit to the momentum transfer. Going farther they claim that this uncertainty is at the root of the complementarity between interference and which-path information.

It is surprising that only a formal similarity and not exactly the uncertainty relation involving variances has prompted the authors to assert that ME in class II complementarity is a consequence of the uncertainty principle. The entire novelty of Storey *et al*'s approach lies in the use of the particular device as a detector of position of the particle. In a double slit experiment the detector state $|D_i\rangle$ refers to position measurement x_i. The above analysis readily leads to the fact that the Welcher weg detector can be perfectly efficient only when d and p_m satisfy the relation $p_m \geq \hbar/d$. Thus the so-called uncertainty relation deduced by Storey *et al* is a property of their detecting device. The inequality in (1) is based on the assumption of a cut off in $\mathcal{O}_\xi(p)$. This restricts the validity of (1) only to special types of detectors. Not all detectors can be claimed to have this property. For example we can have detectors for which O_ξ^1 is equal to 1 near x_1 ($|x - x_1| \geq x_0$) and zero elsewhere. It can be shown that the corresponding Fourier transform $\mathcal{O}_\xi^1(p)$ cannot have a cut off and the detector is efficient when $|x_1 - x_2| \geq x_0$ (see Appendix A).

Two presumptions of the momentum transfer analysis need some scrutiny. One presumption is that when the spatial wave function gets localised, the momentum space wave function is also disturbed at least by an amount required by the uncertainty relation. In the case of a double slit experiment, consider an initial wave packet with zero expectation value of the transverse momentum. The wave function collapses from $[\psi_1 + \psi_2]/\sqrt{2}$ immediately after passing a double slit to ψ_2 (say), where ψ_i's are normalised wave packets at the respective slits. We note that for an unambiguous which-slit detection ψ_1 and ψ_2 are orthogonal:

$$\int \psi_1^* \psi_2 \mathrm{d}\tau = 0.$$

Since the momentum dispersion in this case is given by

$$\sigma_p = \langle p^2 \rangle = \int \psi^* p^2 \psi \mathrm{d}\tau,$$

it follows that:

$\sigma_p = (\sigma_p)_1 = (\sigma_p)_2$ (from symmetry, since ψ_1 and ψ_2 are identical packets).

The momentum dispersion is, therefore, same for the superposed and the collapsed state. Wiseman *et al* [16] have subsequently obtained this general result only for Welcher weg schemes with nonclassical, nonlocal momentum transfer.

The second presumption is, if the momentum space wave function changes, the particle whose state is represented by the function has experienced a momentum transfer from some other system. But the change in σ_p is an ensemble property. In a single event whether a momentum transfer has occurred or not cannot be asserted. In the case of a double slit experiment if the detector near one of the slits clicks, we know it has absorbed a small amount of energy. From the Fermi Golden Rule we can conclude that the interacting particle has exchanged energy/momentum with the detector. When the detector is not triggered it is certain that the detector has not taken part in any energy/momentum transfer process. Since the walls of slits impose only a rigid boundary condition, they cannot exchange energy/momentum with the particle. From these arguments we conclude that in this case no energy/momentum transfer is involved.

Finally we note that at the limit point $\mathcal{V} = 0$, the detector states $|D_1\rangle$ and $|D_2\rangle$ are orthogonal. So it will be possible to find a suitable observable η of which $|D_1\rangle$ and $|D_2\rangle$ are eigenvectors with different eigenvalues. So a measurement of η will lead to unambiguous which-path information and the initial entangled state collapses to either $\psi_1|D_1\rangle$ or $\psi_2|D_2\rangle$ and there is no interference. All further considerations which are introduced by Storey *et al* in the case of imprecise which-path determination becomes redundant. The recent controversy [17–20] between Scully *et al* and Storey *et al* regarding just one specific example of which-slit detection scheme appears, on the whole, somewhat misplaced.

In conclusion we note that the absence of interference in a detection process is due to decoherence arising from entanglement of interfering wave function branches with orthogonal detector states. The detector essentially has a quantum character and has degrees of freedom that get entangled with the system under observation in the larger Hilbert space of the total wave function. The entangled wave function gets irreversibly collapsed to either of the two wave function branches when which-slit information is read out from the detector. A comprehensive treatment of a macroscopic observation, however, would require a large multiparticle system and two essential ingredients of measurement: amplification and irreversibility [21].

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Appendix A

Let $O(x)$ be zero for $|x| \geq x_0$. For the corresponding Fourier transform $\mathcal{O}(p)$, we assume there is a cut off at p_0 i.e.,

$$\mathcal{O}(p) = 0 \text{ for } |p| \geq p_0. \tag{A1}$$

Then

$$\mathcal{O}(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-x_0}^{x_0} O(x) e^{-ipx/\hbar} \mathrm{d}x \tag{A2}$$

and

$$O(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-p_0}^{p_0} O(p) e^{ipx/\hbar} \mathrm{d}p \tag{A3}$$

Substituting for $O(p)$ in (A2) from (A1) we get

$$O(x) = \int_{-x_0}^{x_0} O(x') \left[\frac{\sin p_0(x-x')/\hbar}{\pi(x-x')} \right] \mathrm{d}x'. \tag{A4}$$

For (A3) to be true, it is necessary that the function within the bracket must be $\delta(x-x')$. This implies that $p_0/\hbar \to \infty$, i.e. there cannot be any cut off in $O(p)$.

References

[1] D Sen, A N Basu and S Sengupta, *Helv. Phys. Acta* **67**, 785 (1994)
[2] D Sen, A N Basu and S Sengupta, *Curr. Sci.* **69**, 426 (1995)
[3] D Sen, A N Basu and S Sengupta, *Z. Naturforsch.* **52a**, 398 (1997)
[4] M O Scully, B G Englert and H Walther, *Nature* **351**, 111 (1991)
[5] P Storey, S Tan, M Collet and D Walls, *Nature* **367**, 626 (1994)
[6] L S Schulman, Phys.Lett. **A211**, 75 (1996)
[7] N Bohr in P A Schilpp Ed. A Einstein: *Philosopher-scientist* (Library of Living Philosophers, Evanston, Ill., vol 7, 1949; Harper and Row, NY, 1959) pp.200-241
[8] See for example H P Robertson, Phys.Rev. **34**, 163(1929)
E Schrödinger, Berliner Berichte, 296(1930)
E Merzbacher, *Quantum mechanics* (John Wiley, New York 1970) pp. 157-161. The difficulties involved in treating simultaneous measurement of noncommuting observables within the usual framework of quantum mechanics have been discussed by H Margenau and R N Hill, *Prog. Theor. Phys.* **26**, 727(1961)
L Cohen, *J. Math. Phys.* **7**, 781 (1966)
J L Park and H Margenau, *Int. J. Theor. Phys.* **1**, 211 (1968)
W E Lamb Jr., *Phys. Today* **22**, 23 (1969)
J L Park and H Margenau, in *Perspectives in quantum theory*, edited by W Yourgrau and A van der Merwe (Dover, New York, 1979) pp.37-70
H Margenau, *Ann. Phys.* **23**, 469(1963)
[9] W Heisenberg, *The physical principles of quantum theory* (University of Chicago Press, 1930)
[10] A Tonomura, J Endo, T Matsuda and T Kawasaki, *Am. J. Phys.* **57**, 117 (1989)
[11] A Zeilinger, R Gaehler, C G Shull and W Treimer, in *Proc. of the conference on neutron scattering*, Argonne, 1981, edited by J Faber Jr. (AIP New York, 1982) p-93
[12] M O Scully and K Drühl, *Phys. Rev.* **A25**, 2208 (1982)
[13] S M Tan and D F Walls, Phys. Rev. **A47**, 4663 (1993)
[14] P Kwiat, A Steinberg and R Chiao, *Phys. Rev.* **A45**, 7729 (1992)
[15] N C Petroni and J P Vigier, *Found. Phys.* **22**, 1 (1992)
H Rauch and J P Vigier, *Phys. Lett.* **A151**, 269 (1990)
[16] H Wiseman,F Harrison, M Collet, S Tan, D Walls and R Killip, *Phys. Rev.* **A56**, 55 (1997)
[17] B Englert, H Fearn, M Scully and H Walther, in *Quantum interferometry* (edited by F De Martini, G Denardo and A Zeilinger) pp.103-119 (World Scientific, Singapore, 1994)
[18] P Storey, S Tan, M Collett and D Walls, in *Quantum interferometry*, Edited by F De Martini, G Denardo and A Zeilinger (World Scientific, Singapore, 1994) pp. 120-129
[19] B Englert, M Scully and H Walther, *Nature* **375**, 367 (1995)

[20] P Storey, S Tan, M Collet and D Walls, *Nature* **375**, 368 (1995)
[21] B Gaveau and L S Schulman, *J. Stat. Phys.* **58**, 1209 (1990)
L S Schulman, *Time's arrow and quantum measurement* (Cambridge Univ. Press, Cambridge) in press