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Stability analysis of a model equilibrium for a gravito-electrostatic sheath in a colloidal plasma under external gravity effect

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Abstract. The present contribution tries to find a scientific answer to the question of stability of an equilibrium plasma sheath in a colloidal plasma system under external gravity effect. A model equilibrium of hydrodynamical character has been discussed on the basis of quasi-hydrostatic approximation of levitational condition. It is found that such an equilibrium is highly unstable to a modified-ion acoustic wave with a conditional likelihood of linear driving of the so-called acoustic mode too. Thus, it is reported (within fluid treatment) that a plasma-sheath edge in a colloidal plasma under external gravity effect could be highly sensitive to the acoustic turbulence. Its consequential role on possible physical mechanism of Coulomb phase transition has been conjectured. However, more rigorous calculations as future course of work are required to corroborate our phenomenological suggestions.

Keywords. Colloidal plasma; acoustic waves; levitational equilibrium; acoustic turbulence; Coulomb phase transition; wave turbulence model.

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1. Introduction

The process of electrostatic levitation of dust grains against self gravity [1] or external gravity [2] is relevant to a number of astrophysical phenomena. Pandey *et al* [1] have reported the occurrence of new hybrid oscillatory mode (let us term it as the gravito-electrostatic mode) under exact hydrostatic approximation of the self-gravito-electrostatic coupling when these two forces operate on the same scales of space and time. This situation demands a condition that $(Gm_d^2/q_d^2) \approx 1$ [1], where G is gravitational constant, m_d the mass of the dust grains and q_d the charge on the dust grains. Avinash *et al* [2] have carried out a model calculation to show that an acoustic instability occurs for the case of levitated equilibrium in the context of spoke formation in planetary rings. Here, the required electric field for levitational equilibrium is sustained due to Pedersen conductivity. Anomalous particle diffusion transport co-efficient has also been derived and its consequence on levitational condition has been discussed. It is found that the anomalous transport improves the levitational condition.

It is, however, emphasised that these investigations consider an exact hydrostatic approximation for levitational equilibrium. Very recently we have carried out a theoretical analysis to report the possibility of a quasi-levitational kind of equilibrium to exist as a transition interface lying in between the free fall presheath and Debye sheath in a gravitationally sensitive (active) plasma (GSP) system. Using Riemann's proposal of three scale analysis [3] developed in the context of collisional plasma, it has been deduced that a quasi-levitational kind of equilibrium exists in GSP on an intermediate scale with non-monotonous structure for smooth transition to the main Debye sheath. It is thus highly desirable to study the stability behaviour of such equilibria in the context of plasma sheath under gravity effect.

Before we proceed further, let us have a brief outlook about the latest scientific ideas on dusty plasma from a systematic nomenclature point of view. In fact, a dusty plasma should be defined as a colloidal form of the dust grainlike impurity ions (DGLII) in a background plasma medium termed as the parent plasma [4]. The production of these impurity ions occurs through surface charging mechanism of the neutral dust grains of either external origin or of internal origin [5]. However, the "dust grainlike impurity ions" and the "dust grains" will appear interchangeably in the text. This (the charging of the neutral dust grains) causes a deviation from equilibrium quasineutrality of the parent plasma wherein the dust grains are supposed to be embedded. Thus a net background positive potential at global scale arises to cause the electrostatic confinement of the dust grains with random Brownian like motion [6] of thermal character arising due to Coulomb interactions. Thus in our opinion, it seems worth calling the dusty plasma as a colloidal plasma [7] which was coined about four decades before in the scientific literature to signify the same plasma system as a dusty plasma of recent origin. Details about colloidal plasmas have been given in a review article by Sodha and Guha [7]. Thus in the text "dusty plasma" and "colloidal plasma" will be used interchangeably. Now, if these dust grains are susceptible to the gravitational force fields (external or internal) the resulting colloidal plasmas should be termed as GSP. Such plasmas are bound to sustain a quasi-levitational kind of equilibrium (as discussed above) near the boundary surface.

The present contribution intends to analyse the stability behaviour of such equilibria against the acoustic disturbance. Simple hydrodynamical model has been used to discuss about the equilibrium of interest with linear normal mode analysis of the acoustic turbulence. Section 2 deals with the formulation of the problem and model equations for equilibrium and perturbation dynamics within fluid approximation. For simplicity, a constant charge model for the dust grains has been considered. This section also includes the mathematical derivation of desired dispersion relation for the linear acoustic turbulence. In §3, the results and discussions have been described. Final conclusions and future scope of work have been summarized in §4.

2. Formulation of the problem and mathematical model

As specified earlier, a gravitationally sensitive or to say active colloidal plasma in contact with boundary wall can sustain an equilibrium plasma sheath with an intermediate scale length L_s obeying a levitational condition under hydrostatic approximation. This region exists on a quasi-free fall sheath scale. It is better to term it as an acoustic viscous zone or levitational zone where quasi-neutrality holds good and it acts as a transition interface between the Debye sheath (free from gravitational effect) and purely free fall dominated region. It is assumed that the dust grains contain constant charges and hence the colloidal plasma under consideration looks like a normal multicomponent plasma system containing a highly charged heavier impurity ion species. Since, the above mentioned levitational zone is associated, in general, with non-uniformity in plasma population density and inertial flow of the dust grains to maintain the gravito-electrostatic plasma presheath, the likelihood of plasma turbulence of different possible plasma acoustic modes could not be avoided. This is to emphasise that a final equilibrium condition could be established on the acoustic time scale of the low frequency version of the so-called acoustic mode [8,9]. Let us first discuss about the possible dynamical equations and associated equilibrium structure for a model consideration of the levitational zone of plasma sheath structure of a GSP and its stability against acoustic disturbance.

On the inertial time scale of the dust grainlike impurity ions, in general, the parent electrons and ions could be modelled by Boltzmannian distributions;

$$n_e = n_{e0}(z_0) \exp\left\{\frac{e(\phi - \phi_0(z_0))}{T_e}\right\}, \tag{1}$$

$$n_i = n_{i0}(z_0) \exp\left\{\frac{-e(\phi - \phi_0(z_0))}{T_i}\right\}. \tag{2}$$

Here, z_0 defines the levitational point.

The inertial dynamics of these dust grainlike impurity ions can be described by their momentum and continuity equations as given below:

$$\frac{\partial v_I}{\partial t} + (v_I \cdot \nabla) v_I = -\frac{q_I}{m_I}\nabla\phi + g_E, \tag{3}$$

$$\frac{\partial n_I}{\partial t} + \nabla \cdot (n_I v_I) = 0. \tag{4}$$

Here the subscript 'I' denotes the impurity ions in general.

The assumption of quasi-neutrality in equilibrium and perturbation closes the set of governing model equations.

2.1 Model equilibrium

It is, in general, a rule to describe the equilibrium before carrying out its stability analysis. Under generalised boundary conditions of the levitational kind of gravito-electrostatic plasma presheath in GSP, an equilibrium condition could be written by using Taylor's expansion around the levitational point ($z = z_0$):

$$\left[\frac{1}{2}v_{I0}^2(z_0) + \frac{q_I}{m_I}\phi_0(z_0) - g_E z_0\right]$$

$$+\left[\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}z}v_{I0}^2\bigg|_{z=z_0}+\frac{q_I}{m_I}\frac{\mathrm{d}\phi_0}{\mathrm{d}z}\bigg|_{z=z_0}-g_E\right](z-z_0)=\text{ integral constant.} \tag{5}$$

Similarly, from the equilibrium quasi-neutrality condition, one can derive (again using Taylor's expansion around $z = z_0$) the following relationship between various plasma population densities and their gradients;

$$[n_{e0}(z_0)-n_{i0}(z_0)-Z_I n_{I0}(z_0)]$$

$$+\left[\frac{\mathrm{d}n_{e0}}{\mathrm{d}z}\bigg|_{z=z_0}-\frac{\mathrm{d}n_{i0}}{\mathrm{d}z}\bigg|_{z=z_0}-Z_I\frac{\mathrm{d}n_{I0}}{\mathrm{d}z}\bigg|_{z=z_0}\right](z-z_0)=0. \tag{6}$$

Now considering a weak equilibrium flow gradient as compared to that of space charge potential gradient i.e.

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}z}v_{I0}^2\bigg|_{z=z_0}\ll\frac{q_I}{m_I}\frac{\mathrm{d}\phi_0}{\mathrm{d}z}\bigg|_{z=z_0},$$

one can rewrite the equilibrium condition (5) as,

$$\frac{1}{2}v_{I0}^2(z_0)+\frac{q_I}{m_I}\phi_0(z_0)-\frac{v_{If}^2}{2}=0. \tag{7}$$

Where $v_{If}^2 = 2_{g_E z_0}$ specifies the equilibrium free fall kinetic energy of the dust grain-like impurity ions near levitational point $z = z_0$. This implies the energy conservation equation and indicates the existence of a Bernoulli kind of equilibrium flow under the joint action of gravito-electrostatic potential fields. Similarly, under the above discussed approximation, a force balance equation of a quasi-hydrostatic approximation over a finite spatial extension $(z - z_0)$ results as,

$$\frac{q_I}{m_I}\frac{\mathrm{d}\phi_0}{\mathrm{d}z}\bigg|_{z=z_0}-g_E\approx 0. \tag{8}$$

The term 'quasi' is derived from energy balance equation (7) with almost constant kinetic flow energy.

On the same logical arguments quasi-neutrality condition (6) leads to derive,

$$n_{e0}(z_0)-n_{i0}(z_0)-Z_I n_{I0}(z_0)=0, \tag{9}$$

and

$$\frac{\epsilon_{ne}}{L_{ne}}-\frac{1}{L_{ni}}-\frac{\epsilon_I}{L_{nI}}=0, \tag{10}$$

where,

$$\epsilon_{ne}=\frac{n_{e0}(z_0)}{n_{i0}(z_0)},\qquad \epsilon_I=Z_I\frac{n_{I0}(z_0)}{n_{i0}(z_0)},\qquad Z_I=\frac{q_I}{e},$$

$$L_{ne}=[\nabla_z\{\ln n_{e0}\}|_{z=z_0}]^{-1},\ L_{ni}=[\nabla_z\{\ln n_{I0}\}|_{z=z_0}]^{-1},\ L_{nI}=[\nabla_z\{\ln n_{I0}\}|_{z=z_0}]^{-1}.$$

This is to note that the nature of charge on the dust grainlike impurity ions has not yet been specified and hence ϵ_I contains sign through Z_I.

Now for $\epsilon_{ne} \ll 1$ and $\epsilon_I \leq 1$, (10) reduces to

$$L_{ni} \sim -\epsilon_I L_{nI} = L_s. \tag{11}$$

Considering the equilibrium Boltzmann distribution for the electrons and ions near the levitational potential $\phi_0(z_0) \sim (m_I/q_I)g_E z_0$, quasi-neutrality condition (9) could be solved to estimate the value of L_s as

$$-n_{i0}\exp\left(\frac{-e\phi_0(z_0)}{T_i}\right) - Z_I n_{I0}(z_0) = 0,$$

or

$$\exp\frac{e}{T_i}\frac{m_I}{q_I}g_E z_0 = -\frac{Z_I n_{I0}(z_0)}{n_{i0}}.$$

Here, n_{i0} is the parent ion density in space charge free region dominated by free fall dynamics of the dust grains. Thus the quasi-neutral levitational sheath scale $L_s \sim (C^2_{\mathrm{scam}}/g_E\epsilon_I)$. Furthermore,

$$\frac{L_s}{\lambda_{\mathrm{De}}} \sim \left(\frac{\lambda_{\mathrm{Di}}}{\lambda_{\mathrm{De}}}\right)^2 Z_I\frac{m_i}{m_I}\frac{C_s^2}{g_E\lambda_{\mathrm{De}}}. \tag{12}$$

Here, $C_s^2 = (n_{i0}/n_{e0})(T_e/m_i)$ denotes the sound speed of a modified-ion-acoustic mode [9] in a multispecies plasma containing the dust grainlike impurity ions. Similarly $C^2_{\mathrm{scam}} = Z_I^2(n_{I0}/n_{i0})(T_i/m_I)$ is the sound speed of the low frequency version of the so-called acoustic mode in the same plasma system. Similarly,

$$\frac{L_s}{\lambda_{\mathrm{Di}}} \sim \left(\frac{\lambda_{\mathrm{Di}}}{\lambda_{\mathrm{De}}}\right) Z_I\frac{m_i}{m_I}\frac{C_s^2}{g_E\lambda_{\mathrm{De}}}. \tag{13}$$

3. Stability analysis of the model equilibrium

In §2, mathematical formulation for a levitational kind of plasma presheath equilibrium under external gravity has been discussed. This equilibrium provides a physically consistent model approach to analyse the stability behaviour of the intermediate scale as a plasma-sheath interface in a colloidal plasma under external gravity effect. Since the aforesaid equilibrium is free from ambient magnetic field, only acoustic modes will be considered for the description of possible linear plasma turbulence.

3.1 Linear excitation of the so-called acoustic mode

This mode is characterised by the acoustic approximation [8,9];

$$\epsilon_T \gg \epsilon_n \epsilon_z^2 \gg \epsilon_m, \frac{T_i}{T_e}\frac{m_e}{m_I},$$

where,

$$\epsilon_T = \frac{T_i}{T_I}, \quad \epsilon_n = \frac{n_{i0}}{n_{I0}}, \quad \epsilon_z = \frac{Z_i}{Z_I}, \quad \epsilon_m = \frac{m_i}{m_I}.$$

The perturbational scale length under local approximation of the quasi-neutrality requires the following inequality to hold good;

$$\lambda_{\mathrm{Di}} \ll \lambda \ll L_s.$$

On this acoustic scale of the dust grains, electron's and ion's perturbations will follow the Boltzmannian distributions with linear counterparts as;

$$\tilde{n}_e = n_{e0}(z_0)\frac{e\tilde{\phi}}{T_e},$$

$$\tilde{n}_i = -n_{i0}(z_0)\frac{e\tilde{\phi}}{T_i}. \tag{14}$$

On the other hand, the dust grains will follow the inertial dynamics. Assuming weak density gradient and flow gradient, the perturbed dynamical equations from (3) and (4) could be Fourier transformed to derive the density distribution for dust grain-like impurity ions as

$$\tilde{n}_I = \frac{(1 - (i/kL_{n_I}))}{\Omega^2} k^2 n_{I0} \frac{q_I}{m_I}\tilde{\phi}, \tag{15}$$

where

$$\Omega = \omega - kv_{I0} + iv'_{I0}, v'_{I0} = \frac{\mathrm{d}v_{I0}}{\mathrm{d}z}.$$

In deriving (15), it is assumed that the fluctuation variables vary as $\exp(-i\omega t + ikz)$. Now under the approximations of $\epsilon_{ne} \ll 1$ and quasi-neutrality, the dispersion relation could be derived to yield,

$$\Omega = \pm kC_{\mathrm{scam}}\left(1 - \frac{i}{kL_{nI}}\right)^{1/2}. \tag{16}$$

Again for $kL_{nI} \gg 1$ (local approximation), the above relation could be rewritten as,

$$\omega = kv_{I0} \pm kC_{\mathrm{scam}}\left(1 - \frac{i}{2kL_{nI}}\right) - iv'_{I0}.$$

Thus,

$$\omega_r = (kv_{I0} \pm kC_{\mathrm{scam}}),$$

$$\omega_i = \mp\frac{kC_{\mathrm{scam}}}{2kL_{nI}} - v'_{I0}. \tag{17}$$

Here v'_{I0} represents the weak flow gradient of the dust grains near the plasma-sheath edge in GSP. From equilibrium flux conservation of the dust grains, one can infer that the population density gradient and flow gradient are opposite to each other.

3.2 Linear excitation of the modified-ion acoustic wave

This mode is characterised by the following acoustic approximation [8,9],

$$\frac{T_i}{T_e} \ll \frac{n_{i0}}{n_{e0}} \ll \frac{m_i}{m_e}.$$

The relative inertial response time scale $\tau_i^{\rm eq}$ of the parent ions for equilibrium levitational potential to that of its natural inertial response time scale $\tau_i \sim \omega_{\rm pi}^{-1}$ could be estimated [10] to yield;

$$\frac{\tau_i^{\rm eq}}{\tau_i} = Z_I \frac{m_i}{m_I} \frac{C_s^2}{g_E \lambda_{\rm De}} \tag{18}$$

where $C_s^2 = (n_{i0}/n_{e0})(T_e/m_i)$. The perturbational scale length on this acoustic time scale should satisfy $\lambda_{\rm De} \ll \lambda \ll L_s$. If we consider a typical laboratory hydrogen plasma parameters; $n_{i0} \sim 10^{10}$ cm^{-3}, $T_e \sim 1$ eV, $T_i \sim 0.1$ eV contaminated with dust grains of size $a \sim 1\mu$m = 10^{-4} cm; we can show that

$$\frac{\tau_i^{\rm eq}}{\tau_i} \sim 10^2 \left(\frac{n_{i0}}{n_{e0}}\right) \gg 1.$$

Again,

$$\frac{L_s}{\lambda_{\rm De}} \sim 10^2 \left(\frac{n_{i0}}{n_{e0}}\right)^{-1/2}.$$

It is thus obvious to note that the validity of the equilibrium thermal response of the parent ions even on the modified ion acoustic wave time scale could be ensured in laboratory experiments. Hence, a levitational equilibrium on intermediate transition scale of a plasma sheath under gravity effect could be a model equilibrium for consideration of the linear turbulence of the modified ion acoustic wave too but without equilibrium inertial flow in the parent ions. The above ordering of the inertial time scales $(\tau_i^{\rm eq}/\tau_i)$ of the parent ions justifies the stationarity approximation of the parent ions [10] under the influence of equilibrium plasma sheath potential for the quasi-levitational equilibrium of the dust grains to hold good. On this acoustic time scale, the perturbational dynamics of the dust grains could be ignored. However, the electrons will respond as the Boltzmannian and the parent ions as inertial. Following a linear normal mode analysis, one can directly write the dispersion relation in the form [2].

$$\omega = \pm k C_s \left(1 - \frac{i}{k L_{ni}}\right)^{1/2}. \tag{19}$$

Under local approximation $kL_{ni} \gg 1$, the above dispersion relation could be reduced to;

$$\omega \sim \pm k C_s \left(1 - \frac{i}{2 k L_{ni}}\right). \tag{20}$$

This implies that

$$\omega_r = \pm k C_s,$$

$$\omega_i = \mp \frac{1}{2} \frac{k C_s}{k L_{ni}}.$$

4. Results and discussion

Let us first specify the directional co-ordinates for the sound wave propagation to discuss about the stability analysis of (17) and (20). According to our specification, upper sign indicates the sound wave propagation towards plasma (i.e. away from the levitational point), whereas the lower one for the sound wave propagation towards the Debye sheath. It is thus obvious to see that the density gradient destabilises both the low frequency version of the so-called acoustic mode and the modified ion acoustic mode. The flow gradient stabilises the low frequency version of the so-called acoustic mode and thus imposes a threshold on the scale lengths of the flow gradient and density gradients. Thus for the instability to occur an inequality $(L_{vI}/L_{nI}) > (v_{I0}/C_{\mathrm{scam}}) > 1$ is required. Here, $L_{vI} = [\nabla_z(\ln v_{I0})]^{-1}$ denotes the flow gradient scale length. From the flux conservation of the dust grains, one can derive (by using Taylor's expansion near the levitational point $(z = z_0)$) the following relationship between these gradients;

$$n_{I0}(z_0)v'_{I0}|_{z=z_0} + n'_{I0}|_{z=z_0}v_{I0}(z_0) + 2n'_{I0}|_{z=z_0}v'_{I0}|_{z=z_0}(z - z_0) = 0.$$

Here the dash ($'$) as superscript denotes the space derivative.

This results into

$$(z - z_0) = -L_{vI} - L_{nI}. \tag{21}$$

This is to note that for lowest order flux conservation $L_{vI} \uparrow\downarrow L_{nI}$.

Thus it seems logical to argue that the required threshold for the onset of linear acoustic turbulence may be attended in laboratory plasmas over a finite spatial extension of the levitational equilibrium with weaker flow gradient. This is to add further that due to non-monotonous character of the transition to the Debye sheath, an unfavourable density gradient towards the Debye sheath (near levitational point) may allow the existence of linear acoustic turbulence of both the kinds only in the free fall region. It is therefore conjectured to argue that the site for Coulomb phase transition should be localised near the plasma-sheath boundary if the proposed wave turbulence model [11], is assumed to operate as a possible physical mechanism.

5. Conclusions and future scope of work

It seems worthwhile to conclude that the plasma sheath edge of a colloidal plasma under gravity effect should be an active locality to produce linear and nonlinear acoustic turbulence. If it is so, the proposed wave turbulence model may be an important phenomenological model suggestion for adequate attention to pursue further research on the basis of rigorous mathematical calculations especially under dominant role of correlation effects [10]. However, this is to emphasize that the study of collective plasma dynamics of a strongly non-ideal plasma is a nontrivial problem to carry out. Preliminary arguments based on the model calculations have been forwarded for the possibility of selective excitation of the correlation driven modified ion acoustic wave and its nonlinear counterpart (positive solitons of desired geometrical shape) [11]. It seems logically meaningful to conjecture that even the correlation may also drive modified ion acoustic wave turbulence with subsequent formation of positive solitons to provide virtual particles of finite size with effective positive localised space charge. In the same time the correlation effect may also affect the vertical acoustic turbulence as discussed in the present research contribution. However, these are the speculative considerations in favour of the wave turbulence model and need serious investigations to give physically viable footing to the proposed model for understanding the Coulomb phase transition observed in colloidal plasmas (dusty plasmas) containing the dust grainlike impurity ions.

Finally, it can be discussed that the acoustic waves moving towards the Debye sheath may die out due to adverse conditions beyond the levitational point. This point may be supposed to represent a point of highest value of a quasi-neutral plasma sheath potential on a quasi-free fall sheath scale with nonmonotonous turning towards the Debye sheath. The likelihood of nonmonotonous structure to provide a smooth nonlinear transition to the Debye sheath as discussed in the introduction corroborates our ideas.

Future scope of basic physics work on plasma sheath in colloidal plasmas of practical relevance includes the consideration of dynamical effects of charge on the dust grains, either due to plasma turbulence or due to statistical fluctuation under discrete charging model of the dust grains [12]. This is beyond the limit of analytical approach and hence numerical analysis is needed. Furthermore, the effect of ambient magnetic field on plasma sheath in GSP forms another problem for contamination control in processing plasmas of industrial interest. Recent evidences of dust contamination in fusion devices [13] have provided a new impetus for vigorous research activities on static and dynamical behaviour of the plasma sheath in colloidal plasmas. Thus a wide range applicability of the plasma sheath problem in colloidal plasmas of general nature has given a moral boost for research activities in the field of basic physics of plasma sheath phenomenon under different force field configuration.

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[11] C B Dwivedi and M Bhattacharjee, have proposed a phenomenalogical approach to understand the Coulomb phase transition in correlated plasma systems. This approach is based on the wave turbulence model and considers nonlinear quantisation of the modified ion acoustic wave to generate virtual particles with positive polarity. Subsequently these virtual particles and the real dust grains could form a new thermodynamical state for Coulomb condensation to occur depending on the mutual Coulomb coupling The correlation effect may drive the turbulence through the mechanism as discussed in the text. These ideas have been reported in national and international conferences [6] and also in *Proceedings of XI Plasma Science Society of India (PSSI) Symposium on Plasma Science and Technology* held in Oct. 1996 at Barkatullah Univ., Bhopal, Allied Publishers Ltd., New Delhi, 1998 edited by A K Gwal, p. 239-245 and references therein.

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