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A phenomenological study of a charge-symmetry-breaking component in ΛN interaction

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Abstract. Here, we make an indirect phenomenological study of the possible presence of a CSB component in the ΛN interaction in medium and heavy hypernuclei using a semi-empirical formula for the difference in the ground state B_Λ of hypernuclear isobars. We find that light hypernuclei are better suited than heavier hypernuclei for such information.

Keywords. Semi-empirical; binding energy.

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1. Introduction

One would have hoped that different isobars of the same hypernucleus could provide information on the possible presence or otherwise of a charge-symmetry-breaking (CSB) component of the Λ-nucleon (ΛN) interaction. Clearly, the quantity to look at is the difference between the Λ-binding energy (B_Λ) of isobars. The first apparent evidence for ΛN CSB came from the difference in Λ-binding energies of the $^4_\Lambda$H and $^4_\Lambda$He. Bodmer and Usmani [1] have showed that meson-exchange CSB models are consistent with the phenomenological CSB potential for the triplet but not for the singlet case. The CSB interaction is found to be effectively spin-independent and reduces the charge symmetric attractive strength by $\cong 0.05$ MeV relative to the Λp strength.

CSB in the ΛN interaction may be attributed to a number of effects, the most important of which is probably isospin mixing of baryons and mesons [2,3]. The possibility of having a non-zero CSB coupling $\Lambda\Lambda\pi^0$ due to (Λ, Σ^0) mixing was emphasized by Dalitz and Von Hippel [3]. The mixing between mesons, such as (π^0, η) and (ρ^0, ω), is possible in principle. However, the appropriate contribution to CSB of the ΛN interaction vanishes in the limit of equal admixed meson masses. Thus, (ρ^0, ω) mixing is of no practical interest for ΛN CSB, whereas (π^0, η) mixing contributes, but less than (Λ, Σ^0) mixing to CSB. Mass differences between Σ^+ and Σ^- and between a proton and a neutron in the intermediate states appropriate for the two-pion exchange ΛN interaction [2] are other, though less significant, sources of CSB in the ΛN interaction.

A number of hypernuclear isobars have been identified in the p-shell, however, the information of CSB in the ΛN interaction has not yet been extracted. The contribution of the CSB meson-exchange potentials considered in the literature, which violates the charge symmetry by 1–2%, is not found to increase with mass number (A) to account for the differences in the energies of heavier nuclei. Here, we make an indirect phenomenological study of the possible presence of a CSB component in the ΛN interaction in the medium and heavy hypernuclei using a semi-empirical formula for the difference in the ground state B_Λ of a pair of medium or heavy hypernuclei having same core mass number ($A_c = N + Z$) but different number of protons (Z).

In the folding model approach we obtain a semi-empirical formula [4] for B_Λ in inverse powers of A_c. The detailed procedure is given elsewhere [5]. Assuming that the Λ 'sees' two separate potentials due to the proton and the neutron distributions in a nucleus, we first obtain separate single-parameter, semi-empirical formulae, as given in [6] for $B_{\Lambda N}$, the Λ-binding energy to the proton distribution constituted by the Z protons and $B_{\Lambda Z}$, the Λ-binding energy to the neutron distribution constituted by the N neutrons. It is straightforward to write down the formula for the average of these two binding energies. Earlier, it has been shown in [6] that for almost the same values of the parameters, of the $\Lambda - Z$ and $\Lambda - N$ force, in the two cases, the above average value is same as the Λ-binding given by the average potential. As the formula given here is used only for a study, which is of a rather qualitative nature, we do not care for exact values of the parameters. Therefore, plausible values of the $\Lambda - Z$ and $\Lambda - N$ strengths have been employed.

Over a large range of mass numbers, we test the effect of CSB component with this semi-empirical formula for the differences in the ground state Λ-binding energies of isobars. We find that dependence of Λ-binding energy on Z is small even for relatively large ΔZ, where ΛZ is the difference in the number of protons of the two isobars. Formulae are given in the next section and results and discussion in §3.

2. Derivation of the formula

Assuming that Λ 'sees' two separate $\Lambda - Z$ and $\Lambda - N$ potentials in a nucleus, for zero-range ΛN interaction, in the folding model, using the point proton and point neutron densities available in the literature [7], we get for a hypernucleus having Z number of protons and N number of neutrons

$$V_{\Lambda Z}(r) = -ZV_{0p}\, {}^{Z}\rho_p(r) \quad \text{and} \quad V_{\Lambda N}(r) = -NV_{0n}\, {}^{Z}\rho_n(r). \tag{2.1}$$

The point proton and point neutron densities, denoted here as ${}^{Z}\rho_p(r)$ and ${}^{Z}\rho_n(r)$ respectively, (normalized to unity) are chosen to be of Woods–Saxon (W–S) form. The forms of radius 'R' and diffuseness 'a' parameters along with their parameters for proton and neutron are taken from [7]. With $\Lambda - Z$ and $\Lambda - N$ potentials given above and following the approach of Deloff [5] and Flügge [8], for the condition $e^{-(R/a)} << 1$, we obtain the ground state Λ-binding energy in the two potentials as given in [6].

$$B_{\Lambda Z} = D_{\Lambda Z} - \frac{\hbar^2\pi^2}{2\mu_{\Lambda Z}}\left[C'_{0p}A_c^{-2/3} - C'_{1p}A_c^{-1} + C'_{2p}A_c^{-4/3} - \ldots\right]$$

and

$$B_{\Lambda N} = D_{\Lambda N} - \frac{\hbar^2\pi^2}{2\mu_{\Lambda N}}\left[C'_{0n}A_c^{-2/3} - C'_{1n}A_c^{-1} + C'_{2n}A_c^{-4/3} - \ldots\right], \quad (2.2)$$

where $\mu_{\Lambda Z} = (m_\Lambda m_p Z)/(m_\Lambda + m_p Z)$ and $\mu_{\Lambda N} = (m_\Lambda m_n Z)/(m_\Lambda + m_n N)$ are the $\Lambda - Z$ and $\Lambda - N$ reduced masses. $D_{\Lambda Z}$ and $D_{\Lambda N}$ are the Λ-well depths in $V_{\Lambda Z}(r)$ and $V_{\Lambda N}(r)$ potentials. The coefficients C'_{ip} and $C'_{in}(i = 0, 1, 2, \ldots)$ are expressed in terms of respective radius and diffuseness parameters of proton and neutron and Λ-well depths as shown in [6]. The average Λ-binding energy, for the hypernucleus having A number of baryons and Z number of protons is

$${}^{Z,A}B_\Lambda = \frac{Z}{A_c}B_{\Lambda Z} + \frac{N}{A_c}B_{\Lambda N}. \quad (2.3)$$

Using (2.1), we get

$$D_{\Lambda Z} - D_{\Lambda N} = V_{0n}A_c\left[\left(1 - \frac{\Delta V}{V_{0n}}\right)\frac{Z}{A_c}\,{}^Z\rho_p(0) - \left(1 - \frac{Z}{A_c}\right){}^Z\rho_n(0)\right]. \quad (2.4)$$

Taking only the first two terms of both $B_{\Lambda Z}$ and $B_{\Lambda Z}$ expressions and using (2.3) and (2.4), we get

$$\begin{aligned} {}^{Z,A}B_\Lambda = D_{\Lambda N} - \frac{\hbar^2\pi^2}{2\mu_{\Lambda N}}C'_{0n}A_c^{-2/3} + \frac{Z}{A_c}\Bigg\{V_{0n}A_c\Bigg[\left(1 - \frac{\Delta V}{V_{0n}}\right)\frac{Z}{A_c}\,{}^Z\rho_p(0) \\ - \left(1 - \frac{Z}{A_c}\right){}^Z\rho_n(0)\Bigg] - \frac{\hbar^2\pi^2}{2}A_c^{-2/3}\left[\frac{C'_{0n}}{\mu_{\Lambda N}} - \frac{C'_{0p}}{\mu_{\Lambda Z}}\right]\Bigg\}, \end{aligned} \quad (2.5)$$

where $\Delta V = V_{0n} - V_{0p}$ and $A_c = N + Z$. The Λ-binding energy of hypernuclei having same A_c but Z' number of protons and N' number of neutrons can be obtained by replacing Z and N in the above equation by Z' and N'. The difference in ground state Λ-binding energies of the two isobars of hypernuclei can then be written as

$$\begin{aligned} {}^{Z,Z';A}\Delta B_\Lambda = \frac{{}^Z\rho_p(0)}{A_c}\left[Z^2 - Z'^2\left(\frac{{}^{Z'}\rho_p(0)}{{}^Z\rho_p(0)}\right)\right]V_{0n}\left(1 - \frac{\Delta V}{V_{0n}}\right) + \\ {}^Z\rho_n(0)\left[(A_c - 2Z) - (A_c - 2Z')\left(\frac{{}^{Z'}\rho_n(0)}{{}^Z\rho_n(0)}\right)\right]V_{0n} + \frac{{}^Z\rho_n(0)}{A_c} \\ \times\left[Z^2 - Z'^2\left(\frac{{}^{Z'}\rho_n(0)}{{}^Z\rho_n(0)}\right)\right]V_{0n} - \frac{\hbar^2\pi^2}{2}\Bigg\{C'_{0p}A_c^{-5/3}\left[\frac{Z}{\mu_{\Lambda Z}} - \frac{Z'}{\mu_{\Lambda Z'}}\right] \\ + C'_{0n}\left[A_c^{-2/3}\left(\frac{1}{\mu_{\Lambda N}} - \frac{1}{\mu_{\Lambda N'}}\right) - A_c^{-5/3}\left(\frac{Z}{\mu_{\Lambda N}} - \frac{Z'}{\mu_{\Lambda N'}}\right)\right]\Bigg\}. \end{aligned} \quad (2.6)$$

3. Result and Discussion

The analytical formula for ${}^{Z,Z';A}\Delta B_\Lambda$, the ground state Λ-binding energy difference of the two isobars, is mainly used to draw some qualitative conclusions. The point proton and point neutron densities, both normalized to unity, are chosen to be of W–S form. The forms of radius and diffuseness for proton and neutron and their parameters, as mentioned earlier, are taken from [7].

We choose approximate plausible value of the strength parameters, V_{0p} and V_{0n} of the $V_{\Lambda Z}(r)$ and $V_{\Lambda N}(r)$ potentials, respectively. $(\Delta V/\bar{V})$ is chosen to be $\cong 0.14$, where $\bar{V}$ is an average of V_{0p} and V_{0n}. With these parameters, ${}^{Z,Z';A}\Delta B_\Lambda$, calculated for stable or the near-stable isobars of hypernuclei, over a large mass number range varying from 25 to 250, is found to be very small (< 0.5 MeV). Although, reasonable values of ΔZ are at most 2 or 3 for the stable or near-stable isobars, we do not expect very large difference in the qualitative nature of the results even for relatively larger ΔZ. Now, for large ΔZ, in principle, one would have to consider exotic isobars. First of all, it is most unlikely to create an exotic hypernucleus because exotic nuclei are so short-lived and even, for the sake of an argument, if a lighter exotic hypernucleus could be produced, the value of the crucial term being considered here is still rather small to be of any particular advantage.

We see that the dominant term in ${}^{Z,Z';A}\Delta B_\Lambda$ is

$$ {}^{Z}\rho_n(0)\left[(A_c-2Z)-(A_c-2Z')\left(\frac{{}^{Z'}\rho_n(0)}{{}^{Z}\rho_n(0)}\right)\right]V_{0n}, \tag{3.1}$$

which arises from the difference in $D_{\Lambda Z}$ and $D_{\Lambda N}$ of the two isobars. Except for very small correction terms, the coefficients of the formulae (2.2) have a very small role. This suggests that if accurate data is available for isobars, it might provide a reasonably sensible method for determining ${}^{Z'}\rho_n(0)$ relative to ${}^{Z}\rho_n(0)$ rather than any reliable information on CSB. Since the term involving CSB, $\Delta V/V_{0n}$, occurs with the rather small coefficient and still higher powers of the same, there seems little point in investigating the effect of CSB from studies of the experimental ${}^{Z,Z';A}\Delta B_\Lambda$. Such studies seem mainly to provide information on the relative central neutron density of the two isobars. However, even for this study, since the difference ${}^{Z,Z',A}\Delta B_\Lambda$ is even less than 0.5 MeV for the stable or the near-stable isobars, it would be feasible only if very accurate data are available.

One may also discuss this matter in the light of the semi-empirical formula of Rahman Khan and Shoeb [9], being given below for ready reference

$$B_\Lambda = V_\Lambda - \frac{\alpha_\Lambda}{A_c^{1/3}} + \beta_\Lambda\frac{Z}{A_c} - \frac{\eta_\Lambda}{A_c^{2/3}}. \tag{3.2}$$

Hence,

$${}^{Z,Z';A}\Delta B_\Lambda = \beta_\Lambda\left(\frac{\Delta Z}{A_c}\right). \tag{3.3}$$

For a large number of nuclei, ranging from $A_c = 25$ to 250, stable or near-stable nuclei are considered. The reasonable value of ΔZ for these nuclei is 2 or 3. ${}^{Z,Z';A}\Delta B_\Lambda$ as obtained from (3.3) above is even less than 0.2 MeV.

Thus, on the basis of our analysis of the medium and heavy nuclei, the CSB is not well determined and hence offers no reliable information. However, it may happen that light hypernuclei for which more data is available are more sensitive to the CSB force. Hence, one may conclude that for getting a better idea of a CSB force it is more suitable to study light hypernuclei.

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