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Improved but low-x approximated relation between F_2^p scaling violation and gluon momentum distribution function

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Abstract. We present a brief analysis on the approximate methods for the determination of gluon distribution from the scaling violation of proton structure F_2^p in the low-x limit. In the leading order, a general low-x approximated relation is presented having more accuracy than the previous methods. Next-to-leading order correction is presented incorporating double-log-approximation. The proposed method is found to give good agreement with data. It may also be used to discriminate between the sets of gluon distributions in the low-x.

Keywords. DGLAP equations; structure function and gluon distribution; low-x.

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1. Introduction

Deep-inelastic scattering (DIS) of lepton-hadron provides a direct measurement of quark distribution. The measurement of gluon distribution enters indirectly through DGLAP evolution equation [1]. (A direct determination of gluon distribution was performed recently for $x > 10^{-3}$ [14]). In the small-x limit ($x < 10^{-2}$) the evolution equation connects the scaling violation of proton structure function F_2^p and gluon momentum distribution function G, because of small contribution from quarks. Using this fact, several approximate methods were developed [2–5] in the recent past.

Low-x approximated leading-order (LO) DGLAP evolution equation for proton structure function is

$$\mathrm{d}F_2^p(x,Q^2)/\mathrm{d}\ln Q^2 \approx 2\sum_i e_i^2(\alpha_s(Q^2)/4\pi)\int_x^1 G(x/y)P_{qg}^{(1)}(y)\mathrm{d}y, \qquad (1)$$

where $\alpha_s(Q^2) = \alpha_s$ is the strong coupling constant, e_i is the charge of the i^{th} flavour and $P_{qg}^{(1)}$ is the LO splitting function for $g \to q\bar{q}$. For four flavours (1) can be written as

$$\mathrm{d}F_2^p(x,Q^2)/\mathrm{d}\ln Q^2 \approx \frac{20}{9}(\alpha_s/4\pi)\int_x^1 G(x/y)(y^2 + (1-y)^2)\mathrm{d}y. \qquad (2)$$

Most of the approximate analytical relations [4,5] between scaling violation of F_2^p and gluon density are based on the Taylor's expansion of G in the integral in (2) up to the first derivative term. The validity of such approximation is questionable [6]. The method proposed by Prytz [2] is based not only on the Taylor's expansion but also on the symmetry property of $P_{qg}^{(1)}$. It can be found that all the methods simply evaluate the integral in (2) approximately. Generally, the gluon input distributions are of the form $G(x) = Ax^\delta (1-x)^\eta (1+\gamma x)$. When x is small, $G(x) \approx x^\delta$.

This form of input is used in brief analysis of these approximate methods in §2. Prytz's method [2] introduces different positive contribution to the exact values of the integral for different values of $\delta(-0.5 \le \delta < 0)$ due to which prediction for $\mathrm{d}F_2^P/\mathrm{d}\ln Q^2$ is higher than the exact one. The method proposed by Ducati *et al* [5] introduces different negative contributions for different values of $\alpha(0.65 < \alpha < 1)$ and δ which makes the method phenomenologically successful. Due to presence of different errors (contributions) for different values of $\delta(\alpha)$, loss of informations are there because of which these methods cannot be applied on high precision data for low-x to discriminate different sets of gluon distributions. Dependency of error on $\delta(\alpha)$ proves that these methods are not general. In §3, a new method has been proposed in the LO by the use of complete expanded series. The numerically tested error is found to be nearly 1% for $x \to 0$. In §4, next-to-leading order (NLO) correction is presented in similar fashion as proposed by Prytz [3] without the use of gluon momentum distribution found from the complete QCD analysis of existing data. We avoid such analysis because – (a) the method cannot be applied to lower values of x than the existing data from which experimental gluon distribution is found and (b) data themselves carry errrors which enter the NLO corrections. For this part, the method proposed by Ball and Forte [9] (which is based on double-log-approximation and is a consequence of QCD in the asymptotic limit) is adopted. This method is found to be applicable in HERA regime [9,11] for $\rho(\sim \sqrt{\ln(1/x)/\ln\ln Q^2}) \ge 1.13$ (for four flavours) [12]. This makes the present analytical result to be completely independent of experimental input, making it applicable for analysis of future data having smaller values of x where the low-x approximation holds good. In §5, we present our results together with Prytz's having MRS D_0' and D_-' inputs [7] for $Q^2 = 20\ \mathrm{GeV}^2$. We compare our prediction with H1 [14] and ZEUS [15] data for different Q^2 together with the higher-order (HO) exact results of Gluck, Reya and Vogt (GRV) [19] which are the complete solutions of DGLAP equations up to NLO corrections. In §6, we present our conclusions.

2. Brief analysis of different approximate methods

When x is small, most of the inputs for gluon momentum distribution can be written as $G(x) \approx x^\delta$. So, the integral (2) can be written as

$$\int_x^1 (x/z)^\delta (z^2 + (1-z)^2)\mathrm{d}z = \left(\frac{2}{3-\delta} - \frac{2}{2-\delta} + \frac{1}{1-\delta}\right) x^\delta, \qquad \text{(a)}$$

which is exact for $x \to 0$. The predicted values for this integral are (b) $\frac{2}{3}(\frac{3/2-\alpha}{1-\alpha})x^\delta$ (Ducati *et al* [5]), (c) $\frac{3}{4}(\frac{4}{3}x)^\delta$ (Bora *et al* [4]) and (d) $\frac{2}{3}(2x)^\delta$ (Prytz [2]) for the above limit of x. Most of the recent inputs have δ in the interval $(-0.5, 0)$ [7,13]. So, these methods can be tested for $\delta \approx 0$. Comparison of (a) and (c) for such values of δ indicates that the

Table 1. Comparison between values of the integral in eq. (2) evaluated by different methods for $G(x) = x$ in the limit $x \to 0$ $\left(\text{Percentage of error} = \frac{\text{predicted} - \text{exact}}{\text{exact}} \times 100\right)$

			Prytz		Ducati *et al*			Bora *et al*		Present		
Input	Value of δ	Exact $(\times x^{\delta})$	Prediction $(\times x^{\delta})$	% of Error	Expansion point α	Prediction $(\times x^{\delta})$	% of Error	Prediction $(\times x^{\delta})$	% of Error	α using (8)	Prediction $(\times x^{\delta})$	% of Error
					0.6	0.6248	+1.8					
MRSD'_0	−0.08	0.6137	0.6307	+2.8	0.65	0.621	+1.19	0.7329	+19.42	0.59923	0.6196	+0.97
					0.75	0.6106	−0.5					
					0.6	0.5227	+3.36					
MRSA	−0.3	0.5057	0.5415	+7.08	0.65	0.5109	+1	0.688	+36.05	0.58778	0.511	+1.05
					0.75	0.4795	−5.2					
					0.6	0.4444	+1.4					
MRSD'_0	−0.5	0.4381	0.4717	+7.6	0.65	0.4278	−2.4	0.649	+48.25	0.57263	0.4358	−0.5
					0.75	0.3849	−12.14					

method proposed by Bora *et al* is incorrect. The amount of errors inherent in other two methods (for $\delta \neq 0$) are

$$\left[\left(\frac{2}{3-\delta}-\frac{2}{2-\delta}+\frac{1}{1-\delta}\right)-\frac{2}{3}2^{\delta}\right]x^{\delta}(\text{Prytz})$$

and

$$\left[\left(\frac{2}{3-\delta}-\frac{2}{2-\delta}+\frac{1}{1-\delta}\right)-\frac{2}{3}\left(\frac{3/2-\alpha}{1-\alpha}\right)^{\delta}\right]x^{\delta},$$

(Ducati *et al*). When tested numerically, Prytz's method is found to be better than the Ducati's *et al* one for $\alpha = 0.75$ (table 1) for different MRS inputs [7,13]. The better result of Prytz's method may be attributed to the fact that his approximation on Taylor's series is based on the symmetry of $P_{qg}^{(1)}$. Truncation of Taylor's expansion at the first derivative term to obtain an approximate result, as proposed by Ducati *et al*, for any point of expansion(α), is not analytically justifiable. The errors in both the methods depend on δ and hence are not general. For fixed δ ($\neq 0$) errors are fixed. For two different δs, but for fixed x, errors are different. Hence, there is loss of information in the discrimination process for two different gluon distributions in both the methods.

3. LO low-x approximated relation between F_2^p and G

Replacing y by $(1-z)$, (2) can be written as

$$\mathrm{d}F_2^p(x,Q^2)/\mathrm{d}\ln Q^2 \approx \frac{20}{9}(\alpha_s/4\pi)\int_0^{(1-x)} G\left(\frac{x}{1-z}\right)[z^2+(1-z)^2]\mathrm{d}z, \tag{3}$$

To evaluate the integral on the r.h.s. for low-x, let us consider that G is regular at very small x. It can be expanded around a point $z=\alpha(0<\alpha<(1-x))$, such that $|z-\alpha|<R$; R being the radius of convergence. This expanded series can be integrated term by term and the resultant series converge [16]. The n^{th} term of the expanded integrand is

$$T^n = (z-\alpha)^{(n-1)}G^{(n-1)}[z^2+(1-z)^2]/(n-1)!,$$

where

$$G^{(n-1)} = \mathrm{d}^{(n-1)}G\left(\frac{x}{1-z}\right)\Big/\mathrm{d}z^{(n-1)}\Big|_{z=\alpha} = G^{(n-1)}\left(\frac{x}{1-\alpha}\right),$$

which is independent of z. Therefore,

$$\int_0^{(1-x)} T^n\mathrm{d}z = G^{(n-1)}\sum_{r=0}^{(n-1)}(-\alpha)^n\frac{1}{(n-r-1)!r!}\left[(1-x)^{(n-r+2)}\frac{2}{(n-r+2)}\right.$$
$$\left.-(1-x)^{(n-r+1)}\frac{2}{n-r+1}+(1-x)^{(n-r)}\frac{1}{n-r}\right]$$

for small-x,

$$\int_0^{(1-x)} T^n \mathrm{d}z \approx G^{(n-1)} \sum_{r=0}^{(n-1)} (-\alpha)^r \frac{1}{(n-r-1)!r!} \times \left[\frac{2}{n-r+2} - \frac{2}{n-r+1} + \frac{1}{n-r}\right]. \tag{4}$$

The complete integrated series can be obtained by the following algorithm.

For $(n-1)-r=0$,

$$A_0 = \int_0^{(1-x)} T^n \mathrm{d}z = \frac{2}{3}\left(G^{(0)} - \alpha G^{(1)} + \frac{\alpha^2}{2!}G^{(2)} \cdots\right)$$
$$= \frac{2}{3}G^{(0)}\left(\frac{x}{1-\alpha} - \alpha\right),$$

for $(n-1)-r=1$,

$$A_1 = \int_0^{(1-x)} T^n \mathrm{d}z = \frac{1}{3}\left(G^{(1)} - \alpha G^{(2)} + \frac{\alpha^2}{2!}G^{(3)} \cdots\right)$$
$$= \frac{1}{3}G^{(1)}\left(\frac{x}{1-\alpha} - \alpha\right),$$

and so on. Therefore,

$$\int_0^{(1-x)} G\left(\frac{x}{1-z}\right)[z^2+(1-z)^2]\mathrm{d}z = (A_0 + A_1 + \cdots)$$
$$= \frac{2}{3}\left[G^{(0)}\left(\frac{x}{1-\alpha} - \alpha\right) + \frac{1}{2}G^{(1)}\left(\frac{x}{1-\alpha} - \alpha\right) + \frac{7}{40}G^{(2)}\left(\frac{x}{1-\alpha} - \alpha\right)\right.$$
$$\left. + (11/240)G^{(3)}\left(\frac{x}{1-\alpha} - \alpha\right) + \cdots\right],$$

or

$$\int_0^{(1-x)} G\left(\frac{x}{1-z}\right)[z^2+(1-z)^2]\mathrm{d}z = \frac{2}{3}\left(G\left(\frac{x}{1-\alpha} - \alpha + a\right) + R(a)\right), \tag{5}$$

where 'a' is a parameter and

$$R(a) = \left[\left(\frac{1}{2} - a\right)G^{(1)}\left(\frac{x}{1-\alpha} - \alpha\right) + \left(\frac{7}{40} - \frac{a^2}{2!}\right)G^{(1)}\left(\frac{x}{1-\alpha} - \alpha\right) + \cdots\right],$$

or

$$R(a) = \left[\left(\frac{1}{2} - a\right)G^{(1)}\left(\frac{x}{1-\alpha}\right) + \left(\frac{7}{40} - \frac{a^2}{2!}\right)G^{(2)}\left(\frac{x}{1-\alpha}\right)\right.$$

$$+\left(\frac{11}{240}-\frac{a^3}{3!}\right)G^{(3)}\left(\frac{x}{1-\alpha}\right)\cdots\Bigg]. \tag{6}$$

We may have an error of nearly 2 % in the last step for $a \sim 0.5$ and $\delta \in (-0.5, 0)$.

Let 'a_0' be the value of 'a' for which $R(a) \approx 0$. Then, (5) can be written as

$$\int_0^{(1-x)} G\left(\frac{x}{1-z}\right)[z^2+(1-z)^2]\mathrm{d}z = \frac{2}{3}G\left(\frac{x}{1-\alpha}-\alpha+a_0\right).$$

Now, the function G is sensitive to the variation of x only when $\frac{x}{1-\alpha} \approx \alpha - a_0$. So for $x \to 0$, $\alpha \to a_0$. So,

$$\int_0^{(1-x)} G\left(\frac{x}{1-z}\right)[z^2+(1-z)^2]\,\mathrm{d}z = \frac{2}{3}G\left(\frac{x}{1-\alpha}\right), \tag{7}$$

where α satisfies the condition (using (6))

$$\left(\frac{1}{2}-\alpha\right)G^{(1)}\left(\frac{x}{1-\alpha}\right)+\left(\frac{7}{40}-\frac{\alpha^2}{2!}\right)G^{(2)}\left(\frac{x}{1-\alpha}\right)$$
$$+\left(\frac{11}{240}-\frac{\alpha^3}{3!}\right)G^{(3)}\left(\frac{x}{1-\alpha}\right)+\cdots \simeq 0. \tag{8}$$

If the form of G is known, α can be found out. Prytz's result can be obtained easily using (8) if assumption is made that $G^{(1)} >> G^{(2)}, G^{(3)}\cdots$. The present method fails to recover other results because of the fact that– one [4] is not totally correct and the other one [5] does not take into account the convergency of the Taylor's series. In table 1, comparison is being made between the exact results (i.e. (a)) and the predictions from (7) and other approximate methods [2,4,5], for different inputs of G [7,13] in the limit $x \to 0$. The proposed method is found to be better than the others, having maximum error of 1.05 % for $\delta \in [-0.5,0]$. Since it can produce the result obtained by Prytz [2] and is applicable for any value of δ for which the expansion is convergent, hence can be considered as general. Not only that, its error can be conjectured to be independent of δ because of the fact that (8) is a relation between α and δ. In the table, the variation in error of this method with respect to δ may be due to the truncation of the series in the above equation at the third term. Using (7) in (3), the LO result can be written as

$$\mathrm{d}F_2^p/\mathrm{d}\ln Q^2 = \frac{20}{9}(\alpha_s/4\pi)G\left(\frac{x}{1-\alpha}\right), \tag{9}$$

where α is found by solving (8).

4. Next-to-Leading Order (NLO) correction

Neglecting the quark contribution, the NLO contribution for F_2^p scaling violation can be written as

$$\mathrm{d}F_2^{p(2)}/\mathrm{d}\ln Q^2 = 2\sum_i e_i^2(\alpha_s/4\pi)\int_x^1 G(x/z)P_{qg}^{(2)}(z)\mathrm{d}z, \tag{10}$$

where $P_{qg}^{(2)}$ is the NLO splitting function [8]. It has been demonstrated by Prytz [3], that the contribution to the integral on the right for $z \in (0.1,0.5)$ is small. So, (10) can be written as

$$\mathrm{d}F_2^{p(2)}/\mathrm{d}\ln Q^2 \simeq 2\sum_i e_i^2(\alpha_s/4\pi)^2(I_1+I_2), \tag{11}$$

where

$$I_1 = \int_{0.5}^{1} G(x/z)P_{qg}^{(2)}z\mathrm{d}z,$$

and

$$I_2 = \int_{x}^{0.1} G(x/z)P_{qg}^{(2)}(z)\mathrm{d}z.$$

The integral I_1 is found to be independent of G and is equal to 3.58G $(\frac{x}{1-\alpha})$ [3] when the complete NLO splitting function is used. To evaluate the integral I_2, Prytz used the gluon distribution function obtained from the complete QCD analysis of existing data. It presents constraint on the application of the method to lower values of x unprobed by DIS experiments. Not only that – the experimental uncertainty enters the method. Hence, such approach has been avoided in this proposed method. Since the integration limits of I_2 are small, the term containing z^{-1} in $P_{qg}^{(2)}$ (which sums the double logarithms [9]) can well approximate its contribution in the small-x limit. Writing

$$\gamma = \sqrt{\frac{52}{3\beta_0}},$$

($\beta_0 = 11 - 2n_f/3$) and double asymptotic scaling variable

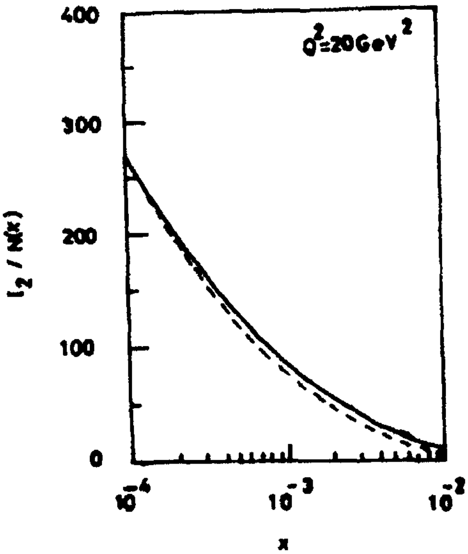
$$\sigma = \sqrt{\ln(x_0/x)\ln[\ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2)]},$$

the sum of the double logarithms can be approximated as [10],

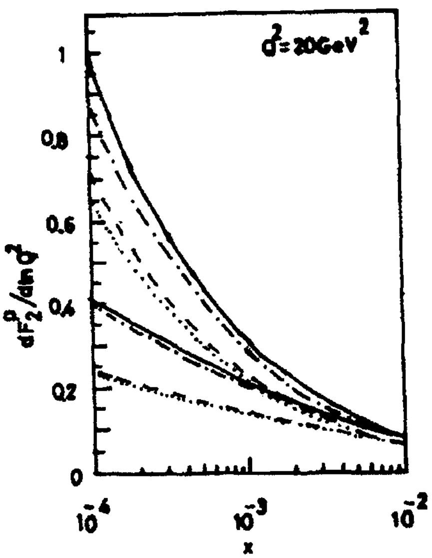
$$I_2 \simeq AI_0(2\gamma\sigma), \tag{12}$$

where x_0 and Q_0^2 are the starting scales of perturbative QCD, n_f is the number of flavours, Λ is the IR cut-off parameter, A is a constant which is to be fixed by comparison with other results and I_0 is the modified Bessel function of the first kind and zeroth order. For prediction from the above equation, the asymptotic form of I_0 is being used.

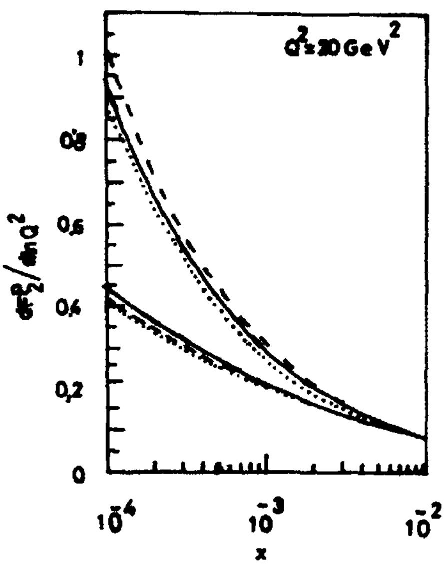
In figure 1, comparison is being made between the predictions from (12) (using $x_0 = 0.1, Q_0^2 = 1$ GeV^2 and $\Lambda = 0.23$ GeV as in [11] and $n_f = 4$) and $N(x, Q^2)$ of [3] having D_0' [7] input at $Q^2 = 20$ GeV^2. A is being fixed from the comparison of these two predictions in the low-x limit where both of them are assumed to be valid. From such analysis the value A is found to be approximately equal to 3. The differences are found to be nearly 30% at $x = 10^{-2}$, 7% at $x = 10^{-3}$ and no difference at $x = 10^{-4}$ and below. Since, it is α_s^2 correction, hence the maximum error (in comparison with $N(x, 20)$) is $\sim 4\%$ validating double-log approximation, at least, in the small-x limit. Thus, the NLO correction can be written as



(1)



(2)



(3)

Figure 1. Comparison between I_2 (text) and $N(x, Q^2)$ (Prytz [3]) at $Q^2 = 20$ GeV^2.

Figure 2. Dashed/dotted upper(lower) line represents the prediction of $\mathrm{d}F_2^p/\mathrm{d}\ln Q^2$ from Prytz/our method having MRS D'_- (D'_0) input for LO; solid/dotted-dashed upper (lower) line represents the prediction of $\mathrm{d}F_2^p/\mathrm{d}\ln Q^2$ from Prytz/present method having MRS D'_- (D'_0) input for (LO+NLO) at $Q^2 = 20$ GeV^2.

Figure 3. Solid/dashed/dotted upper (lower) line represents (LO+NLO) prediction of $\mathrm{d}F_2^p/\mathrm{d}\ln Q^2$ from exact (as in [3])/Prytz/present method having MRS D'_- (D'_0) input at $Q^2 = 20$ GeV^2.

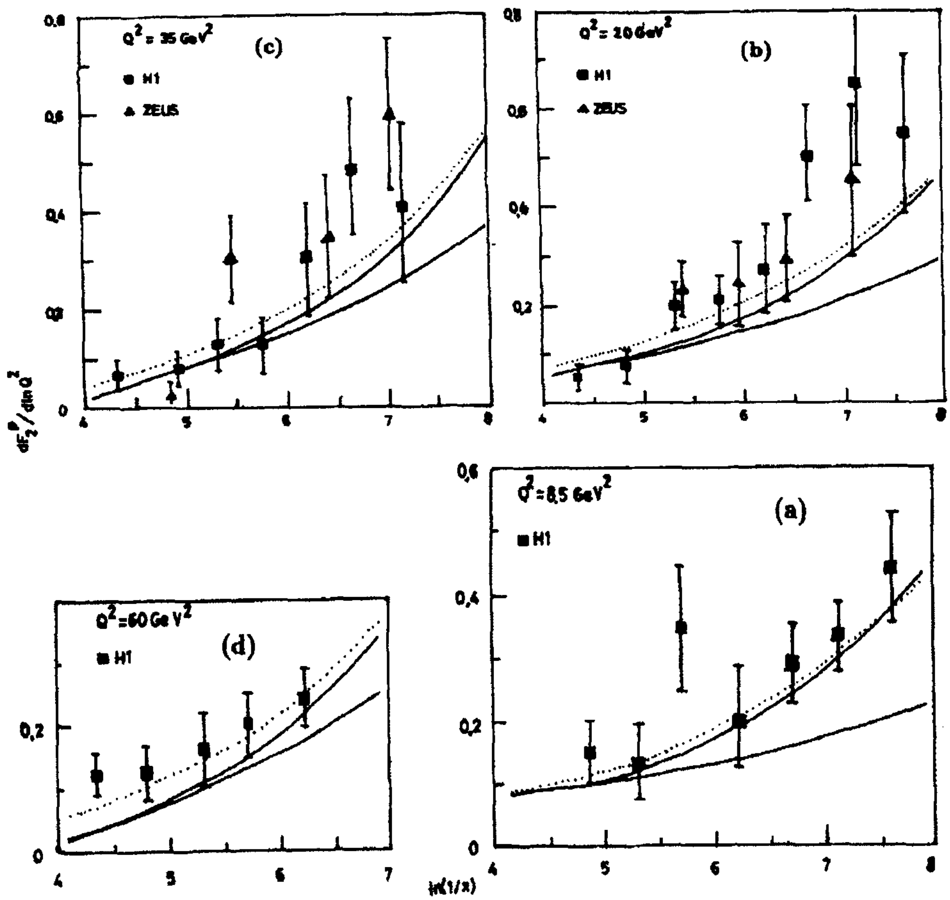


Figure 4(a-d). Solid (dotted) lines represent the (LO+NLO) predictions for $\mathrm{d}F_2^p/\mathrm{d}\ln Q^2$ from the present (exact [19]) method at $Q^2 = 8.5, 20, 35$ and $60\ \mathrm{GeV}^2$. Upper (lower) lines are for D'_- (D'_0) input. Data are obtained by H1 [14] and ZEUS [15] collaborations.

$$\mathrm{d}F_2^{p(2)}/\mathrm{d}\ln Q^2 \simeq \frac{20}{9}(\alpha_s/4\pi)^2\left[3.58G\left(\frac{x}{1-\alpha}\right)+3I_0(2\gamma\sigma)\right]. \tag{13}$$

The complete (NLO corrected) low-x approximated result is

$$\begin{aligned}\mathrm{d}F_2^p/\mathrm{d}\ln Q^2 \simeq{}& \frac{20}{9}(\alpha_s/4\pi)G\left(\frac{x}{1-\alpha}\right)[2/3+3.58(\alpha_s/4\pi)]\\ &+\frac{20}{3}(\alpha_s/4\pi)^2 I_0(2\gamma\sigma),\end{aligned} \tag{14}$$

which is simple, general and independent of any experimental constraint. The total expected error inherent in the method is nearly 6%, without taking into account the contribution of quark.

5. Results

In figure 2, we compare our predictions with predictions of Prytz for LO and LO+NLO at $Q^2 = 20$ GeV^2, using MRS D'_- and D'_0 inputs. In both the cases our predictions are lower than Prytz's. For D'_0 input the differences can be neglected for both LO and NLO. But, for D'_- input the difference is nearly 5% in LO and 10% for LO+NLO at $x = 10^{-4}$. In figure 3, we compare our predictions for (LO+NLO) using D'_0 and D'_- inputs with Prytz and the exact solution (as in [3]) of DGLAP equation (taking into account of quark contribution) for $\mathrm{d}F_2^p/\mathrm{d}\ln Q^2$ at $Q^2 = 20$ GeV^2. In both the cases our predictions are slightly lower than the exact. It is interesting to note that in case of Prytz's method D'_- input indicate negative contribution from quark, but which comes out to be positive from the present method. In figure 4, we compare our predictions on $\mathrm{d}F_2^p/\mathrm{d}\ln Q^2$ for $Q^2 = 8.5$, 20, 35 and 60 GeV^2 having D'_- and D'_0 inputs with data obtained by H1 [14] and ZEUS [15] collaborations together with the exact predictions of GRV (HO) [19]. We use GRV because it is easy to handle. The slopes are evaluated using the linear relation [17,20]

$$F_2^p(x, Q^2) = F_2^p(x, Q_1^2) + \ln(Q^2/Q_1^2)\mathrm{d}F_2^p/\mathrm{d}\ln Q^2,$$

where $Q_1^2 = 20$ GeV^2. This linear relation may not be valid for $Q^2 >> 20$ GeV^2 and $Q^2 << 20$ GeV^2. All the analysed data have $x \leq 1.3 \times 10^{-2}$ (where small-x approximation is supposed to be valid) and 8.5 GeV$^2 \leq Q^2 \leq 60$ GeV^2 ($\rho > 1.5$). It has been found that the data prefer D'_- input for $Q^2 = 8.5$, 20 and 35 GeV^2. But for $Q^2 = 60$ GeV^2, most of the data lie nearly parallel to the predictions having D'_0 indicating a change in preference. But, comparison with exact result indicates that the contribution of quark may become important in this region (i.e. in high Q^2 and relatively high-x ($\sim 10^{-2}$)).

6. Conclusions

We discuss briefly on available different analytical methods [2,3,5] for extraction of gluon distribution from F_2^p scaling violation at low-x. None of these methods is found to be stable for different input distributions because errors are different for different inputs.

We propose a method which is found to be general in low-x. When compared with data obtained by H1 [14] and ZEUS [15] collaborations, agreement is found to be good having MRS D'_- input for $Q^2 < 60$ GeV^2. For high Q^2 and moderately high-x, contribution of quark is to be considered. Otherwise, interpretation on data regarding input distributions may be wrong. Since it is a low-x approximated solution of DGLAP equation (neglecting the quark contribution) for $\mathrm{d}F_2^p/\mathrm{d}\ln Q^2$, there is no scope to study other low-x effects – such as, recombination, analytically. This can be considered along the line proposed by Gribov, Levin and Ryskin (GLR) [10]. So, the present method can accomodate those low-x effects which are accompanied by change in δ (e.g. soft pomeron, hard pomeron phenomena). At very low-x it has been conjectured that BFKL equation [18] may be effective and hence the present method may break down.

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