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On quark matter in a strong magnetic field

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Abstract. The effect of strong magnetic field on the bulk properties of quark matter is reinvestigated taking u, d and s-quarks as well as electrons in the presence of magnetic field. Here the bag pressure is chosen such that in the absence of magnetic field and at zero temperature the binding energy of the uds-system is < 930 MeV while that of ud-system is greater than 940 MeV. It is observed that the equation of state changes significantly in a strong magnetic field. At finite temperature the electron chemical potential varies between 6 and 50 MeV. Thus the expansion of thermodynamical quantities in powers of $T/(\mu_i^2 - M_\nu^{(i)2})^{1/2}$ is valid only up to few MeV. For high temperatures $\sim$40 MeV the exact integral expressions are to be taken.

Keywords. Magnetic field; quarks; equation of state.

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1. Introduction

With the discovery of intense outbursts of low energy γ-rays from neutron stars, there have been lot of interest in the study of neutron stars and their intense magnetic fields. These soft γ-ray repeaters (SGRs) are associated with the young supernova remnants (SNRs) and therefore most probably with neutron stars. Very recently G Kouveliotou *et al* [1] reported the discovery of pulsations in the persistent X-ray flux of SGR 1806–20, with a period of 7.47s and a spin down rate of 2.6×10^3 syr^{-1} suggesting the pulsar age and (dipolar) magnetic field strength to be $\sim$ 1500 years and 8×10^{14} gauss respectively. Their observations demonstrate the existence of 'magnetars', neutron stars with magnetic fields of about 100 times stronger than those of pulsars and support the suggestion that SGR bursts are caused by neutron-star 'crust-quakes' produced by magnetic stresses [2].

Numerous authors [3] have studied the effect of intense magnetic fields on the weak decay rates that has effects on the big bang nucleosynthesis. Bezchasinov and Haensel [4] have studied the cross-section of neutrino scattering in the presence of a strong magnetic field and found significant modification on the angular and energy dependence of the scattering cross-section. Very recently Chakrabarti [5] has studied the effect of strong magnetic field on dense quark matter consisting of u, d and s quarks and electrons in beta equilibrium. He considered the effect of magnetic field on u, d quarks and electrons but treated s-quark free due to its high mass. The value of the bag pressure was chosen to be about $BB^{1/4} \simeq 67$ MeV which does not reproduce the requisite binding energy for ud as well as uds system [6] in the absence of a magnetic field. On the other hand it would be desirable if we have ud as unbound and uds system as the most stable than Iron [6] i.e. the binding energy of uds system is to be less than 930 MeV while that of ud system to be greater than 940 MeV. In beta equilibrium the chemical potential of d and s quarks being equal, it is expected that due to finite s-quark mass, it will be affected more than the d-quark in the presence of magnetic field. Guided by these ideas we have re-examined the effect of magnetic field and temperature on the bulk properties of u, d, s quarks and electrons.

In §2 we give a very brief formalism for this study and in §3 the results are given along with the conclusion.

2. Formalism

The thermodynamical properties of the bulk SQM system in the presence of a strong magnetic field is described by the thermodynamical potential given by

$$\Omega = \sum_i \Omega_i - \frac{8}{45}\pi^2 T^4, \tag{1}$$

where the second term is the contribution due to gluons.

The general expression for the thermodynamical potential Ω_i in the presence of magnetic field is

$$\Omega_i = -T\frac{q_i g_i}{2\pi^2} B \sum_{\nu=0}^{\infty}(2-\delta_{\nu 0}) \int_0^{\infty} \mathrm{d}K_z \ln\left\{1+\exp[\beta(\mu_i - \epsilon_i^{(\nu)})]\right\}, \tag{2}$$

where the single particle energy $\epsilon_\nu^{(i)}$ is given by

$$\epsilon_\nu^{(i)} = (K_z^2 + m_i^2 + 2\nu q_i eB)^{1/2},$$

and $i = u, d, s$ and e. The pressure and number densities are given by

$$p = -\sum_i \Omega_i - BBn_i = -\left(\frac{\partial \Omega_i}{\partial \mu_i}\right)_T, \tag{3}$$

and finally

$$E = \sum_i \left[\Omega_i + \mu_i n_i - T\left(\frac{\partial \Omega_i}{\partial T}\right)_{\mu_i}\right] + BB. \tag{4}$$

Assuming β-equilibrium we have

$$\mu_d = \mu_s = \mu \qquad \text{(say)}; \tag{5}$$

and

$$\mu_d = \mu_u + \mu_e. \tag{6}$$

The charge neutrality condition gives

$$2n_u - n_d - n_s - 3n_e = 0. \tag{7}$$

whereas the baryon number density of the system is

$$n_B = \frac{1}{3}(n_u + n_d + n_s) \tag{8}$$

Thus for given values of n_b and BB, there are two independent parameters out of μ_d, μ_s, μ_u and μ_e, which can be fixed by (7) and (8).

For $T = 0$, the number densities are:

$$n_i = \frac{q_i g_i B}{2\pi^2} \sum_{\nu=0}^{\nu_{\max}^{(i)}} (2 - \delta_{\nu 0})[\mu_i^2 - M_\nu^{(i)^2}]^{1/2}, \tag{9}$$

where

$$M_\nu^{(i)} = (m_i^2 + 2\nu q_i B)^{1/2}, \tag{10}$$

where $\nu_{\max}^{(i)}$ is equal to $\left[(\mu_i^2 - m_i^2)/(2q_iB)\right]$. The ν limit is finite for $T \to 0$ limit only when the maximum available energy of a particle is approximately equal to its Fermi energy. The expressions for pressure and energy density of the ith species are given by

$$p_i = \frac{q_i g_i B}{2\pi^2} \sum_{\nu=0}^{[\nu_{\max}^i]} (2 - \delta_{\nu 0}) \left\{ \frac{1}{2}\mu_i(\mu_i^2 - M_\nu^{(i)^2})^{1/2} \right. \\ \left. - M_\nu^{(i)^2} \ln \left[\frac{\mu_i + (\mu_i^2 - M_\nu^{(i)^2})^{1/2}}{M_\nu^{(i)}} \right] \right\}, \tag{11}$$

and

$$E_i = \frac{q_i g_i B}{2\pi^2} \sum_{\nu=0}^{[\nu_{\max}^i]} (2 - \delta_{\nu 0}) \left\{ \frac{1}{2}\mu_i(\mu_i^2 - M_\nu^{(i)^2})^{1/2} \right. \\ \left. + \frac{1}{2} M_\nu^{(i)} \ln \left[\frac{\mu_i + (\mu_i^2 - M_\nu^{(i)^2})^{1/2}}{M_\nu^{(i)}} \right] \right\}. \tag{12}$$

We have also studied the effect of magnetic field in the low temperature limit. The expressions used are the same as in [5].

3. Results and conclusions

The variations of the number densities, pressure and energy density of the system at $T = 0$ with magnetic field needs the fixing of the values of m_u, m_d, m_s and the bag pressure BB.

We have chosen $m_u = m_d = 5$ MeV and $m_s = 150$ MeV. The choice of the bag pressure was made in such a way that at $T = 0$ and $B = 0$ the ud energy/baryon > the energy per baryon of the uds system (i.e. which is less than 930 MeV). This was achieved by fixing $BB^{1/4}$ as 150 MeV. It is found that the energy/baryon for bag pressure up to 80 MeV increases monotonically with baryon density.

The behaviour of the number of densities with magnetic fields is similar to those as reported in [5]. The only difference being that n_u becomes about 1.5 times its density when it is free which is slightly different from [5]. For ud, uds systems at $B = 0$ and uds at $eB/m_e^2 = 1.0 \times 10^5$ where it is seen that uds is the true ground state of the matter as suggested by Witten [6].

In figure 1, we have plotted energy per baryon vs. n_B/n_0 for $BB^{1/4} = 150$ MeV. The energy per baryon decreases with magnetic field implying that the SQM is more stable in the presence of magnetic field. It is interesting to note that the position of minimum changes with configuration of the quark/strange star as well as with magnetic field.

In figure 2, we plot pressure vs. energy per baryon at $T = 0$ for ud, uds at $eB/m_e^2 = 0$ and uds at $eB/m_e^2 = 1.0 \times 10^5$ and 5.0×10^5. It is noted that for small magnetic fields the equation of state change slightly. But the change is significant at high magnetic fields.

This is due to the fact that at high magnetic fields only a few states will contribute for d and s-quark states. This fact persists at finite temperatures also. In this analysis the electron chemical potential varies between 6 to 50 MeV and so an expansion in powers of $T/(\mu_i^2 - M_\nu^{(i)^2})^{1/2}$ is not justified for temperatures as large as 40 MeV [5]. Therefore we have chosen $T = 5$ MeV and plotted in figure 3, the pressure vs. energy per baryon for $eB/m_e^2 = 0, 10^4, 10^5, 2 \times 10^5, 5 \times 10^5$. For higher temperatures we have to evaluate n_i, p_i and E_i from exact equations (2–4) which will be reported soon.

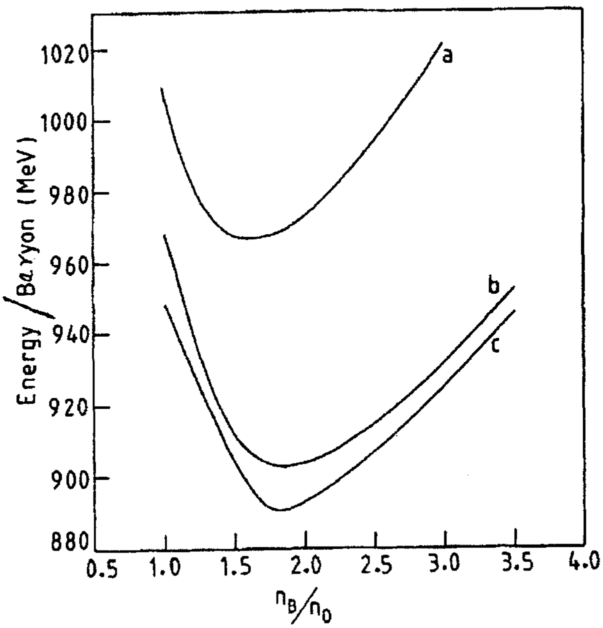

Figure 1. Plot of energy per baryon against baryon number density, n_B/n_0, where $n_0 = 0.17$ fm^{-3}. (a) represents ud system at $eB/m_e^2 = B^* = 0$ while (b) represents uds system at $B^* = 0$ and (c) is for $B^* = 10^5$.

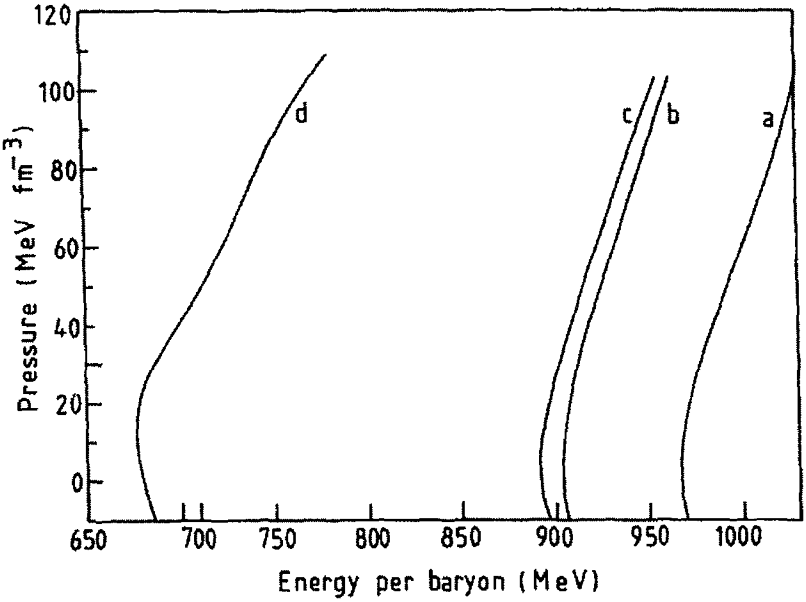

Figure 2. Plot of pressure vs. energy per baryon. (a), (b) and (c) represent the same as in figure 1 while (d) is for magnetic field = 5×10^5.

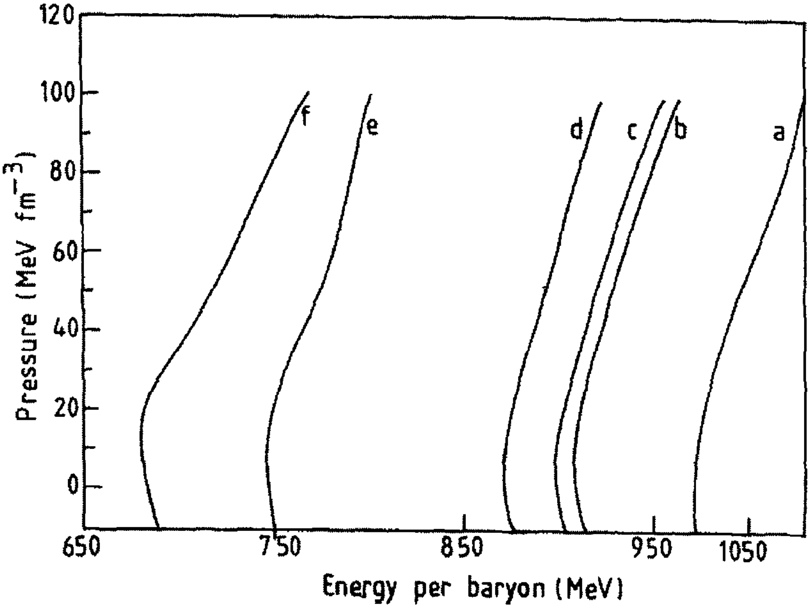

Figure 3. Plot of pressure vs. energy per baryon at a temperature of 5 MeV for SQM. a, b, c, d and e are for magnetic fields, $B^* = 0$, 10^4, 9×10^4, 2×10^5, 4×10^5 and 5×10^5 respectively.

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