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Particle production, back reaction and singularity avoidance

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Abstract. We consider particle production in Robertson–Walker spacetime as particle-antiparticle rotation. We thereby obtain a scale factor that guarantees particle production. We then study quantum field effects in spatially flat homogeneous and isotropic spacetime with energy density of created particles and one loop quantum correction as back reaction. In the numerical solution initial values are determined from particle production simulated scale factor and obtain the evolution of the universe both at early and late times having a bounce.

Keywords. Particle production; back reaction; singularity avoidance.

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1. Introduction

There are some physical consequences of physical cosmology of quantum field in curved spacetime. It is suggested that vacuum polarization and particle production can alter the classical evolution considerably through their contributions to the energy momentum tensor in the classical Einstein equations. Various authors [1–8] invoke quantum field effects as an attempt to find a mechanism through which the universe might avoid an initial singularity. In Robertson–Walker(R–W) spacetime the Hawking–Penrose singularity theorem [9] allows a requirement $\rho + 3P > 0$ for a singularity to occur. However, quantum field effects might allow a violation of this condition such that the universe may bounce at some dense epoch, rather than encounter a singularity. This type of bounce (replacement of regular contraction by expansion) could only occur at Planck regime. Attempts have been made [10–12] to solve the gravitational field equation with a source term given by $\langle T_{ab} \rangle$, and in addition a classical radiation density term. But the proponents of this bounce near Planck era take one loop quantum correction term in $\langle T_{ab} \rangle$, i.e. consider $aR + bR^2 + cR^2$ log R/R_0 type Lagrangian to set up the semiclassical Einstein field equation though close to Planck regime the one loop approximation is no longer reliable.

It is therefore guessed that within the region of validity of the semiclassical equation with a $\langle T_{ab}\rangle$ generated from an action just mentioned above, one may look for a bounce from a different mechanism. The quantum matter field in curved space generate a curvature dependent action $W(g_{ab})$

$$W(g_{ab}) = \int \mathrm{d}^4x(-g)^{1/2}\left(a + bR + cR^2 + \ldots\right) , \qquad (1)$$

where R is the curvature scalar and the constants $a, b, c, \ldots$ contain mass dependent term. At early times (near Planck time) R^2 and higher order terms are dominating. But at a much larger time i.e. away from Planck time the quantum gravitational effects fade away, implying $bR > cR^2$. Knowing [13] a, b and c this condition implies $m^2 > R \sim t^{-2}$, where t is the particle Compton time. When this transition occurs near about the Compton time there is a rapid change of scale factor, it attains a small value; 'bounces' and expands again. This causes a large particle production and there is a sort of resonance in the energy density of created particles. The works in this direction start with a scale factor where the bounce is inbuilt [14,15] and look for particle production. The energy density of created particles indeed show the resonance, as expected, near the Compton time.

In the present work we study the mechanism of second type of bounce and investigate whether the mechanism works in the framework of Einstein field equation when the quantum effects of particle production are taken into account. For the purpose we consider a Robertson–Walker (R–W) spacetime and calculate the stress tensor of massless conformal fields in one loop quantum correction. To effect the bounce, the conformal invariance must be somehow broken to ensure particle production.

In curved spacetime vacuum is a delicate object, the exact characteristics of which are yet to be resolved, not only that the particle production is very much dependent on the choice of vacuum (both $|\mathrm{in}\rangle$ and $|\mathrm{out}\rangle$ vacuum). In semiclassical approximation to quantum gravity, one loop quantum corrections due to massless conformal fields along with a classical radiation density term allows only vanishing particle production. However, the trace anomaly, that breaks the conformal invariance in general leads to vacuum polarization and indicates no allowance of particle production into the theory. The vacuum and the effect of particle production are related through $\langle \mathrm{out}|\mathrm{in}\rangle \propto \exp\left(Im\, W\right)$ where W is a measure of vacuum instability and is related to particle production amplitude. In curved spacetime the metric g_{ab} and the Ricci scalar R are real and a action with a imaginary W is hard to simulate. The convenient approach is to calculate the energy density of the created particles and include it in the rhs of the Einstein equation. As the scale factor is not known a priori and hence also the vacuum, we adopt Parker's definition of particle and antiparticle states and guarantee particle production through particle-antiparticle rotation [16]. We discuss this mechanism with a toy example in §2, using the method adopted in our earlier works [17, 18, 19]. Our discussion remains valid both in conformally flat and non-flat spacetime. In §3 we consider a realistic R–W spacetime and follow a equivalent description [17, 18] of antiparticle to particle rotation as reflection in complex time. The approach allows us to evaluate the scale factor and also the energy density of created particles. The scale factor thus evaluated and also the energy density of the created particles are then used as boundary conditions to solve the back reaction problem along with one loop quantum corrections for scalar fields (assuming $R > m^2$). This is discussed in §4. To circumvent the gap in the analytic analysis, we take up numerical integration of back reaction equation in §5. In §6 we end up with a concluding discussion on the basis of the results obtained in this work. Recently the study of catastrophic particle production under periodic perturbation has gained importance [23,24] through the parametric resonance formalism. In cosmological application of reheating after inflation [25] it is supposed that there will be no explosive creation of fermions due to Pauli exclusion principle. However, our method of particle production clearly indicates a large particle production due to denominator in eq. (36). Our method can also be generalized to study parametric resonance converting one dimensional temporal eq. (32) to Mathieu type equation. The result obtained in this paper clearly indicates the importance of particle production to understand the reheating phase as well as the radiation dominated era after the bounce. Though there has been an extensive study in this direction [11,12,26,27], it is now realized that weakly interacting spinless fields predicted in modern unified theories might change the standard scenario of nucleo-synthesis or baryo-genesis through their huge production and decay at least at some epoch of the early universe.

2. Bounce mechanism – a toy example

To study particle production-induced bounce one must know a priori the scale factor and the vacuum involved in calculating the production amplitude. But in curved spacetime the vacuum is a delicate object which decides the definition of particle-antiparticle states as well as the mode of particle production. To circumvent this difficulty we start with a motion of a spin $1/2$ Dirac particle in a R–W spacetime

$$\mathrm{d}s^2 = C^2{}_1(\eta)\left[\mathrm{d}\eta^2 - \mathrm{d}r^2 - f(r)\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2\right)\right], \tag{2}$$

with Dirac equation

$$\left[i\gamma^\mu(x)\frac{\partial}{\partial x^\mu} - i\gamma^\mu(x)\Gamma_\mu(x)\right]\Psi(x) = m\Psi(x). \tag{3}$$

Here $\gamma^\mu(x)$ are curvature dependent Dirac matrices, Γ_μ the spin connection and m the mass of the particle. To study particle production the temporal equations corresponding to eq. (3) are only important. It is found that [17,18] with the substitution

$$\Psi_{\lambda jlm} = C^{-3/2}B_\lambda(\eta)M_{\lambda jl}(r)S(\theta,\varphi)Z_{jlm}, \tag{4}$$

where $0 < \lambda < \alpha, j = 1/2, 3/2, \ldots, l = j + 1/2$ and

$$B_\lambda(\eta) = \begin{pmatrix} f_{\lambda+}(\eta)\mathbf{I} & 0 \\ 0 & f_{\lambda-}(\eta)\mathbf{I} \end{pmatrix}, \tag{5}$$

$\Psi_{\lambda jlm}$ is a solution of Dirac equation if $f_{\lambda\pm}$ satisfies the equation

$$\begin{aligned} &\tfrac{\partial f_{\lambda+}}{\partial \eta} + i\lambda f_{\lambda-} + imC_1(\eta)f_{\lambda+} = 0, \\ &\tfrac{\partial f_{\lambda-}}{\partial \eta} + i\lambda f_{\lambda+} - imC_1(\eta)f_{\lambda-} = 0. \end{aligned} \tag{6}$$

It should be pointed out that eq. (6) remains valid both in conformally flat and non-flat spacetime. Introducing

$$X = \begin{pmatrix} f_{\lambda+} \\ f_{\lambda-} \end{pmatrix}, \tag{7}$$

we write the temporal part of the Dirac equation in R–W spacetime eq. (2) in a two dimensional form

$$i\,\partial_\eta X \;=\; H\,X, \tag{8}$$

with

$$H \;=\; \sigma_3 m C_1(\eta) + \sigma_1\lambda, \tag{9}$$

where σ_i are the Pauli 2×2 spin matrices. To define the free orthonormal we assume

$$\lim_{|\eta|\to\infty} C_1(\eta) \;=\; C_{\rm in} \;=\; C_{\rm out} < \infty, \tag{10}$$

and write in the form

$$[i\partial_\eta - \sigma_3 m C_1(\eta) - \sigma_1\lambda]\, X_j(\eta) \;=\; 0. \tag{11}$$

As eq. (11) involves only two anticommuting spinors, the free orthonormals are defined as

$$X_{10} \;=\; \begin{pmatrix} \cos\frac{1}{2}\theta \\ \sin\frac{1}{2}\theta \end{pmatrix} e^{-i\,\omega\,\eta}, \tag{12}$$

$$X_{20} \;=\; \begin{pmatrix} \sin\frac{1}{2}\theta \\ \cos\frac{1}{2}\theta \end{pmatrix} e^{i\,\omega\,\eta}, \tag{13}$$

where

$$\cos\theta \;=\; \frac{mC_{\rm in}}{\omega}, \quad \sin\theta \;=\; \frac{\lambda}{\omega}, \quad 0<\theta<\frac{\pi}{2}. \tag{14}$$

The motion of spin $\frac{1}{2}$ particle given by eq. (11) is analogous to the motion of an electron in e.m. field $A^\mu \;=\; (0,0,0,A)$ in which m is replaced by p_3 and $C_1(\eta)$ by $p_3 + eA$. The particle production in electromagnetic case is very well known and as in [16], we consider the pair production as reflection process, not in space but in conformal time. According to this, the solution of Dirac equation which approaches a multiple of X_{20} as $\eta \to -\infty$ contains information on R, the pair production amplitude, as $\eta \to +\infty$. To understand the particle production using this technique we consider an idealized example of R–W spacetime

$$C_1(\eta) = \begin{cases} C_1, & 0 \le \eta \le \eta_A \\ C_i, & \text{otherwise.} \end{cases} \tag{15}$$

In order to see particle production for the scale factor given in eq. (15) we require the evolution of the function $X_2(\eta)$, subject to the boundary condition

$$X_2(\eta) = (1+iT)X_{20}(\eta), \quad \eta < 0. \tag{16}$$

The pair production amplitude R_p is identified from the behaviour of $X_2(\eta)$ for $\eta > \eta_A$ and can be obtained from eq. (16) through a unitary transformation (i.e. rotation) on eq. (16):

$$X_2(\eta) = (1+iT)e^{-iH_0\eta}e^{-iH_A\eta_A}X_{20}(0), \tag{17}$$

$$X_2(\eta) = X_{20}(\eta) + iR_p X_{10}(\eta), \quad \eta > \eta_A, \tag{18}$$

where

$$H_0 = \sigma_1 \lambda + \sigma_3 m C_i, \tag{19}$$

$$H_A = \sigma_1 \lambda + \sigma_3 m C_1. \tag{20}$$

Invoking charge conservation

$$\int \mathrm{d}^3x\, \psi_2{}^\dagger\, \psi_2|_{\eta=-\infty} = \int \mathrm{d}^3x\, \psi_2{}^\dagger\, \psi_2|_{\eta=+\infty}, \tag{21}$$

we get

$$1 + |R_p|^2 = |1 + iT|^2. \tag{22}$$

Using eq. (21) and eq. (22) it is found that the evolution of eq. (17) to eq. (18) occurs if $\exp(-iH_A\eta_A)$ is a rotation of odd multiple of π about an axis defined in eq. (20) and is orthogonal to the axis defined in eq. (19). Such a rotation takes X_{20} into X_{10} and leads to the conditions

$$\omega_A \eta_A = \left(N + \frac{1}{2}\right)\pi, \qquad N = 0, 1, 2, 3 \ldots, \tag{23}$$

$$m^2 C_1 C_i + \lambda^2 = 0, \tag{24}$$

$$\omega_A{}^2 = m^2 C_1{}^2 + \lambda^2. \tag{25}$$

In a realistic spacetime both C_1 and C_i are positive. Writing

$$mC_1 = mC_i + A, \tag{26}$$

the reality of mC_i requires

$$|A| \geq 2\lambda \tag{27}$$

and

$$mC_i = -\omega^2. \tag{28}$$

If $C_1 > C_i$, A remains positive and eq. (28) implies ω imaginary. Hence we must have $C_1 < C_i$ always. This is a clue to the bounce. Reality of λ^2 requires

$$1 - \left[1 - \frac{4\lambda^2}{A}\right]^{\frac{1}{2}} < 2\left[\frac{\left(N + \frac{1}{2}\right)\pi}{A\eta_A}\right]^2 < 1 + \left[1 - \frac{4\lambda^2}{A^2}\right]^{\frac{1}{2}}. \tag{29}$$

Near threshold $A \simeq 2\lambda$ hence for $N = 0$ resonance

$$\lambda\eta_A \simeq 2^{\frac{1}{2}}\left(N + \frac{1}{2}\right) \simeq 1. \tag{30}$$

Equation (30) indicates the expected occurrences of bounce near Compton time for $\lambda \simeq m$. In realistic spacetime conditions eq. (24) and eq. (25) are violated and the physical region poles (occuring in R_p and T and affecting the rotation $X_{20} \to X_{10}$) become a resonance. This fact signifies an instability of vacuum resulting in dominant particle production. We proceed with this aspect in the next section for a realistic spacetime with a smooth scale factor.

3. Antiparticle to particle rotation for smooth scale factor

In order to define the vacuum we assume

$$\lim_{|\eta|\to+\infty} C_1(\eta) < \infty \tag{31}$$

and at some given time η_0, it is possible to define Minkowski-like vacuum as in eq. (12) and eq. (13). The second order form of Dirac equation is

$$[\partial_\eta{}^2 + \lambda^2 + m^2 C_1{}^2(\eta) - i\,m\,\dot{C}_1(\eta)]f_1 = 0. \tag{32}$$

Here $\dot{C}_1(\eta) = \partial_\eta C_1(\eta)$ and the solution f_2 is obtained from f_1 as $f_2(m,\eta) = f_1(-m,\eta)$. Equations (16) and (18) are now translated in the following way. We find the turning points of eq. (32). The turning points are identified as points where the particle turns back, not in space but in time, which according to Feynmann–Stuckleberg prescription must be identified as an event of pair production. Using the method of CWKB [19, 20] we find that eq. (23) and eq. (24) are now replaced by

$$S(\eta_1,\eta_2) \equiv \int_{\eta_1}^{\eta_2} W(\eta)\,\mathrm{d}\eta = \left(N + \frac{1}{2}\right), \tag{33}$$

$$W(\eta) \equiv \left[\lambda^2 + m^2 C^2{}_1(\eta) - i\,m\,\dot{C}_1(\eta)\right] = 0. \tag{34}$$

Equation (34) determines the turning points η_1 and η_2, that in general are complex. The pair production amplitudes are now given by

$$X(\eta) \underset{\eta\to\infty}{\longrightarrow} \exp[iS(\eta,\infty)] + iR_p \exp[-iS(\eta,\infty)], \tag{35}$$

where pair production amplitude

$$R_p = -\frac{\exp[iS(\eta_1,\infty)]}{1+\exp\left[2iS(\eta_1,\eta_2)\right]}. \tag{36}$$

In eq. (31) we have neglected the WKB pre-exponential factor for convenience. Equation (36) shows the existence of poles in R_p as resonances because $\eta_{1,2}$ are now complex. When $W(\eta)$ becomes constant, eq. (33) will give back the results eq. (23) of §2. The enhancement of particle production through resonance and hence the instability of vacuum is quite obvious from eq. (36).

To show antiparticle to particle rotation we consider X_j to be any normalized solution of

$$[i\partial_\eta - \sigma_3 m C_1(\eta) - \sigma_1\lambda]X_j(\eta) = 0. \tag{37}$$

Define a current

$$J_i = X^\dagger \sigma_i X, \quad J^2 = 1. \tag{38}$$

In the notation of eq. (12) and eq. (13) a free particle corresponds to $J_+ = (\sin\theta, 0, \cos\theta)$ and a free antiparticle corresponds to $J_- = (-\sin\theta, 0, -\cos\theta)$. Now for pair production J, defined in eq. (38) must go from J_- at $\eta = -\infty$ to J_+ at $\eta = +\infty$ for a smooth $C_1(\eta)$. To obtain a smooth rotation from J_- to J_+, we define

$$J_1 = -\sin\theta\cos\frac{1}{2}\varphi(\eta) - \xi\cos\theta\sin\varphi(\eta), \tag{39}$$

$$J_2 = \sin\frac{1}{2}\varphi(\eta)\ \left[1 - 4\xi^2\cos^2\frac{1}{2}\varphi(\eta)\right]^{1/2}, \tag{40}$$

$$J_3 = -\cos\theta\cos\frac{1}{2}\varphi(\eta) + \xi\sin\theta\sin\varphi(\eta). \tag{41}$$

Here ξ is constant and $\varphi(\eta)$ is a function such that it varies smoothly from 0 at $\eta = -\infty$ to 2π at $\eta = +\infty$. This variation takes J_- at $\eta = -\infty$ to J_+ at $\eta = +\infty$, ensuring particle to antiparticle rotation. The currents defined in eq. (39) and eq. (41) satisfies $J^2 = 1$ and should satisfy eq. (37) and eq. (38). Using eq. (37) and eq. (38) we get

$$\dot{\vec{J}} = 2\vec{W}\times\vec{J}, \qquad \vec{W} = (\lambda, 0, mC_1(\eta))\,. \tag{42}$$

Using eq. (41), and eq. (39) in eq. (42) we get

$$\dot{\varphi} = 4\lambda J_2 K^{-1}, \tag{43}$$

$$C_1(\eta) = -\frac{\lambda L}{mK}, \tag{44}$$

where

$$K = \cos\theta\sin\frac{1}{2}\varphi(\eta) + 2\xi\sin\theta\cos\varphi(\eta), \tag{45}$$

$$L = \sin\theta\sin\frac{1}{2}\varphi(\eta) - 2\xi\cos\theta\cos\varphi(\eta), \tag{46}$$

We solved eq. (43) numerically (for $\xi = 0.25$) and found that it is represented by an analytic expression

$$\varphi = \pi[1 + \tanh(\lambda\eta - 1)]. \tag{47}$$

In eq. (47), λ has the dimension of mass and sets the scale of η. It should not be confused with eigenmode frequency λ occurring in eq. (37). Actually we evaluated eq. (47) for $\tan\theta = 1$ and this gives $\lambda = mC_{\rm in}$ from eq. (14). The analytic expression is a very idealized case. For a realistic situation, the scale factor would depend on m $C_{\rm in}$ and θ and is quite reasonable (see also eq. (73)).

Equations (43), (44) and (47) now allow us to determine $C_1(\eta)$ in terms of η. For a realistic spacetime the scale factor must be positive otherwise there will be no particle production. We will show it shortly. We are free to redefine the vacuum and write

$$C(\eta) = C_0 + C_1(\eta), \tag{48}$$

where C_0 is a constant and $C(\eta)$ represents a realistic spacetime. The constant C_0 is fixed such that $C(\eta)$ is positive; the violation would then make particle production vanish. For nonvanishing particle production we get singularity avoidance in the theory. Taking $\frac{\lambda}{m} = 1,\ \xi = 0.25$, we find from eq. (48)

$$C(\eta) = \begin{cases} C_0 + C_i\,, & |\eta| \to \infty \\ C_0 - 3C_i\,, & \lambda\eta \to 1\,,\ (\varphi \to \pi) \end{cases}. \tag{49}$$

Equation (49) shows that the bounce occurs at $\lambda\,\eta = 1$ and the minimum value of the scale factor is $C_0 - 3C_i$. Thus the mechanism of bounce proposed in §2 also works in most general case of conformally flat or non-flat spacetime.

Let us now calculate the energy density of created particles for the scale factor given in eq. (44). To get an idea of the analytic nature of the energy density we approximate eq. (48) around $\lambda\eta = 1$. With $\frac{\lambda}{m} = C_i, C_0 - 3C_i = b$ we get

$$C^2(\eta) = b^2 + 2C_i\pi^2 b\,(\lambda\eta - 1)^2. \tag{50}$$

Equation (50) indicates that at $\eta \to \infty$, it approaches the radiation dominated cosmology with

$$a(t) = C^{\frac{1}{2}}(t) \propto t^{\frac{1}{2}}.$$

To get an idea about the energy density of created particles, we evaluate eq. (36) to get

$$|R_p|^2 \approx \frac{e^{-\pi\beta_0}}{(1 + e^{-\pi\beta_0})^2}, \tag{51}$$

where

$$\beta_0 = \frac{mb^{\frac{3}{2}}}{(2C_i)^{\frac{1}{2}}} + \frac{\lambda^2}{m(2C_i b)^{\frac{1}{2}}}, \tag{52}$$

From eqs (51) and (52) we find that if at some time $C(\eta) \to 0$ we must have $b = 0$. This makes $\beta_0 \to \infty$ and $|R_p|^2 \to 0$. So in order to have dominant particle production about the region $bR > cR^2$, we should have $b \neq 0$ and the singularity is avoided. Thus the particle production provides a mechanism for the avoidance of singularity. Using the general expression [15] in Birrell and Davies

$$\rho = (2\pi)^2 C^{-4}(\eta) \int |R_p|^2 \omega \lambda^2 \mathrm{d}\lambda, \tag{53}$$

we find

$$\rho = \frac{\rho_r}{C^4(\eta)}. \tag{54}$$

Because of $\exp(-\pi\beta_0)$ factor, eq. (53) remains finite and ρ_r in eq. (54) depends on parameters b and C_i of the scale factor. The expression eq. (51) was obtained by Audrestch and Schäfer [21] but without the factor in the denominator. In Audrestch and Schäfer [21] production amplitude can tend to zero even when the the scale factor remains finite but in our case the production amplitude goes to zero when the scale factor goes to zero. In a realistic cosmological model the initial vacuum will be greatly affected by the R^2 term. Our results eq. (50) and eq. (54) will therefore be valid near about the bounce in which these expressions remain workable. Actually ρ given in eq. (54) is an approximation because we have assumed the vacuum both at early and late times that correspond to a radiation dominated cosmology, which is not actually the case in reality. Just before the onset of bounce the dominating R^2 term causes an instability in the vacuum resulting in particle production and after the bounce the matter production will halt the growth of R^2 term to a power law asymptotic behaviour of the background spacetime. In the next section, to justify the claim just mentioned, we try to obtain a realistic model of spacetime with eq. (50) and eq. (54) as initial conditions to start the numerical solution of Einstein field equation. To correct the approximate form eq. (54) allowance of term proportional to R^2 should be added to eq. (54) to halt the growth of R^2 term at late times. However in the present work we work with eq. (54) only and for this reason we take care in numerical calculation not to continue in the region where R^2 like term begins dominating. In the next section we take up the back reaction problem where energy density of created particles plays a dominant role to ensure $bR > cR^2$ transition as well as to halt the growth of R^2 term at late times. It is worthwhile to point out that the above mechanism of bounce i.e., $J \to -J$ rotation works also for scalar field. We would like to put it in a future publication.

4. Back reaction and avoidance of singularity

We start from Einstein equation

$$R_{ab} - \frac{1}{2}g_{ab}R = -\frac{l^2}{2}\langle 0|T_{ab}|0\rangle + T_{ab}{}^{(N)}, \tag{55}$$

in which $l = (16\pi G)^{\frac{1}{2}}$ is the Planck length, g_{ab} is the metric tensor, $\langle 0|T_{ab}|0\rangle$ is the expectation value of the stress-energy tensor operator for the quantum fields supposed to arise from $0\left(R^2\right)$ type term in eq. (1) and with a contribution $T_{ab}{}^{(N)}$ coming from eq. (54). Equation (55) is called the semiclassical back-reaction equation and is a semiclassical approximation to quantum gravity. At high curvature $R > m^2$, we can neglect the mass and take the standard results of one-loop approximation as [10,11,15]

$$\begin{aligned}\langle 0|T_{ab}|0\rangle &= \frac{1}{3}\alpha\left(R_{||ab} + RR_{ab} - g_{ab}R^{||c}{}_{||c} - g_{ab}\frac{R^2}{4}\right)\\ &\quad +\beta\left(R_a{}^{c}R_{bc} - \frac{2}{3}RR_{ab} - \frac{1}{2}g_{ab}R^{cd}R_{cd} + \frac{1}{4}g_{ab}R^2\right),\end{aligned} \tag{56}$$

where R_{ab} is the Ricci tensor, $R = g^{ab}R_{ab}$ is the scalar curvature, and α, β are constants which are to be fixed by experiment or by some regularisation scheme. It fixes in our model the second length scale, apart from Planck scale. $T_{ab}{}^{(N)}$ is not known a priori but near about the bounce we replace it by the approximation eq. (54). It should be pointed out that the bounce mechanism proposed in the previous section applies both to spinor and scalar fields as well. The expression eq. (54) serves as energy density of produced particles and eq. (56) is the one loop quantum corrections for $R >> m^2$. Thus in our model we have massive particle production inbuilt through eq. (54).

Let us try to understand eq. (56) in the light of particle production. Equation (55) is conserved in conformally flat spacetime i.e. $\langle 0|T_{ab}|0\rangle_{||b} = 0$. As the universe enters the bounce region this conservation is violated and we get large particle production. At late time $\eta \to \infty$, the produced particles try to behave classically so that universe emerges in such a way that rhs of eq. (55) restores conservation again. In other words the emergence of bounce avoiding singularity and resulting in particle production maintains the energy conservation to emerge in a state dictated by classical cosmology consistent with eq. (55). We find from the discussion of the previous section that it is probable for the universe to emerge with a radiation dominated cosmology where the quantum corrections due to eq. (56) become less and less effective gradually. However, we expect to get other solutions with $\langle T_a{}^b \rangle_{||b} = 0$ but they will not be asymptotically classical solutions (ACS) and will not represent the realistic spacetime. But at early times there are no particles present. As a result the vacuum changes in such a way that eq. (56) becomes dominating with $\langle T_b{}^a \rangle_{||a} = 0$ i.e. the initial vacuum again becomes a conformal vacuum. Neglecting $T_{ab}{}^{(N)}$, eq. (56) gives a de-Sitter solution and we know that in de-Sitter vacuum there is no particle production. Whether this mechanism works has to be verified by explicit calculations. This is the basic difference with the other works [10,11] in this direction. In the R–W spacetime the only variable is $C(\eta)$ and hence all the nontrivial components of eq. (55) must be linearly independent. Therefore, we choose only the time-time component of eq. (55). Using the notation of Birrel and Davies[15], we find

$$\langle 0|T_{00}|0\rangle = \frac{\alpha}{6}{}^{(1)}H_{00} + \beta\ {}^{(3)}H_{00}, \tag{57}$$

where

$${}^{(1)}H_{00} = a^{-1}\left[9D\ddot{D} - \frac{9}{2}\dot{D}^2 - \frac{27}{8}D^4 - 9kD^2 + 18k^2\right], \tag{58}$$

$${}^{(3)}H_{00} = a^{-1}\left[\frac{3}{16}D^4 + \frac{3}{2}kD^2 + 3k^2\right], \tag{59}$$

$$a(\eta) = C^2(\eta)\ ,\ D = \frac{\dot{a}}{a}\ ,\ \dot{D} = \frac{\partial D}{\partial \eta}. \tag{60}$$

We take $T_{00}{}^{(N)}$ as in eq. (54). This implies that we should start our calculations near-about the bounce point to fix the initial values. We set our scale as

$$b = l^{-1}\rho_r{}^{-\frac{1}{4}}C(\eta), \tag{61}$$

$$\chi = \frac{1}{\sqrt{6}}\sqrt{\rho_r}(\eta - 1), \tag{62}$$

with $\lambda = 1$. The eq. (55) then reduces to

$$\dot{b}^2 + \frac{6kb^2}{\sqrt{\rho}} = 1 + \frac{1}{3}\alpha\left[\frac{\ddot{b}b}{2b^2} - \frac{\ddot{b}\dot{b}^2}{b^2} - \frac{1}{4}\left(\frac{\dot{b}}{b}\right)^2 - \frac{3k}{\sqrt{\rho}}\left(\frac{\dot{b}}{b}\right)^2 + \frac{9k^2}{\rho_r}\right] + \beta\left[\frac{1}{12}\left(\frac{\dot{b}}{b}\right)^4 + \frac{k}{\sqrt{\rho_r}}\left(\frac{\dot{b}}{b}\right)^2 + \frac{3k^2}{\rho_r}\right], \tag{63}$$

where $\dot{b} = db/d\chi$. Given the parameters α, β and ρ_r, to fix initial values of $b, \dot{b}$ and $\ddot{b}$, one should have a reasonable approximation to eq. (62) to obtain a solution. For $k = 0$ and $\alpha > 0$, FHH [10] obtained a one parameter family of solutions

$$f = \left\{1 + \frac{\beta}{6\alpha} y^{-4/3} + \ldots\right\} + \Psi_{10} y^{1/6} e^{-3^{1/2} y^{2/3}} \{1 + \ldots\}$$
$$-\frac{1}{2} {\Psi_{10}}^2 y^{1/3} e^{-12^{1/2} y^{2/3}} \{1 + \ldots\}, \qquad y \to \infty \tag{64}$$

where $f = |b|^{3/2}$, $y = |\alpha|^{-3/2} b^3$ and Ψ_{10} is an arbitrary constant. By eq. (64) we have an one parameter family of solution. Equation (64) when integrated gives

$$b = b_0 + \chi + \frac{\beta}{72} \chi^{-3} + \ldots, \qquad \chi \to \infty, \tag{65}$$

where b_0 is an integration constant. In [11], Ψ_{10}^2 term is set to zero to find initial values of $\dot{b}, \ddot{b}$ and to fix b from eq. (65). A subsequent numerical integration backward in time cannot then guarantee vanishing particle production because the first term unity in eq. (64) also arises from eq. (54) which is in the form of classical radiation density term. The fact that there is vanishing particle production as well as $f \to 1$ when $\chi \to \infty$ is not borne by other works in this direction [10, 20]. We verified through numerical calculation, taking $\beta = 6\alpha$, that

$$f \to \begin{cases} 0 & \text{for } y < y_c, \\ \infty & \text{for } y > y_c, \end{cases} \tag{66}$$

where y_c is a critical value with y defined in eq. (64) and suggest the possibility of having a $f \to 1$ behaviour at late time. To obtain $f \to 1$ behaviour one needs to halt the growth of f and is arbitrarily done by choosing Ψ_{10}^2 term in eq. (64) to zero [11]. In other words quantum corrections greatly regulate the $f \to 1$ behaviour at $\chi \to \infty$. Thus vanishing particle production cannot lead to $f \to 1$ as claimed in [11,12]. We will discuss on this point shortly in the concluding discussion. This is the basic difference between our work and that of Anderson's work [11,12,26,27].

For massive particle production, we need more terms to eq. (54) or to follow the procedure adopted by Anderson. Actually our objective is to introduce the effect of particle production numerically without a detailed study of the evolution of the quantum vacuum.

In order to establish that particle production really matters in obtaining $f \to 1$ behaviour simulating asymptotically classical solutions, we evaluate the behaviour of $b(\chi)$ from particle production-dominated scale factor

$$b(\chi) = b_0 - b_i \frac{\sin\varphi/2 - 2\zeta\cos\varphi}{\sin\varphi/2 - 2\zeta\cos\varphi}. \tag{67}$$

Using the results of the previous section ($\zeta = 0.25,\ \lambda = 1$), eq. (67) gives

$$b(\chi) = (b_0 - 3b_i) + b_i \pi^2 (\chi - 1)^2. \tag{68}$$

Here b_0 and b_i are constants and the bounce occurs at $\chi = 1$ with the scale factor $b_m = b_0 - 3b_i$. It should be pointed out that eqs (67) and (68) are only cited to elucidate the role of particle production. To be consistent we obtain solution of eq. (63) around $\chi \approx 1$ to find the parameters in eq. (68). We write $H = (\mathrm{d}b/\mathrm{d}\chi)/b$ and neglecting H^4 term (since near bounce $H \approx 0$) we get

$$2H\ddot{H} - \dot{H}^2 - M^2H^2 + \rho^2 = 0, \tag{69}$$

where

$$\rho^2 = \frac{12}{\alpha}\left(1 + \frac{3k^2(\alpha+\beta)}{\rho_r} - \frac{6kb^2}{\sqrt{\rho_r}}\right), \tag{70}$$

$$M^2 = \frac{12}{\alpha}\left(b^2 + \frac{\alpha k}{\sqrt{\rho_r}} - \frac{\beta k}{\sqrt{\rho_r}}\right). \tag{71}$$

Near the region $H \approx 0$, we set $b = b_m$ in eq. (70) and eq. (71) and find the solution of eq. (69) as

$$H = \frac{\rho}{M}\sinh M(\chi - \chi_0). \tag{72}$$

For $k = 0$ we then obtain for $\chi \sim \chi_0$

$$b = b_m\left\{1 + 3^{1/2}\alpha^{-1/2}(\chi - \chi_0)^2\right\}. \tag{73}$$

From eq. (73) and eq. (52), it is found that as $\alpha \to 0$, implying vanishing of quantum corrections, β_0 of eq. (52) goes to infinity and hence $|R_p|^2 \to 0$. This makes the difference with the other works [11,12] though we arrive at similar eq. (63) with that of [11] and [12] for the back reaction equation. From eq. (72) we get

$$b(\chi) = b_m e^{(\rho/M^2)(\cosh M(\chi-\chi_0)-1)}. \tag{74}$$

It is evident from eq. (74) that to halt the exponential growth, we should also consider H^4 term in eq. (69). At $b = b_m$ we have

$$\frac{6kb_m^2}{\sqrt{\rho_r}} = 1 - \frac{\alpha}{12}\frac{\ddot{b}_m^2}{b_m^2} + \frac{3k^2}{\rho_r}(\alpha+\beta). \tag{75}$$

Fixing b_m, we can determine $\ddot{b} \simeq \ddot{b}(\chi \approx \chi_0)$ from (75) and use eq. (74) for $\chi \to \chi_0$ to fix b, $\dot{b}$ and $\ddot{b}$. It should be pointed out that eq. (74) for $\chi \to \chi_0$ leads to the form eq. (68) from which the constants b_0, b_i can be obtained. However we will work with eq. (74) to fix the initial values that guarantee the avoidance of singularity and look at the evolution dictated by eq. (63) through numerical solutions. The other type of solutions will be found in the works of Anderson [11,12]. Hence we proceed with eq. (74) to fix b, $\dot{b}$ and $\ddot{b}$ adjusting b_m such that asymptotically classical solutions (ACS) consistent with eq. (63) or eq. (64) can be obtained. To establish that the particle production could lead to classical solution as well as bounce avoiding singularity, we discuss in the next section $k = 0$ and $\beta = 6\alpha$ case only for elucidation.

5. Asymptotically classical solutions

To obtain the asymptotically classical solutions we take

$$\alpha = \frac{\beta}{6} = \frac{1}{2880\pi^2}, \quad k = 0 \ \text{ and } \ \rho_r = 1. \tag{76}$$

To make allowance for a correction to the approximate form eq. (54), we give some initial values to $\ddot{b}(\chi = \chi_0)$.

In the out asymptotic region $f \to 1$, hence $\dddot{b} = 0$ such that $b(\chi = \chi_0)$ is given by (73) and amounts to no particle production. If we allow $\dddot{b} \neq 0$ even at $\chi \to \chi_0$, we take contribution of particle production as if the vacuum gets modified. In the context of the model, we searched (instead of an analytic treatment) near $\chi \to \chi_0$ for a suitable $\dddot{b}$ for given b_m and $\ddot{b}_m$, so that $f \to 1$ behaviour is achieved at $\chi \to \infty$. The justification of this approach is established through our numerical calculation.

We fix the initial values of $b, \dot{b}, \ddot{b}$ from eq. (73) and calculate $\dddot{b}$ using eq. (62) for $\chi = 1.0001$ so that (73) remains valid in this region. We then solve eq. (63) carrying out the solution both forward and backward in time. The figure 1 shows the numerical solution that approaches $\dot{b}(\chi) \to 1$ at χ large. The upper, middle and lower curves corresponds $b_m = 0.015996, 0.015997, 0.01599359$ and it is evident that a slight change in b_m value greatly affects the early time behaviour leading to de-Sitter type, symmetric bounce type and singularity reaching type solutions respectively. As soon as the solution reaches ACS at $\chi \approx 1.13$, we find that the quantum corrections again dominate to destroy the ACS behaviour (shown in figure 1 for $\chi > 1.13$). In order to regulate this behaviour, one should modify eq. (54) as [22]

$$\rho = \frac{C_1 \rho_r}{C^4(\chi)} \int_{\eta_0}^{\eta} R^2 a^5 \mathrm{d}\eta, \tag{77}$$

where R is the Ricci scalar. We observe that fixing $\dot{b} \approx 1$ at late times needs to give some initial values to $\dddot{b}$ at $x \approx x_0$ which in turn requires to incorporate H^4 term in eq.(68). In our numerical calculation (figure 1) it is found that the term like (77) or initial values of $\dddot{b}$ would be crucial to obtain the asymptotic behaviour of Friedmann cosmology. As it is evident from eqs (73), (74) and (75), one expects to get Friedmann behaviour at late time fixing b_m, $\ddot{b}_m$ from eq. (74) or eq. (75), but numerical calculations do not reveal this behaviour. But giving some initial values to $\dddot{b}$ at $\chi \approx \chi_0$ imply a contribution like eq. (77) or H^4 term in eq. (69). Our conclusion is that it is the particle production, contrary to the claim in [11,12], that leads to the Friedmann behaviour at late time.

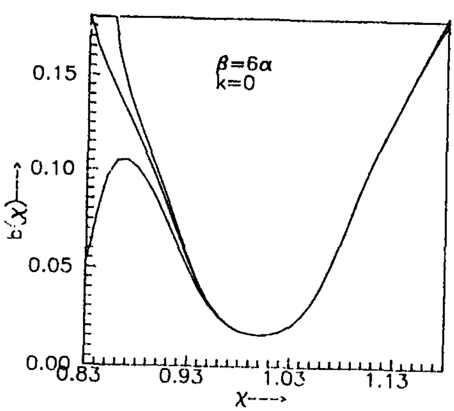

Figure 1. This figure shows marginal ACS for $\beta = 6\alpha$. The minimum scale factor for the three curves are shown in the text. The value of b(min.) is fixed by eq. (63) at the bounce.

To understand the origin of particle production, we can write from Parker's general arguments [1], in a R–W spacetime, the relation between trace anomaly and the complete stress energy tensor as

$$\rho(t_2)a^4(t_2) = \rho(t_1)a^4(t_1) - \int_{t_1}^{t_2} a^3 a' \langle T_\mu{}^\mu \rangle_{\text{ren}}\, \mathrm{d}t. \tag{78}$$

Here prime denotes derivative with respect to t, the proper time. Whenever trace of the energy momentum tensor is not zero, we get particle production. In our work the $T_\mu{}^\mu$ is not zero and breaks the conformal invariance. As it is hard to obtain analytical solution in the region where conformal invariance breaks, we obtain it in a different way and treat avoidance of singularity through particle production mechanism. One way to incorporate the effects of H^4 term as well as the R^2 term in eq. (77) is to give some nonzero value to $\dot{b}$ near about the bounce. We search for it but find that our solutions become marginally Friedmann. In view of this fact we also carried out numerical integration backward in time using eq. (64) to fix the initial values. Doing so we find that the value of $\ddot{b}$ does not satisfy the requirement $\ddot{b} = \sqrt{(3/\alpha)}b_m$ necessary to satisfy eq. (63) i.e., the analytical solution near about the bounce. To get the Friedmann behaviour at late time the value of b_m and the values of $\dot{b}, \ddot{b}$ have to be adjusted and differs from our results remarkably. This is why we carried out the calculation with the approximate form of the energy density of created particles as defined in eq. (54) of §3.

In our numerical calculation we fix b, $\dot{b}$ and $\ddot{b}$ giving suitable values to b_m and χ_0 and find the behaviour at late time as is found in the works [11,12]. However, if we continue further at large χ values, it is found that the solutions either fall down or show some oscillations. To restore the Friedmann behaviour, eq. (64) has to be modified suggesting the importance of particle production in the back reaction analysis.

Our conclusion is that one loop quantum corrections along with particle production contribution avoid the singularity and lead to marginal Friedmann behaviour at late times.

6. Discussion

In the present work we have studied the effect of particle production on the evolution of the universe due to one loop quantum corrections. We observed that the universe avoids the singularity and reaches asymptotically to a reasonable ACS of radiation density cosmology. The result obtained in this work is consistent with the investigation [11,12,26,27] done earlier. However, there are some basic differences. The one loop quantum correction is identical with that of massless conformal fields and the particle production density is also the same as the radiation energy density thereby as if apparently implying no particle production in our model. It would be worthwhile to point out that even in [26], the dominant effect of mass terms (that break conformal invariance) is to effectively change the amount of classical radiation in the universe because $\langle T_0^0 \rangle$ for mass terms diverges like b^{-4}. At the present level we have no way to distinguish between our work (demanding particle production) and that of Anderson [11,12] where no particle production is assumed a priori, where a classical radiation density term is included to ensure the avoidance of singularity and ensuring radiation density regime at late time.

In any work describing particle production in curved spacetime the choice of quantum vacuum is very delicate and important. In our work we have seen that if $b \to 0, |R_p|^2$ in eq. (51) goes to zero. Thus for $b = b_m, \beta_0$ in eq. (52) will be large and hence near about the bounce, the solution of the field equation can now be easily obtained. If we neglect the spinorial complication and wrote eq. (50) as

$$C^2(\eta) = a^2 + d^2(\lambda\eta - 1)^2,$$

the solution of eq. (32) is given as

$$\chi_k^{(0)} \underset{\beta_0 \to \infty}{\longrightarrow} (4md\beta_0)^{-1/4} \exp[-i(md\beta_0)^{1/2}\eta'], \tag{79}$$

where η fixed and $\lambda\eta - 1 = \eta'$. The exact solution

$$\chi_k{}^{(\text{in})} = (2m\, d)^{-1/4} \exp[-\pi\beta_0/8] \times D_{-(1-i\beta_0)/2}[(i-1)(m\, d)^{1/2}\, \eta'], \tag{80}$$

reduces to eq. (79) plus term of $0({\beta_0}^{-1/2})$. Thus for η' negative eq. (79) corresponds to no particle state. Thus we take eq. (79) as our "in" vacuum state as $b \to b_m$. In the limit of $m \to 0$, $m\beta_0$ tends to λ [see eq. (52)] hence the vacuum chosen is a conformal vacuum. In [26], the vacuum is chosen for $b \to 0$ and this makes the basic difference between our work with that of [11,12]. The out vacuum is fixed through

$$\chi_k{}^{(\text{out})}(\eta') = \chi_k{}^{(\text{in})}(-\eta'), \qquad \text{for } \eta > 0,$$

where $\chi_k{}^{(\text{in})}$ is given in eq. (80). These results are very standard [15] and hence we do not elaborate it.

To know the evolution of quantum vacuum at a very late time we must solve the field equation of particle with explicit construction of $\langle {T_0}^0 \rangle$ along with the vacuum definition as is taken in this work. We would like to present it in a future publication. As the vacuum definition (in our case $b \to b_m$) is mostly like [27], we expect also that the particle production will cause the ACS to expand like classical-matter-dominated universe at late times. The objective of the present work is to simulate a de-Sitter universe at early time and matter dominated or classical radiation dominated spacetime at late time in view of the ambiguity in defining the vacuum in curved spacetime. In other words we like to dictate the late time behaviour through the restrictions on early time behaviour contrary to the investigations carried out in [11,12,26,27]. Fixing a conformal vacuum at late time and use of massless conformal fields have forced the author in [11,12] to assume no particle production in their work. But for a curvature scalar >> mass of the field, the expression for $\langle 0|T_{\mu\nu}|0\rangle$ introduced in our work serves equally well for massive fields also. Apart from these technical differences, it is worthwhile to restudy the back reaction equation in light of the antiparticle to particle rotation dominated contribution in the energy density of the produced particles and investigate the nature of late time behaviour corresponding to eq. (63).

It is worthwhile to point out the differences with our works and that of Anderson's [11,12,26,27]. We concentrate on bounce solutions avoiding singularity. We show that the singularity avoidance takes place due to particle production as emphasized in the introduction. The energy density of the produced particles calculated in §3 comes out to be similar to the radiation density term as was obtained in [11,12,26,27] making the two approaches very identical with respect to the Einstein equation. To stress our claim we started from a scale factor simulated from particle-antiparticle rotation and find that eq. (74) resulting from back reaction equation around the bounce region is consistent with (68). Though we obtain almost similar results with that of Anderson, we proceed in our numerical calculation from initial conditions at an early time in contrary to late time initial conditions as were taken in [11,12,26,27]. The new results we obtain are that the particle production greatly effects the late time behaviour and has not been reasonably tackled in the work of Anderson [11,12] and the evolution of the vacuum has a definite role in deciding the late time behaviour. Unlike Anderson, we concentrate only on those solutions that have a de-Sitter behaviour, at early time and emerging asymptotically as classical universes due to particle production.

One may argue about the form of the scale factor obtained through the $J \to -J$ rotation. To clarify our claim, we carried out calculations of the current density in some standard expanding spacetime with known solutions of massive spinor fields and found that the currents show expected oscillations signifying particle to antiparticle rotation and thereby signaling the presence of particle production as is carried out in this paper. Our approach also indicates a way to study the evolution of vacuum of the quantum fields as it evolves according to Einstein equation. This trend of investigation has also been elaborated in [23,24,25] in the context of particle production.

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