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Possibility of weakly charged nonlinear solitary dust clouds near the continuum threshold of dust population in a quasi-neutral dusty plasma

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Abstract. Considering the Boltzmann response of the plasma ions and electrons and inertial dynamics of the charged dust grains, the possibility of very weak compressive soliton near the continuum limit of the dust population has been inferred. It is concluded that the behaviour of such coherent structures could be well described by the numerical analysis of the derived nonlinear classical energy integral equation for bounded solutions. These seem to be higher order dispersive structures within acoustic limit of the nonlinear turbulence. It is observed that the dust density enhancement beyond the continuum threshold causes regular increment in width and amplitude of the soliton structures. It is found that the soliton amplitude sensitively depends on the massive impurity's population. These coherent structures could be visualized as weakly charged solitary dust clouds of finite extension ($\sim$ plasma Debye length) within Boltzmann environment of plasma particles in their local surroundings. The seeding mechanism of such clouds may be attributed to some plasma instabilities driven by either internal or external free energy sources. Numerical analysis of the problem concludes that the experimental observations of such clouds could be possible in low density plasma regime. It is deduced that for plasma density $\sim 10^6\,\mathrm{cm}^{-3}$ at temperatures of a few electron volts and for micron to 10 nm sized dust grains, the observation of such structures could be possible within wide range variability of the dust population density.

Keywords. Dusty plasmas; soliton; dust grains; solitary dust clouds; colloidal suspension; coherent structures.

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1. Introduction

Dusty plasma is an unique multicomponent plasma system and is composed of dispersed macroscopic charged dust grains in any given parent plasma background to form a colloidal type suspension [1]. The finite size of the dust grains (typically of the order of micron and submicron sizes) causes a qualitative modification over the conventional charging mechanism (i.e. electronic transitions on atomic or molecular scales) of the plasma particles. The dust grains are charged through surface-plasma interaction processes on macroscopic scale size of the order of the dust surface area. In the absence of any radiative environment in the given parent plasma background, the dust grains normally acquire negative charges to high order of magnitude with respect to normal electronic charge ($q_d \sim 10^5$–$10^6 e$). Thus the dust charge to mass ratios are different (by orders of magnitude) than the normal ion species in general multicomponent plasmas. Furthermore, the dust charge, mass and size, in general, behave as dynamic variables [2, 3, 4] and produce novel effects on collective degrees of plasma dynamics especially on longer time scale phenomena. The dust charging mechanism being governed by the surface-plasma interaction process on collective scales of space and time depends on the ambient plasma (parent plasma) conditions and dust grain size and its electrical properties. The dust charging phenomena could be modelled as a capacitor charging process [5–7] and, accordingly estimations of dust charging/discharging time scales (τ_c) and the maximum charge (q_d) on it can be given as [8]:

$$\tau_c \sim \omega_{\rm pi}^{-1} \frac{\lambda_{\rm D}}{a}, \qquad q_d \sim C\Phi_{\rm fo}, \tag{1}$$

where $\omega_{\rm pi}$ is the parent ion oscillation frequency, $\lambda_{\rm D}$, the plasma Debye length, a, the size of the dust grains, $C(\approx a)$, the capacitance of the dust surface and $\Phi_{\rm fo}$, the equilibrium floating potential of the dust surface.

Dusty plasma is ubiquitous and occurs in many natural conditions of astrophysical environments [9], including the earth's environment, planetary environments and in laboratory experiments of basic practical interests [10–12]. Most of the earlier investigations were confined around the dust charging model development [5–7] and the effects of dust grains on high frequency collective plasma properties [13–15] under isolated approximation of dust grains. However, theoretical activities on dynamical effects of the dust grains within collective approximation of dusty plasmas picked up momentum at around the eighties [9, 16]. However, a couple of articles were written even earlier [17]. During the period of late eighties a novel plasma sound wave namely the so-called acoustic mode was reported by Dwivedi and co-workers [18–20] in a theoretical proposal of a quasi-neutral three component plasma system with hot and cold ions. It is reported that such sound waves occur on inertially stretched acoustic time scales and the stretching produces a qualitative modification in electrodynamical behaviour of the parent plasma species. The distinct qualities of these sound waves have been described in later publications [20, 21]. In fact, a dusty plasma system provides a practical replica of the proposed model by Dwivedi *et al* [18, 21]. It is worth mentioning that the same characteristic sound wave was published a year later by Rao *et al* [22] in the context of dusty plasmas and was named as the dust acoustic wave (DAW). However, the later communications by Dwivedi [23] include the arguments that the DAW should be visualized as a low frequency version of the so-called acoustic mode in plasmas containing dust like heavier impurity ions in any given parent plasma background.

If we estimate the relative scaling of the wave time scale (τ_ω) and the dust charging time scale (τ_c) for a given parent plasma, we find

$$\frac{\tau_\omega}{\tau_c} \sim p\left(\frac{a}{\lambda_{\rm Di}}\right)\frac{1}{k\lambda_{\rm Di}} \quad \text{for } k^2\lambda_{\rm Di}^2 \ll 1, \tag{2}$$

where $p = [(m_d/m_i)(n_{io}/n_{do}Z_d)(1/Z_d)]^{1/2}$, $\lambda_{\rm Di}$ is the parent ion Debye length. Other quantities are defined elsewhere. This is to note that the above scaling has been derived for the low frequency version (DAW) of the so-called acoustic mode which is a natural normal sound wave in plasmas with dust-like heavier impurity ions. From the general linear charging equation, one can see that for $\tau_\omega \gg \tau_c$, which requires $p \gg 1$, the effect of the steady state charge dust charge fluctuation could be nominal within the quasi-neutrality limit of the parent plasma fluctuations (i.e. $\tilde{n}_e/n_{eo} \sim \tilde{n}_i/n_{io}$). However, as reported by Dwivedi and Pandey [21] in the context of shock structure formation in dusty plasmas, the steady state dust charge fluctuations in the above limit does not yield the damping to the normal plasma mode of interest. Rather, it rescales the linear normal mode frequency by a factor of the order of unity. In the other extreme limit ($\tau_\omega \ll \tau_c$) too the average dust charge fluctuation effect could be negligible and this can occur for $p < 1$. In the intermediate cases, especially when $\tau_\omega \approx \tau_c$ non-collisional damping arises due to non-steady state dust charge fluctuation dynamics and it cannot be avoided. Hence, for comparable wave time scale and dust charging time scale, the significant damping of the plasma mode under considerations should occur. Thus theoretical plasma modelling of the dusty plasmas for collective plasma dynamics and its instabilities, requires, in general, the consideration of dust charge fluctuation dynamics. However, under certain conditions as discussed above, a constant dust charge model could be justified [8, 24–26] for selective dust-plasma parameter domain.

In many of the earlier theoretical studies on collective modes in dusty plasmas [22, 27–31] a constant dust charge model was adopted wherein the dust charge variation was ignored. The inclusion of dust charge fluctuation dynamics leads to a novel type of non-collisional damping to the plasma modes [2–4, 21] in their linear limits of fluctuation amplitudes. Several workers [21, 32, 33] have considered the dust charge fluctuation effect to analyse the possibility of nonlinear coherent structures of the low frequency version (DAW) of the so-called acoustic mode in dusty plasmas. Within very weak limit of the dispersion strength, a shock structure [21] was reported to occur due to appropriate balancing of the nonlinearity and collisionless damping induced by the dust charge fluctuation dynamics. Since then a few more research articles have been written especially on the existence conditions and internal structure properties of the nonlinear coherent shock wave structures in dusty plasmas [34–36].

Recently, a review article has been written by Verheest [37] on waves and instabilities in dusty space plasmas to give a comprehensive and informative outlook about how rapidly the field of dusty plasma is growing in various context of its natural existences. The present article revisits the formation of nonlinear coherent structures in dusty plasmas in a very specified domain of the dust-plasma parameters. This domain lies around and above the dust density threshold beyond which the dust dynamics suffers a transition from isolated behaviour to the collective behaviour. It is found that in this limiting zone of dust-plasma parameters, the weak dispersion approximation does not hold good and it dictates to carry out a full numerical analysis of the exact nonlinear energy integral equation to describe the properties of possible nonlinear coherent structures, the low frequency version of the so called acoustic mode. The numerical analysis leads to conclude that shorter scale acoustic solitary waves ($\sim$ ion Debye length) with quite weak potentials are likely to exist near the above mentioned transition zone of charged dust population density. It is emphasized to note that such nonlinear short scale structures could better be termed as the short scale macroscopic solitary dust clouds with small but finite negative charging. Thus these clouds (with $\mathrm{Sech}^2\xi$ profiles) could be treated as the lowest order steady state nonlinear macroscopic structures of dust distribution on associated dust acoustic time scale. These clouds could be visualized as collective Coulomb inertial suspensions of the charged dust particles in an ambient parent plasma environment with appropriate collective responses of the parent plasma species. These solitary dust clouds form localized colloidal like states of the charged dust grains and these may exist either in random distributions or in some ordered distributions under given conditions [38, 39]. Section 2 deals with mathematical derivation of *KdV* equation whereas the § 3 includes the exact numerical analysis of full Poisson's equation. Results and discussions form a part of § 4.

2. Mathematical formulation and basic equations

We consider a plasma consisting of isothermal electrons, isothermal ions as well as cold but heavier charged dust particles. However, to begin with, we will take the general notations of the charge on electrons, ions and dust grains as q_e, q_i and q_d respectively. On the inertial time scale of the dust grains within cold approximation, the electrons and ions of the parent plasmas can be well described by their respective thermal Boltzmann distributions:

$$n_e = n_{eo}\exp(-q_e\phi/T_e), \tag{3}$$

$$n_i = n_{io}\exp(-q_i\phi/T_i), \tag{4}$$

where n_{eo} and n_{io} are the respective equilibrium number densities of the parent electrons and ions and ϕ is the self consistent ambipolar potential. Now the one dimensional propagation of the low frequency version of the so-called acoustic mode in dusty plasmas could be described by the following basic equations:

$$\frac{\partial n(d)}{\partial t} + \frac{\partial}{\partial x}(n(d)v(d)) = 0. \tag{5}$$

$$\frac{\partial v(d)}{\partial t} + v(d)\frac{\partial v(d)}{\partial x} = -\frac{q_i^2}{q_e q_d}\frac{T_e}{T_i}\frac{n_{io}}{n_{do}}\frac{\partial \Phi}{\partial x}. \tag{6}$$

$$\frac{\partial^2 \Phi}{\partial x^2} = -\frac{T_i}{T_e}\left[\frac{q_e^2}{q_i^2}\frac{n_{eo}}{n_{io}}\exp(-\Phi) + \frac{q_e}{q_i}\exp[-\Phi(q_i/q_e)x(T_e/T_i)] + \frac{q_e q_d}{q_i^2}\frac{n_{do}}{n_{io}}n(d)\right]. \tag{7}$$

In the above equations, lengths and times have been expressed in units of ion Debye length $[\lambda_{\mathrm{Di}} = (T_i/4\pi n_{io}q_i^2)^{1/2}]$ and dust oscillation period $[\omega_d^{-1} = (m_d/4\pi n_{do}q_d^2)^{1/2}]$ where m_d and n_{do} are the mass and the equilibrium number density of the dust particles. We have normalized the fluid velocity $v(d)$ by a reference sound speed of the low frequency version of the so-called acoustic mode $C_s = [Z_d^2(n_{do}/n_{io})(T_i/m_d)]^{1/2}$ for $\lambda_{\mathrm{De}} \gg \lambda_{\mathrm{Di}}$, $Z_d = |q_d/q_i|$. The number densities $n(e)$, $n(i)$ and $n(d)$ for electrons, ions and the charged dust grains are normalized by their own respective equilibrium densities. The electrostatic wave field potential Φ has been normalized by electron thermal potential T_e/q_e.

The charge neutrality condition at the equilibrium requires

$$q_e n_{eo} + q_i n_{io} + q_d n_{do} = 0. \tag{8}$$

In order to first characterize the weak amplitude solition associated with the low frequency version of the so-called acoustic mode, we derive the usual *KdV* equation [40] by the standard reductive perturbation method (RPM). In doing so the usual stretched co-ordinates as given below have been employed to carry out the perturbational analysis.

$$\xi = \varepsilon^{1/2}(x - \mathrm{M}t),$$
$$\tau = \varepsilon^{3/2}t, \tag{9}$$

where ε is a small expansion parameter and M (Mach number) is the dimensionless speed of the nonlinear stationary structure normalized by the speed C_s of the normal sound wave as mentioned above.

We define the following power series expansion about the respective unperturbed states to carry out the reductive perturbation analysis:

$$n(d) = 1 + \varepsilon n^{(1)}(d) + \varepsilon^2 n^{(2)}(d) + \dots, \tag{10}$$

$$\Phi = \varepsilon\Phi^{(1)} + \varepsilon^2\Phi^{(2)} + \dots, \tag{11}$$

$$v(d) = \varepsilon v^{(1)}(d) + \varepsilon^2 v^{(2)}(d) + \dots. \tag{12}$$

Following the usual standard procedure of RPM, we finally derive the associated *KdV* equation for density and potential evolutions

$$\frac{\partial n^{(1)}(d)}{\partial\tau} + An^{(1)}(d)\frac{\partial n^{(1)}(d)}{\partial\xi} + B\frac{\partial^3 n^{(1)}(d)}{\partial\xi^3} = 0, \tag{13}$$

$$\frac{\partial\Phi^{(1)}}{\partial\tau} + C\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi} + B\frac{\partial^3\Phi^{(1)}}{\partial\xi^3} = 0. \tag{14}$$

The lowest order (linear) solution reduces to give $\mathrm{M} = \frac{1}{\sqrt{(1+\lambda_{\mathrm{Di}}^2/\lambda_{\mathrm{De}}^2)}} \sim 1$ and hence the constants A, B and C are respectively as

$$A = \frac{1}{2}\left[3 + Z_d\frac{n_{do}}{n_{io}}\left(1 + \frac{q_e}{q_i}\frac{T_i}{T_e}\frac{\lambda_{\mathrm{Di}}^2}{\lambda_{\mathrm{De}}^2}\right)\right],$$
$$B = 1/2, \tag{15}$$
$$C = \frac{1}{2}\frac{T_e}{T_i}\left[\frac{3n_{io}}{Z_d n_{do}} - \left(1 - \frac{n_{eo}}{n_{io}}\frac{T_i^2}{T_e^2}\right)\right].$$

If we consider an extreme mathematical limiting case of $Z_d n_{do}/n_{io} \to 0$ (i.e. $n_{eo} \sim n_{io}$) and of course within the continuum limit of plasma approximation, the self consistent existence of a bounded solution of eq. (14) demands that $k \to \infty$ to avoid the violation of the approximation of weak amplitude soliton as because $C \to \infty$ and the C_s goes to zero in the above dust density limit. However, the limit requirement on $k \to \infty$ for self consistent weak amplitude soliton solution near continuous threshold puts a question mark on the validity of weak dispersion approximation for the *KdV* soliton. Thus the above preliminary arguments motivate us to believe that the solution of the nonlinear energy integral equation is required to characterize a nonlinear coherent structure near the continuum threshold of dust population density. Accordingly, the next section deals with the exact solution of the associated nonlinear energy integral equation for the low frequency version of the so-called acoustic mode in dusty plasmas.

3. Exact analysis

As motivated by the arguments in the previous section, it is necessary to develop a full nonlinear energy integral equation for the self consistent characterization of the possible solitary wave structure in the vicinity of the continuum threshold of the dust population density. Let us consider that all the dependent variables in equations (5)–(7) are dependent on a single variable $\zeta = x - \mathrm{M}t$, where ζ is again normalized by λ_{Di} and M is the Mach number (soliton velocity$/C_s$). Thus in the stationary frame of the stretched co-ordinate system, the dust momentum eq. (6) and the dust continuity eq. (5) could be solved under the approximations of energy and flux conservations to yield the inertial dust density distribution as

$$n(d) = \left(1 - \frac{2Q\Phi}{\mathrm{M}^2}\right)^{-1/2}, \tag{16}$$

where $Q = (q_i^2/q_e q_d)\,(T_e/T_i)\,(n_{io}/n_{do})$.

In deriving the inertial dust density distributions, usual boundary conditions for the localized space charge potential profile (viz. $\Phi \to 0, v(d) \to 0$ and $n(d) \to 1$ at $\zeta \to \pm\infty$) have been implemented. Now substitution of the plasma particle's density distributions in the Poisson's eq. (7) and doing some mathematical manipulations, one can derive the desired energy integral equation as follows:

$$\frac{1}{2}\left(\frac{\mathrm{d}\Phi}{\mathrm{d}\zeta}\right)^2 + \mathrm{V}(\Phi) = 0, \tag{17}$$

where the Sagdeev potential (the so-called pseudo potential) $\mathrm{V}(\Phi)$ is defined as

$$\begin{aligned}\mathrm{V}(\Phi) = {} & \beta(q_{ie})^{-2} n_e[1 - \exp(-\Phi)] + \beta^2 (q_{ie})^{-2}[1 - \exp(-\Phi q_{ie}/\beta)] \\ & + \mathrm{M}^2 (q_{ie} q_{id})^{-2} \beta^2 n_d^2 \left[1 - \left(1 - \frac{2 q_{ie} q_{id} \Phi}{\beta n_d \mathrm{M}^2}\right)^{1/2}\right],\end{aligned} \tag{18}$$

where $\beta = T_i/T_e$, $n_d = n_{do}/n_{io}$, $n_e = n_{eo}/n_{io}$, $q_{ie} = q_i/q_e$, $q_{id} = q_i/q_d$.

Now the existence of bounded solution to eq. (17) requires that $\mathrm{V}(\Phi)$ be negative between two points $\Phi = 0$ and some extreme value $\Phi = \Phi_m$. The other boundary conditions read as

$$\begin{aligned}& \mathrm{V}(\Phi) = \frac{\mathrm{dV}(\Phi)}{\mathrm{d}\Phi} = 0, \quad \frac{\mathrm{d}^2\mathrm{V}(\Phi)}{\mathrm{d}\Phi^2} < 0 \quad \text{at } \Phi = 0, \\ & \frac{\mathrm{dV}(\Phi)}{\mathrm{d}\zeta} = 0 \text{ at } \Phi = 0, \quad \mathrm{V}(\Phi) = 0 \quad \text{at } \Phi = \Phi_m, \\ & \frac{\mathrm{dV}(\Phi_m)}{\mathrm{d}\Phi} > 0 (< 0) \quad \text{if } \Phi_m > 0 (< 0).\end{aligned} \tag{19}$$

In the language of the classical mechanics these conditions ensure that a particle (which is fictitious in nature in this case) starting at $\Phi = 0$ makes a single transit to the point Φ_m and comes back to the initial position, so that $|\Phi_m|$ represents the maximum amplitude of the soliton.

For numerical analysis of eq. (17), we specify the charge of the plasma species as

$$q_e = -e,\ q_i = e \text{ and } q_d = -e|Z_d|.$$

The quasineutrality condition at equilibrium (eq. 8) yields

$$\varepsilon_d = Z_d n_d = (1 - n_e). \tag{20}$$

Thus the pseudopotential V(Φ) reduces to

$$V(\Phi) = \beta(1-\varepsilon_d)[1-\exp(-\Phi)] + \beta^2[1-\exp(\Phi/\beta)] + M^2\varepsilon_d^2\beta^2\left[1-\left(1-\frac{2\Phi}{\beta\varepsilon_d M^2}\right)^{1/2}\right]. \tag{21}$$

From eq. (20) we see that $\varepsilon_d \approx 1$ for $n_{eo} \ll n_{io}$ and $\varepsilon_d \ll 1$ for $n_{eo} \sim n_{io}$. The continuum or the fluid approximation in asociation with point dust charge approximation restricts the dust population density to lie in the limit, $\lambda_{\rm Di}^{-3} \ll n_{do} \ll a^{-3}$. Under the validity limit of the continuum approximation, the likelihood of short scale acoustic solitons near the continuum threshold exists. For numerical appreciation we consider a model plasma with $n_{io} = 10^6$/cc and $T_i = 0.1$ eV, $T_e = 1$ eV. For these plasma parameters, $\lambda_{\rm Di} = 740(T_i\,{\rm eV}/n_{io})^{1/2} = 0.234$ cm. The continuum limit restricts the dust density population to obey the following scaling:

$$n_{do}(4/3)\pi\lambda_{\rm Di}^3 \geq 1,$$

or

$$n_{do} \geq 1/4.12 \times \lambda_{\rm Di}^3 = 18.5/\text{cc}.$$

Assuming the spherical capacitor model, the micron sized grains (for $T_e \sim 1$ eV) could develop the maximum charge $Z_d \sim aT_e/e^2 \sim 700$.

Thus the minimum value of ε_d, beyond which the dust behaves as a fluid is $\varepsilon_d > \varepsilon_{\rm dmin} = Z_d n_{do}/n_{io} \sim 10^{-2}$. For the lowest possible dust size ($\sim$ 10 nm) in the same plasma environment, the minimum threshold of the dust charge density comes out to be $\varepsilon_{\rm dmin} \sim 10^{-4}$.

Using Runga–Kutta method the exact energy integral equation has been solved. We have plotted the pseudopotential profile (V(Φ) vs Φ) as obtained from eq. (21) for $\varepsilon_d = 0.9$ in figure 1a and for $\varepsilon_d = 0.9 \times 10^{-4}$ (the limiting value within the fluid regime) in figure 1b. The Mach number is varied from 1.1 to 1.3 in step of 0.1. It is seen that the Sagdeev potential is never zero for any negative value of Φ (i.e. any positive value of ϕ). The lower limit of Mach number which ensures that V(Φ) is a potential well between Φ and Φ_m is $M = 1/(1+\lambda_{\rm Di}^2/\lambda_{\rm De}^2)^{1/2}$. This is easily seen from the expansion of V(Φ) in Φ [eq. (21)]. Thus the possibility of any rarefactive soliton with positive polarity is ruled out. So, only the compressive solitons with negative polarity are the possible structures in the Boltzmann environment of the parent plasma species. Similar conclusions are reported by others [34] for *KdV* solitons in dusty plasmas. However, our case differs in the parameter domain of dust plasma parameters where the solution of full Poisson's equation is needed to characterize the possible soliton structures.

In figure 2a we have plotted the soliton profile (Φ vs ζ) for $\varepsilon_d = 0.9$ and $\varepsilon_d = 0.9\times 10^{-1}$ for M = 1.01. It is seen that the amplitude Φ_m as well as the width Γ of the soliton

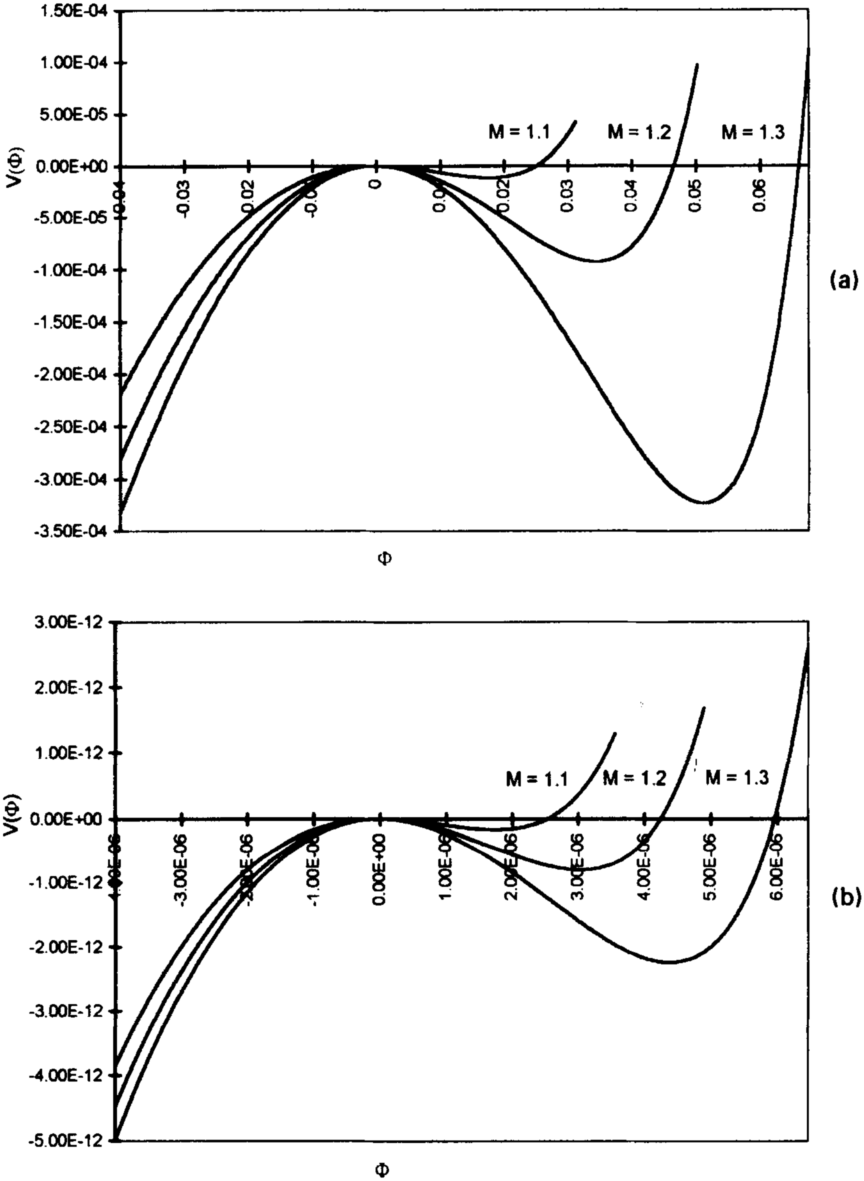
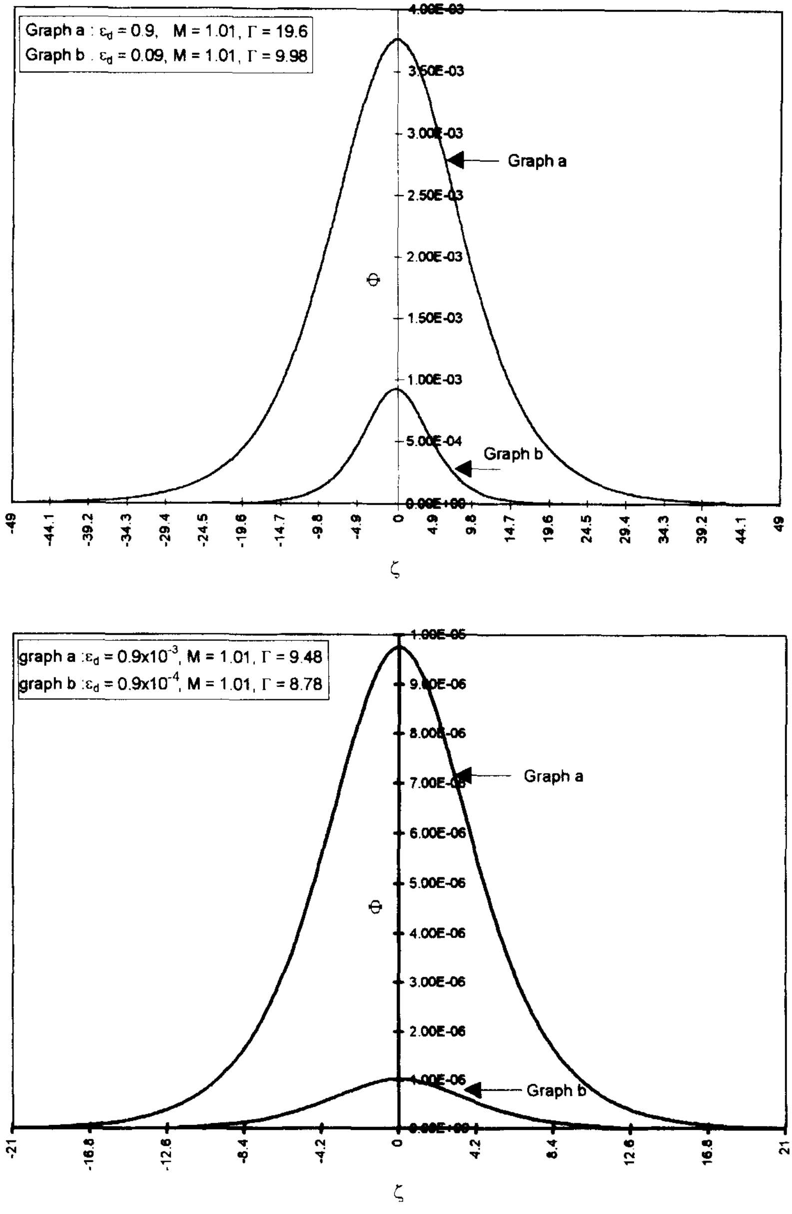


Figure 1(a). Sagdeev potential V(Φ) versus Φ for M = 1.1, 1.2 & 1.3; $\varepsilon_d = 0.9$, $T_e = 1.0\,\text{eV}$, $T_i = 0.1\,\text{eV}$. **(b).** Sagdeev potential V(Φ) versus Φ for M = 1.1, 1.2 and 1.3; $\varepsilon_d = 0.9 \times 10^{-4}$, $T_e = 1.0\,\text{eV}$, $T_i = 0.1\,\text{eV}$.

decreases with ε_d. The same conclusion can be arrived at from the corresponding plot for $\varepsilon_d = 0.9 \times 10^{-3}$ and for $\varepsilon_d = 0.9 \times 10^{-4}$ (figure 2b).

In the figures 3(a) and 3(b) we have plotted the potential profile of the solitons for $\varepsilon_d = 0.9$ and for $\varepsilon_d = 0.9 \times 10^{-4}$ respectively. The Mach number is varied from 1.1 to 1.3 in step of 0.1. It is observed that the amplitude of the solitons (Φ_m) increases with Mach number with a simultaneous decrease in the width of the solitons. To see if the *KdV*



(a)

(b)

Figure 2(a). Soliton profile Φ versus ζ for $\varepsilon_d = 0.9$ and $\varepsilon_d = 0.9 \times 10^{-1}$; M = 1.01, $T_e = 1.0\,\text{eV}$, $T_i = 0.1\,\text{eV}$. (**b**). Soliton profile Φ versus ζ for $\varepsilon_d = 0.9 \times 10^{-3}$ and $\varepsilon_d = 0.9 \times 10^{-4}$; M = 1.01, $T_e = 1.0\,\text{eV}$, $T_i = 0.1\,\text{eV}$.

conservation rule $\Phi_m\Gamma^2$ = constant holds, we have plotted $\Phi_m\Gamma^2$ vs M for $\varepsilon_d = 0.9$ (figure 4a) and for $\varepsilon_d = 0.9 \times 10^{-4}$ (figure 4b) and then compared with the corresponding plot obtained from the *KdV* eq. (14). (However, the solution of the eq. (14) gives the perturbed potential $\Phi^{(1)}$; to obtain $\Phi_{KdV} = \varepsilon\Phi^{(1)}$, we compared the solution $\varepsilon\Phi^{(1)}$, with the solution of eq. (17) obtained by expanding it in Φ and M, assuming small values

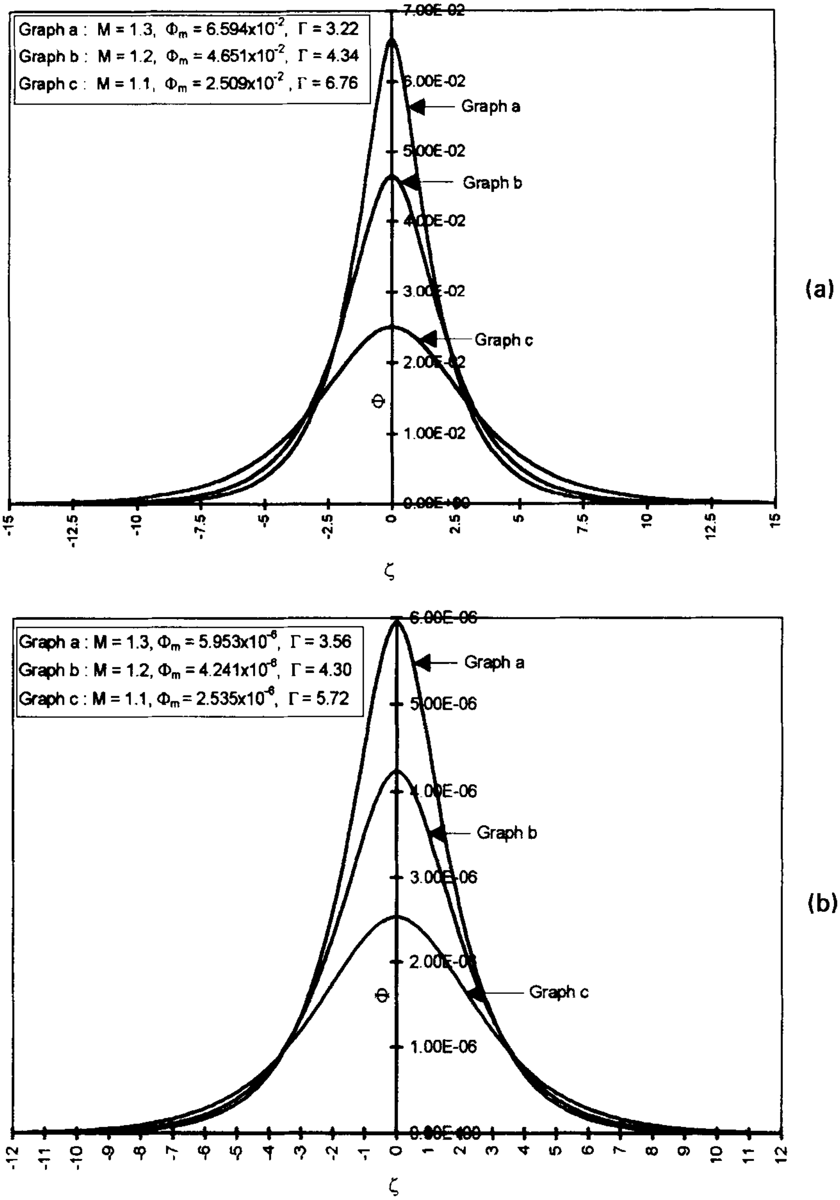


Figure 3(a). Soliton profile Φ versus ζ for $\mathrm{M} = 1.1, 1.2$ and 1.3; $\varepsilon_d = 0.9$; $T_e = 1.0\,\mathrm{eV}$, $T_i = 0.1\,\mathrm{eV}$. (**b**). Soliton profile Φ versus ζ for $\mathrm{M} = 1.1, 1.2$ & 1.3; $\varepsilon_d = 0.9 \times 10^{-4}$; $\mathrm{M} = 1.01$, $T_e = 1.0\,\mathrm{eV}$, $T_i = 0.1\,\mathrm{eV}$.

for both. It is seen that both the solutions match with each other for small Φ and M). From the graphs of figure 4a and 4b we can conclude that (1) $\Phi_m\Gamma^2$ is not constant and it actually decreases with increase in Mach number, (2) $\Phi_m\Gamma^2$ approaches the corresponding *KdV* value for small Mach numbers, as expected, (3). The difference between $\Phi_m\Gamma^2$ and the corresponding *KdV* value is more pronounced when $\varepsilon_d = 0.9\times$

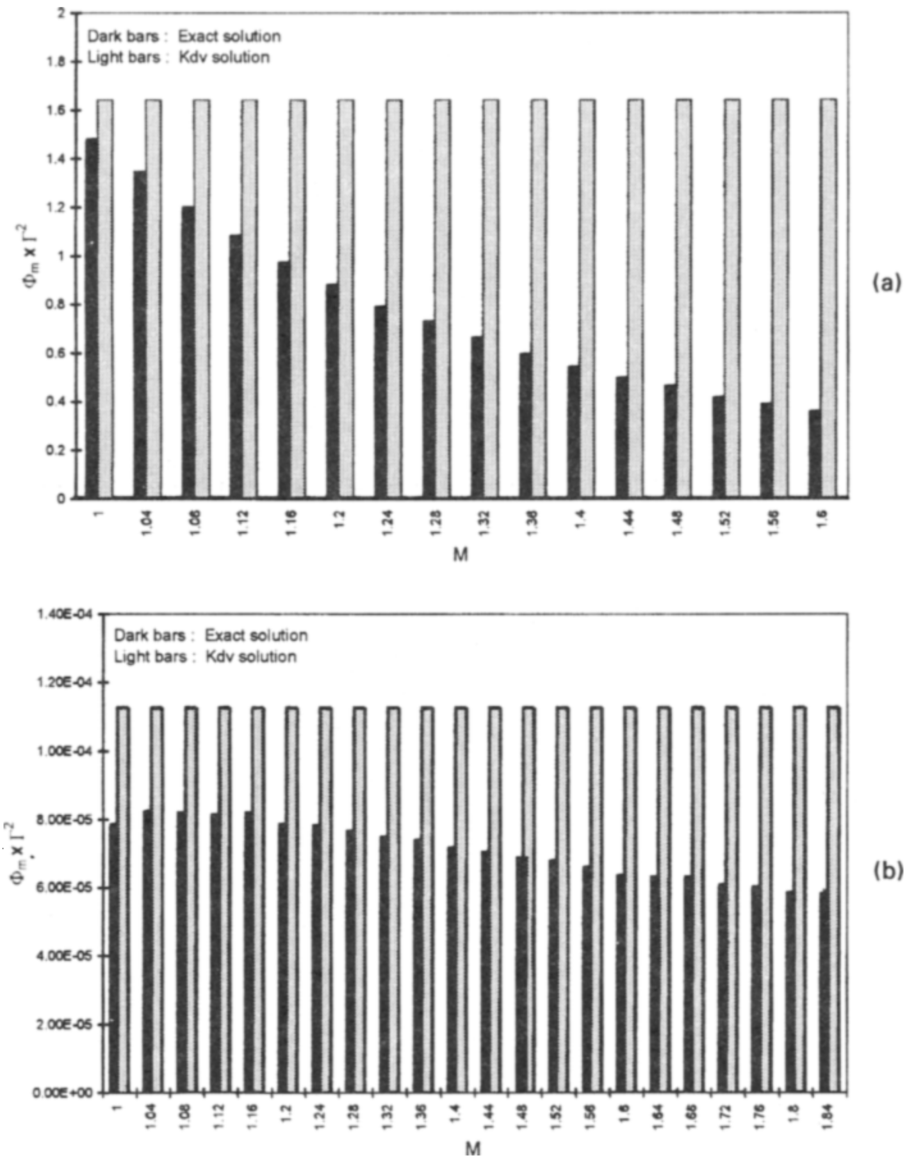
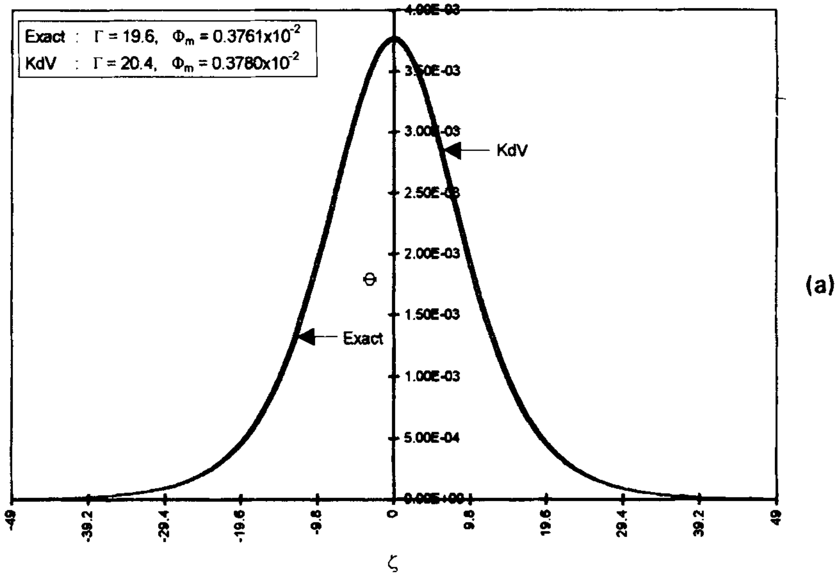


Figure 4(a). $\Phi_m \times \Gamma^2$ versus M for $\varepsilon_d = 0.9$; $T_e = 1.0\,\text{eV}$, $T_i = 0.1\,\text{eV}$. (**b**). $\Phi_m \times \Gamma^2$ versus M for $\varepsilon_d = 0.9 \times 10^{-4}$; $T_e = 1.0\,\text{eV}$, $T_i = 0.1\,\text{eV}$.

10^{-4} than when $\varepsilon_d = 0.9$. Thus the solitons seem to deviate from *KdV* nature when ε_d approaches the continuum threshold. In figure 5a the soliton profiles for $\varepsilon_d = 0.9$ and $\text{M} = 1.01$ is displayed. The lower curve represents the result of exact calculation while the upper curve represents the *KdV* soliton. It is seen that the profiles nearly match with each other which is as usual. The figure 5b represents profiles for the same ε_d but for $\text{M} = 1.60$. The two profile do not match as expected. The reason is obvious because the higher order



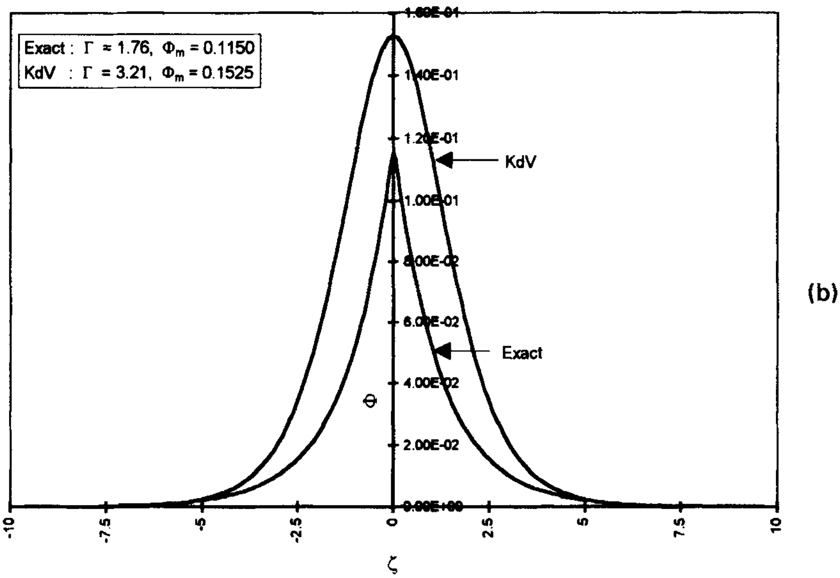


Figure 5(a). Soliton potential Φ versus ζ from exact solution and from *KdV* solution; $\varepsilon_d = 0.9, \mathrm{M} = 1.01$, $T_e = 1.0\,\mathrm{eV}$, $T_i = 0.1\,\mathrm{eV}$. (**b**). Soliton potential Φ versus ζ from exact solution and from *KdV* solution; $\varepsilon_d = 0.9$, $\mathrm{M} = 1.60$, $T_e = 1.0\,\mathrm{eV}$, $T_i = 0.1\,\mathrm{eV}$.

nonlinear effects produce spiky character. Figure 6a and figure 6b are the corresponding plots for $\varepsilon_d = 0.9 \times 10^{-4}$. In this case it is seen that the exact soliton amplitudes do not match with that of the *KdV* soliton even for small M. Thus, it reinforces the argument that the solitons depart from *KdV* nature when ε_d approaches the continuum threshold. Furthermore, for higher M values again the spiky behaviour is pronounced. Thus the

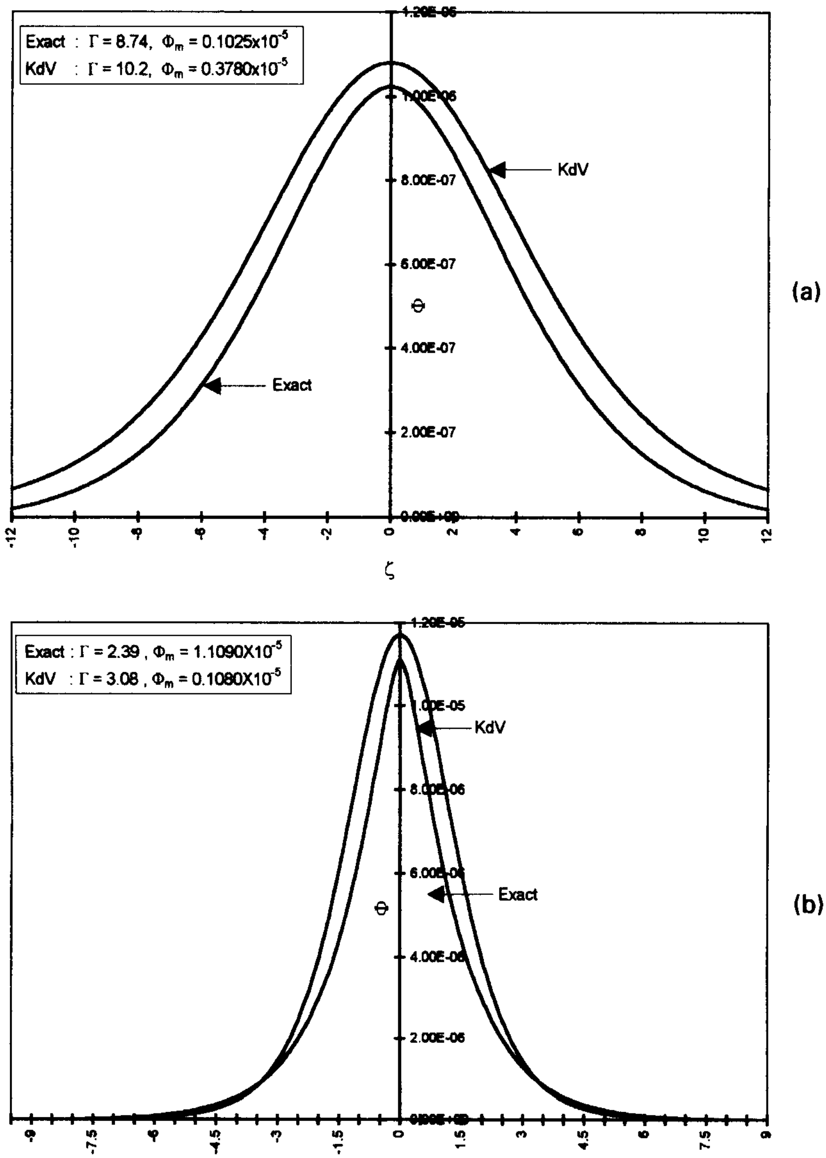


Figure 6(a). Soliton potential Φ versus ζ from exact solution and from *KdV* solution; $\varepsilon_d = 0.9 \times 10^{-4}$, M $= 1.01$, $T_e = 1.0\,\text{eV}$, $T_i = 0.1\,\text{eV}$. (**b**). Soliton potential Φ versus ζ from exact solution and from *KdV* solution; $\varepsilon_d = 0.9 \times 10^{-4}$, M $= 1.60$, $T_e = 1.0\,\text{eV}$, $T_i = 0.1\,\text{eV}$.

compositional dependence of the soliton properties could be attributed to the non-*KdV* type character of the possible acoustic solitons near the continuum threshold. In brief one can say the following; near the continuum threshold (which we have explored) very weak amplitude solitons with higher dispersion effect (shorter scale) are likely to exist; higher order nonlinear effects are pronounced, as usual, only for higher M values. Furthermore, near the continuum threshold *KdV* equation cannot describe the accurate quantitative properties of the solitary structures.

4. Results and discussion

The present theoretical investigation deals with the possible nonlinear acoustic coherent structures in plasmas containing dust-like heavier charged impurity ions. It is found that within plasma fluid approximation, the finite size (typically of the order of parent plasma Debye lengths) solitary dust clouds (with short scale potential and density distributions) are likely to exist near the continuum threshold of the dust population density. It is noted that such short scale nonlinear structures could not be accurately described by usual *KdV* equation because it loses its validity as weak dispersion effect near the transition zone of the dust population density. The transition zone stands for the dust population density paramter domain where the isolated dust grain approximation i.e. discrete particle behaviour breaks down and the continuum behaviour begins to reveal. In this limiting case, the self consistent validity of weak amplitude *KdV* approximation i.e. $M \sim 1$, dictates to infer that short scale ($\sim$ parent ion Debye length) nonlinear space charge distribution through coherent structure formations are the likely candidates. However, the higher dispersion effect in turn may become non-negligible and hence the numerical solution of the full energy integral equation has been carried out to describe the properties of these solitary wave structures.

Our calculation differs from the earlier works in dust density parameter domain of interest and concludes that both the *KdV* type and spiky short scale structures of weakly charged dust clouds could be formed near the continuum threshold of the dust fluid approximation. Numerical calculations lead to suggest that such weakly charged dust clouds near the defined transition zone could be observed in low density plasmas with $n_{io} \sim 10^6\,\text{cm}^{-3}, T_e \approx T_i \approx 1$ eV for micron to 10 nm sized dust contaminations. Wide range variability in dust population density for such observations could be adjusted by suitable choice of the parent plasmas for given dust parameters. The choice of these parameters is arbitrary that gives a scope for wide range variation of the dust population density within fluid approximation. It is suggested that experimental observations should be carried out to verify our theoretical findings. In the last it is emphasized that in the Maxwellian environment of parent electrons and ions only compressive solitons with negative polarity (for negative dust grains) and with positive polarity (for positive dust grains) are the only solutions. However, it is not so for the modified ion acoustic wave on parent ion acoustic time scale as recently reported by Dwivedi *et al* [41] in the context of their proposal for the wave turbulence model (WTM) as a possible physical mechanism for Coulomb phase transitions in dusty plasmas. It should be clarified that the term "solitary dust cloud or clouds" included in the text has been purposely used to emphasize for such nomenclature because these nonlinear structures resemble the colloidal suspension of the inertial dust grains in the electrostatic force field distributed within finite space.

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