Abstract. In the $O(36)$ limit of the interacting boson model including spin-isospin degrees of freedom (IBM-4), starting with a group chain that preserves $s$ and $d$ boson spins and isospins together with a simple mixing Hamiltonian, it is shown that the model generates, for heavy $N = Z$ nuclei, even-even to odd-odd staggering in the number of $T = 0$ pairs in the ground states for moderate difference in the basic $T = 0$ and $T = 1$ s-boson pair energies; the staggering disappears when the energy difference is large.

Keywords. Interacting boson model; IBM-4; isospin; $T = 0$ pairing; $T = 1$ pairing; dynamical symmetries; drip line nuclei.

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1. Introduction

One of the declared goals of the radioactive ion beam facilities that are going to become available in the near future, is to study proton–neutron ($pn$) pairing in nuclei near the proton drip-line in the mass range $A \sim 60–100$ [1]. An important question here is $T = 0$ versus $T = 1$ pairing in the ground states of heavy $N \sim Z$ odd-odd nuclei, with the $T = 0$ pairing arising only from $pn$ pairs, and the change in the pairing strength from the neighbouring even-even nuclei. So far, the simple isovector $O(5)$ pairing model with protons and neutrons in a single-$j$ shell [2], Monte Carlo shell model method [3], a cranked mean-field model with $T = 0$ and $T = 1$ pairing interactions [4] and the $U(6) \otimes U(6)$ limits of IBM-4 [5] are used to study $T = 0$ versus $T = 1$ pairing in heavy $N = Z$ nuclei. The purpose of this brief report is to present results of further study of this problem using IBM-4 symmetry limits. The spectrum generating algebra (SGA) for IBM-4, with six spin-isospin degrees of freedom for the $s$ and $d$ bosons is $U_{sdST}(36)$ [6]; note that $(ST) = (10) \oplus (01)$. Recently [7] all the symmetry limits of IBM-4 are classified and at the primary level of the $U(36)$ group-subgroup lattice of the model, there are four symmetry limits: (i) $U(6) \otimes U(6)$; (ii) $U(18) \oplus U(18)$; (iii) $U(6) \oplus U(30)$; (iv) $O(36)$. The structure of the $T = 1$ and $T = 0$ bands in $^{74}$Rb (the nucleus $^{74}$Rb is the heaviest known $N = Z$ odd-odd nucleus in $A > 60$ region that has been studied experimentally with any spectroscopic detail [8]) is described successfully for the first time using IBM in [7] and here a group chain starting from $O(36)$ is used (see (1) ahead). The same group chain is employed in this report for investigating the problem of $T = 0$ vs $T = 1$ pairing in heavy $N = Z$ nuclei.
2. \( O(36) \supset O(6) \oplus O(30) \) group chain and number of \( T = 0 \) and \( T = 1 \) pairs

Let us begin with the \( O_{sdST}(36) \) group chain considered in [7],

\[
\begin{align*}
\{N\} & \quad \{\omega\} \quad \{\omega_1, \omega_2\} \quad S, \quad T, \\
O_{sdST}(36) & \supset O_{sdST}(36) \supset \{O_{sT}(6) \supset O_{s}(3) \oplus O_{T}(3)\} \oplus \\
O_{sdST}(30) & \supset \{O_{d}(5) \supset O_{L}(3)\} \oplus \{O_{sdT}(6) \supset O_{s}(3) \oplus O_{T}(3)\} \\
& \supset O_{L}(3) \oplus \{O_{s}(3) \oplus O_{T}(3)\} \supset O_{(3)} \oplus O_{T}(3) \supset \left\{\begin{array}{ccc}
L & S & T \\
\bar{J} = L + S & T
\end{array}\right\}.
\end{align*}
\]

(1)

The generators of all the groups in (1) are given in [7]. As all the \( N \) boson states (note that the boson number \( N \) is even for \( N = Z \) even-even nuclei and \( N \) is odd for \( N = Z \) odd-odd nuclei; the ‘\( N \)’ used for denoting boson number should not be confused with the ‘\( N \)’ used for neutron number) correspond to the totally symmetric irrep \( \{N\} \) of \( U_{sdST}(36) \), the \( O_{sdST}(36) \) irreps are labelled by the seniority quantum number \( \omega \) in the total \( sdST \) space \((\omega = N, N - 2, N - 4, \ldots 0 \text{ or } 1)\). The \( O_{sdST}(6) \) and \( O_{sdST}(30) \) quantum numbers \((\omega_s, \omega_d)\) are given by the rule [9] \( \omega = 2r_s + \omega_s + \omega_d \) where \( r_s = 0, 1, 2, \ldots \). All other quantum numbers in (1) will be specified as and when they are needed. Following the applications of good \((T, T_d)\) symmetry limits of IBM-3 [10], it is assumed that for both even-even and odd-odd nuclei \( \omega = N \) and \( \omega_d = 0 \) for the ground state (GS) and the later assumption guarantees that \( L = 0 \) for the GS. In this situation except for the \( O_{s}(3) \) irreps \( S_s \) and \( T_s \), irrep labels for the remaining groups in (1) need not be specified. Then in the symmetry limit (1),

\[
|\text{GS}\rangle = |N; \omega = N, (\omega_s, \omega_d = 0), S = S_s, T = T_s, L = 0\rangle.
\]

(2)

In order to calculate number of \( T = 0 \) pairs and number of \( T = 1 \) pairs in the GS, the state (2) is transformed into a basis with good \( s \) and \( d \) boson numbers. To this end results of [9] are used where a compact formula for transformation brackets \((C's \text{ in (3) ahead})\) between \( U(N) \supset O(N) \supset O(N_a) \oplus O(N_b) \) and \( U(N) \supset U(N_a) \oplus U(N_b) \supset O(N_a) \oplus O(N_b) \), with \( N = N_a + N_b \), for symmetric \( U(N) \) irreps \( \{n\} \) and for any \( N_a \) and \( N_b \) is derived. Denoting the basis states for these two chains by \( |n_\omega(\omega_a \omega_b) \alpha\rangle \) and \( |n(n_a n_b)(\omega_a \omega_b) \alpha\rangle \) respectively,

\[
|n_\omega(\omega_a \omega_b) \alpha\rangle = \sum_{n_a} C_{n_a,n_b}^{n_\omega(\omega_a \omega_b)} (N_a, N_b) |n(n_a n_b)(\omega_a \omega_b) \alpha\rangle; \ n = n_a + n_b.
\]

(3)

The results in [9] give for example the formulas \( |n_\omega(\omega_a \omega_b) \alpha\rangle = \omega_a + (n - \omega_a + \omega_b + N_b - 2) \frac{n - \omega_a - \omega_b}{2n + N - 4} \) and \( |n(n_a n_b)\omega(\omega_a \omega_b) \alpha\rangle = \omega_a + (n - \omega)(2\omega_a + N_a)/(2\omega + N). \) Using (3), the states (2) are transformed into states with number of \( T = 0 \) and \( T = 1 \) pairs being good quantum numbers,

\[
\begin{align*}
|N; \omega = N, (\omega_s, \omega_d = 0), S = S_s, T = T_s, L = 0\rangle & = \sum_{n_a} C_{n_a,n_d}^{N;\omega=N,\omega_d=0}(6, 30)C_{n_a,S,T}^{2,2S,T}(3, 3)C_{n_d,T}^{0,0,0,0,0} \times |N; (n_d,S_{n_d,T})(S_d=0, T_d = 0); n_s + n_d = N, n_s; S = n_s, n_d; T = n_d. \\end{align*}
\]

(4)
$T = 0$ versus $T = 1$ pairing in $O(36)$ limit of IBM-4

Figure 1. (a) Single $s$ and $d$ boson energies and (b) energies of lowest $T = 0$ ($S = 1$) and $T = 1$ ($S = 0$) states for five boson ($N = 5$) system as a function of $\beta/\alpha$; the hamiltonian is defined by (5).

where $\omega_{ds}$ and $\omega_{dt}$ are the quantum numbers of the groups $O_{ds}(15)$ and $O_{dt}(15)$ respectively and they take trivial values $\omega_{ds} = 0$, $\omega_{dt} = 0$ as $\omega_{d} = 0$ for the states (2). Note that in deriving (4) we used $O_{dST}(30) \supset O_{ds}(15) \oplus O_{dt}(15)$ (but not $O_{dST}(30) \supset O_{d}(5) \oplus O_{sTd}(6)$ as chosen in (1)) as this is more convenient and because the final results do not depend on this choice for $\omega_{d} = 0$ as in (2). For the basis states on the r.h.s of (4), the number of $T = 0$ pairs is $N_{T=0} = n_{s} = n_{s,s} + n_{d,s}$ and similarly number of $T = 1$ pairs is $N_{T=1} = n_{T} = n_{s,T} + n_{d,T}$; $N = N_{T=0} + N_{T=1}$ and the fraction of $T = 0$ pairs is $f(T = 0) = N_{T=0}/N$. Using (4), it is straightforward to calculate $f(T = 0)$ in the states defined by (2) or mixtures of them. For the GS of odd-odd $N = Z$ nuclei, the boson number $N$ is odd and $\omega_{s} = 1$ in (2) giving $(ST) = (10)$ or $(01)$. In the symmetry limit ignoring the $S(S + 1)$ and $T(T + 1)$ contributions to the energies, the $T = 0$ and $T = 1$ GS energies are degenerate and using the formulas given below (3), it is seen easily that $f(T = 0) = (9N^{2} + 162N + 101)/(16N + 16)$ for $T = 0$ GS and $f(T = 0) = (7N^{2} + 94N - 101)/(16N + 16)$ for $T = 1$ GS. For example for $N = 5$, $f(T = 0) = 0.676$ for $T = 0$ GS and $f(T = 0) = 0.324$ for $T = 1$ GS. For the GS of even-even $N = Z$ nuclei $f(T = 0) = 0.5$ as the boson number $N$ is even, $\omega_{s} = 0$, $S = S_{s} = 0$ and $T = T_{s} = 0$. Therefore, in the $O_{dST}(36)$ symmetry limit (1,2) there is even-even to odd-odd staggering in $f(T = 0)$.

3. Results of mixing calculations

In reality the $T = 0$ and $T = 1$ states for odd-odd nuclei will not be degenerate and a simple hamiltonian (appropriate for the basis defined by (2)) that generates a
Figure 2. Fractional number of $T = 0$ pairs $f(T = 0)$ as a function of the boson number $N$ for various values of $\beta/\alpha$: (a) for $T = 0$ GS (i.e, lowest $S = 1$, $T = 0$ state) of $N = Z$ odd-odd nuclei ($N$ odd) and $T = 0$ GS (i.e, lowest $S = 0$, $T = 0$ state) of $N = Z$ even-even nuclei ($N$ even); (b) for $T = 1$ GS (i.e, lowest $S = 0$, $T = 1$ state) of $N = Z$ odd-odd nuclei ($N$ odd) and $T = 0$ GS (i.e, lowest $S = 0$, $T = 0$ state) of $N = Z$ even-even nuclei ($N$ even). See text for further details.

splitting is,

$$H = \alpha C_2(O_{SST}(6)) + \beta C_2(SU_{SS}(3)) + \gamma \left[ \frac{1}{31} C_2(SU_{dST}(30)) \right]. \tag{5}$$

In (5), $C_2$'s are quadratic Casimir operators and $SU(N)$ instead of $U(N)$ and the factor $1/31$ are used for convenience. The term with $\alpha$ is diagonal in the basis (2) with eigenvalues given by $\alpha \omega_s (\omega_s + 4)$. The other two terms are diagonal in the basis defined by the states on the r.h.s of (4) with eigenvalues $\beta n_{SS}(n_{SS} + 3)$ and $(\gamma/31)n_d(n_d + 30)$. In the following discussion it is assumed that $\alpha > 0$. Single boson energies ($\epsilon's$) defined by (5) are, $\epsilon(T_s = 0)/\alpha = 5 + 4(\beta/\alpha)$, $\epsilon(T_s = 1)/\alpha = 5$ and $\epsilon(T_d = 0)/\alpha = \epsilon(T_d = 1)/\alpha = 5 + \gamma/\alpha$; see figure 1a. Thus for one boson system, assuming $\gamma/\alpha > 0$ and $\alpha > 0$, $T_s = 1$ is GS for $\beta/\alpha > 0$ and $T_s = 0$ is GS for $\beta/\alpha < 0$.

For $N > 1$, the hamiltonian (5) mixes the basis states (2) (i.e, $\omega_s$ is mixed). For a given $N$, the matrix for $H$ is constructed for various values of $\beta/\alpha$ using (3) and after diagonalizing $f(T = 0)$ is calculated for: (i) lowest $S = 1$, $T = 0$ state (i.e, $T = 0$ GS) for $N$ odd (odd-odd nuclei); (ii) lowest $S = 0$, $T = 1$ state (i.e, $T = 1$ GS) for $N$ odd (odd-odd nuclei); (iii) lowest $S = 0$, $T = 0$ state (i.e, $T = 0$ GS) for $N$ even (even-even nuclei). Numerical calculations showed that the energies $\epsilon(T_d)$ of $d$-boson states do not significantly alter the behaviour of $f(T = 0)$ and therefore in all the calculations $\gamma/\alpha = 2$ is
chosen; this part gives a contribution of about 0.08 – 0.1 for \( f(T = 0) \). Varying \( \beta/\alpha \) (which is a measure of the competition between \( T = 0 \) and \( T = 1 \) pairs), it is seen that the GS energy of \( N \) boson system is in direct correlation with the single \( s \)-boson energies \( \epsilon(T_s) \); figure 1b shows this for \( N = 5 \) case. The results for \( f(T = 0) \) as a function of the boson number for various values of \( \beta/\alpha \) are shown in figure 2; results for the cases (i) and (iii) are shown in figure 1a and for the cases (ii) and (iii) in figure 1b. The most significant result that follows from figure 2 is that there is even-even to odd-odd staggering in \( f(T = 0) \) in \( N = Z \) nuclei for moderate difference in \( T = 0 \) and \( T = 1 \) \( s \)-boson pair energies (i.e. for \( |\beta/\alpha| \leq 0.5 \)) and the staggering disappears when this energy difference is large. Comparing figure 1 with figure 2 it is seen that, in the situation that \( T = 0 \) and \( T = 1 \) \( s \)-boson pairs compete (i.e. their energies are close) there is staggering and absence of staggering implies dominance of one of them. This result of IBM-4 is consistent with the results obtained from the shell model [2, 3]. More importantly, using the \( U(6) \otimes U(6) \) chains of IBM-4 and a hamiltonian similar to (5) without the \( \gamma \)-term but in total six dimensional \( ST \) space, \( f(T = 0) \) is studied in [5] and this scheme also produces the staggering effect. Thus the feature of even-even to odd-odd staggering in \( f(T = 0) \) in the GS of heavy \( N = Z \) nuclei is a robust prediction of IBM-4. It should be remarked that the physical sub-spaces chosen in the present work and in ref. [5] are quite different but in both cases the \( C \)-coefficients of (3) enter.

4. Conclusion

Results reported in this paper for even-even to odd-odd staggering in the number of \( T = 0 \) pairs in the ground states of heavy \( N = Z \) nuclei together with the description of the observed ground \( T = 1 \) and excited \( T = 0 \) bands in \( ^{74}\text{Rb} \) [7] by the \( O(36) \) group chain (1) establish that the \( O(36) \) dynamical symmetry limits of IBM-4 are relevant for near proton drip line nuclei.

References

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