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Motion of test particles around global monopoles in Brans–Dicke theory

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Abstract. A detailed analysis of the motion of test particles around global monopole in Brans–Dicke theory of gravity has been prescribed using the Hamilton–Jacobi (H–J) formalism. The trajectory of the test particles are trapped by the monopole with some restriction on the coupling parameter ω.

Keywords. Global monopole; Brans–Dicke theory; Hamilton–Jacobi formalism.

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1. Introduction

Monopoles are point-like topological objects that may arise during phase transitions in the early universe [1, 2]. In particular, if $\pi_2(\mu) \neq I(\mu$ is the vacuum manifold) i.e. μ contains surfaces which cannot be continuously shrunk to a point [1] then monopoles are formed. The gravitational field of a monopole exhibits some interesting properties, particularly in relation to the appearance of non-trivial space-time topologies. In general relativity, using weak field approximation there are solutions of Barriola–Vilenkin [3] corresponding to the metries generated by global monopoles. Recently, Barros and Romero [4] have described solutions for global monopole in Brans–Dicke theory using the same approximation for weak-field.

In this letter we study the motion of test particles in the gravitational field of global monopoles with Brans–Dicke theory using H–J formalism and examine whether Brans–Dicke coupling parameter can be restricted to obtain bound orbits.

2. Study of geodesic using H–J formalism

The line element for static spherical symmetric space-time is

$$\mathrm{d}S^2 = -B(r)\mathrm{d}t^2 + A(r)\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\theta^2). \tag{1}$$

Suppose a relativistic particle of mass m is moving in the gravitational field described by (1). The H–J equation is

$$-\frac{1}{B(r)}\left(\frac{\partial S}{\partial t}\right)^2 + \frac{1}{A(r)}\left(\frac{\partial S}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial S}{\partial \theta}\right)^2 + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial S}{\partial \phi}\right)^2 + m^2 = 0. \tag{2}$$

As there is no explicit dependence of t and ϕ, so S can be taken in the form [5]

$$S(r,\theta,\phi,t) = -E\cdot t + S_1(r) + S_2(\theta) + J\phi, \tag{3}$$

where E and J are constants and are identified as the energy and angular momentum of the particle. Now, substituting this ansatz for S in the H–J equation, the unknown functions S_1 and S_2 have the following integral form

$$S_1(r) = \epsilon \int [E^2/B(r) - m^2 - p^2/r^2]^{1/2} A^{1/2} \mathrm{d}r, \tag{4}$$

and

$$S_2(\theta) = \epsilon \int [p^2 - J^2 \mathrm{cosec}^2\theta]^{1/2} \mathrm{d}\theta. \tag{5}$$

Here p is the separation constant and $\epsilon = \pm 1$, the sign changing whenever r [or θ] passes through a zero of the integrand in (4) [or (5)].

According to H–J formalism the trajectory of the test particle is characterised by [6]

$$\frac{\partial S}{\partial E} = \text{constant}, \ \frac{\partial S}{\partial J} = \text{constant and } \frac{\partial S}{\partial p} = \text{constant}.$$

Hence we obtained,

$$t = \epsilon \int \frac{\sqrt{A}E}{B\sqrt{E^2/B - m^2 - p^2/r^2}} \mathrm{d}r \tag{6}$$

$$\phi = \epsilon \int \frac{J\cdot \mathrm{cosec}^2\theta}{\sqrt{p^2 - J^2 \cos c^2 v}} \mathrm{d}\theta \tag{7}$$

and

$$\cos^{-1}\left(\frac{\cos\theta}{\sqrt{1 - J^2/p^2}}\right) = \epsilon \int \frac{\sqrt{A}p\,\mathrm{d}r}{r^2\sqrt{E^2/B - m^2 - p^2/r^2}} \tag{8}$$

with the constants to be zero without any loss of generality.

Thus from (6) the radial velocity of the particle is

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{B\sqrt{E^2/B - m^2 - p^2/r^2}}{E\sqrt{A}} \tag{9}$$

The turning points of the trajectory are given by $\mathrm{d}r/\mathrm{d}t = 0$ and consequently the potential curves are

$$\frac{E}{m} = B^{1/2}\left(1 + \frac{p^2/m^2}{r^2}\right)^{1/2}. \tag{10}$$

Now if we take the metric coefficient $B(r)$ to be that of Barros and Romero for Brans–Dicke monopole after neglecting the mass term then we have [4]

$$B(r) = 1 - \frac{8\pi\eta^2}{\phi_0} + \frac{16\pi\eta^2}{\phi_0(2\omega+3)} \mathrm{In}\frac{r}{r_0}.$$

Thus the extremals of the potential curve are the solution of the equation

$$\frac{m^2r^2}{p^2} = \ln(r^2/r_0^2) - (2\omega + 4) + \frac{(2\omega+3)\phi_0}{8\pi\eta^2}. \tag{11}$$

It has two real solutions if

$$\frac{8\pi\eta^2}{\phi_0} < \frac{2\omega+3}{2\omega+4} \tag{12}$$

and hence the trajectory of the particle can be trapped by the monopole due to the restriction on the coupling parameter ω.

On the other hand, we now take the space-time metric to be [4]

$$dS^2 = \alpha(r)[-dt^2 + dr^2 + \beta r^2 d\Omega_2^2], \tag{13}$$

which describes, in the weak field approximation the space-time generated by a global monopole in Brans–Dicke theory and is analogous to the solution of Barriola–Vilenkin for $\omega \to \infty$. In this case the radial component of the velocity has the form

$$\frac{dr}{dt} = \frac{1}{E}\sqrt{E^2 - \alpha(m^2 + p^2/\alpha\beta r^2)} \tag{14}$$

with $\alpha(r) = (1 + 16\pi G\eta^2/2\omega + 4)\ \ln r/r_0$, $\beta = 1 - 8\pi\eta^2 G(2\omega + 3/2\omega + 4)$

So the potential curve is characterised by

$$\frac{E}{m} = \sqrt{\alpha + \frac{p^2}{\beta r^2 m^2}}. \tag{15}$$

The extremals are given by

$$\gamma = \pm\frac{p}{m}\sqrt{\frac{(2\omega+4)}{8\pi G\eta^2\beta}}. \tag{16}$$

Thus we have two real extremals and there are bound orbits for test particles.

3. Discussion

We have seen in both the cases that particles can be trapped by the gravitational field of global monopole. In the first case there is restriction on ω for bound orbit while in the second case there are no restrictions. However, in the second case, as $\omega \to \infty$, we see from eq. (16) that $r \to \pm\infty$, and hence the particles cannot be trapped by monopoles. Also the metric (13) reduces to the metric of Barriola–Vilenkin (B–V) when $\omega \to \infty$ and we have seen previsouly [5] that there are no bound states for B–V monopole. Therefore, we can conclude that Brans–Dicke field has effect on the trajectory of the test particle around monopole.

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