PRAMANA
—journal of
physics

Vol. 51, Nos 3 & 4
September/October 1998
pp. 481–488

Amplification of extra-ordinary mode wave in the presence of kinetic Alfvén wave turbulence

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MS received 21 March 1997; revised 9 June 1998

Abstract. The amplification mechanism of extra-ordinary mode radiation in the presence of kinetic Alfvén wave turbulence driven by an electron beam is studied. It is shown that plasma maser process may be responsible for the amplification of the extra-ordinary mode through up-conversion of turbulent energy via nonlinear wave particle interaction. The relevance of this investigation to space plasmas is discussed.

Keywords. Weak turbulence theory; nonlinear wave particle interaction; resonant and non-resonant waves; growth rate.

PACS No. 52.35

1. Introduction

The consideration of anomalous electromagnetic radiation from a plasma with enhanced turbulent fluctuation can be regarded as a central problem in attempting to interpret various astrophysical and laboratory plasma phenomena. The plasma maser interaction [1] is one of the possible mechanisms for up-conversion of energy from low frequency fluctuation to high frequency electromagnetic radiation. The plasma maser is recognized as one of the lowest order mode-coupling processes in weak turbulence theory alongwith three-wave resonance and nonlinear scattering resonance.

The plasma maser interaction takes place when resonant as well as non-resonant plasma waves are present in a system [2]. The resonant waves are those for which the linear Landau resonance condition is satisfied ($\omega - \mathbf{k} \cdot \mathbf{v} = 0$), while the non-resonant waves are those for which neither the linear Landau resonance condition nor the nonlinear scattering resonance condition is satisfied (i.e. $\Omega - \mathbf{K} \cdot \mathbf{v} \neq 0$ and $\Omega - \omega - (\mathbf{K} - \mathbf{k}) \cdot \mathbf{v} \neq 0$). Here ω, Ω are the frequencies of the resonant and non-resonant waves with wave vectors $\mathbf{k}$ and $\mathbf{K}$, respectively.

Recent investigation [3] shows that energy and momentum conservation relations are exactly satisfied by the plasma maser interaction, while the standard Manley–Rowe relation is violated; as a result an efficient up-conversion of energy takes place from the low frequency resonant mode to the high frequency non-resonant mode. Instead of the standard Manley–Rowe relation, the plasma maser satisfies a generalized Manley–Rowe relation [3] which takes into account the effective action due to particles in addition to the wave actions. The plasma maser works at least for two cases. The first one is the open system for which a free energy source from outside the system is available [4,5]. The second case is the magnetized plasmas with a symmetry breaking factor [2,6].

Recent numerical simulations have verified almost all the theoretical predictions of plasma maser. Use of a two-dimensional electromagnetic and relativistic particle code shows that electromagnetic waves are effectively generated by plasma maser effect from whistler mode turbulence driven by temperature anisotropy [7], and from Langmuir turbulence driven by electron beam instability [8]. There is complete agreement between theoretical predictions and numerical findings so far as the growth rate, polarization and direction of propagation of the waves are concerned.

In the present paper we study the amplification of extra-ordinary mode radiation in presence of kinetic Alfvén wave (KAW) turbulence through plasma maser interaction to obtain a growth rate for Saturnian kilometric radiation (SKR) in the R-X mode. Detailed analysis of Voyager 1 data near day-side equatorial plasma sheet by Sittler *et al* [9], have revealed that beyond the innermost boundary of the outermost part of the plasma sheet ($R \sim 16R_S$) there exists a number of detached plasma regions. Goertz [10] showed that MHD waves are produced in the Saturn magnetosphere as a result of centrifugal flute instability and such type of MHD waves can accelerate field aligned electrons to energies of several keVs. On the other hand, Hasegawa [11] has shown that the mechanism converting the field aligned MHD wave free energy to field aligned electron energy is nothing but the wave-mode conversion mechanism. The conversion of MHD waves to kinetic Alfvén wave allows in turn a coupling of energy from the macro or MHD scale to the micro or kinetic scale. The field aligned electron fluxes can then generate kilometric radiation in RH extra-ordinary mode. It is also stated that these field aligned fluxes may be produced as a consequence of the kinetic Alfvén wave's electric field in the direction of the average local magnetic field. MHD waves have a large scale size because the parallel wavelength is comparable to the system size.

In § 2 the nonlinear dispersion relation for extra-ordinary mode is derived. The growth rate of the extra-ordinary mode through the form of linear Landau resonance is obtained in § 3. Finally, the applications and discussions of our result is presented in § 4.

2. Nonlinear dispersion relation

We derive the nonlinear dispersion relation of the extra-ordinary mode in the presence of stationary KAW turbulence which propagates in the $x-z$ plane with wave vector $\mathbf{k} = (k_\perp, 0, k_\parallel)$. The wave fields are $\mathbf{E}_l = (E_{l\perp}, 0, E_{l\parallel}$ and $\mathbf{B}_l = (0, B_{ly}, 0)$. Basic equations are the Vlasov–Maxwell set of equations:

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}} - \frac{e}{m}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\frac{\partial}{\partial \mathbf{v}}\right] f_e = 0, \quad (1)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{J},$$

$$\mathbf{J} = en_0 \int \mathbf{v} f_e(\mathbf{r}, \mathbf{v}, t)\, \mathrm{d}\mathbf{v}. \tag{2}$$

The unperturbed electron distribution function, electric and magnetic fields are

$$f_e = f_{0e} + \epsilon f_{1e}, \qquad \mathbf{E}_{0l} = \epsilon \mathbf{E}_l, \qquad \mathbf{B}_{0l} = \mathbf{B}_0 + \epsilon \mathbf{B}_l, \tag{3}$$

where ϵ is a smallness parameter and is a measure of the perturbed quantities due to the presence of KAW and $\mathbf{B}_0 = (0, 0, B_0)$ is the external magnetic field.

We assume that the system is open and stationary KAW turbulence is produced by dc current. Now we consider the interaction between the low frequency KAW turbulence and a high frequency extra-ordinary mode through resonant electrons. We also assume that the density gradient is very small. Then the linear response of the electron distribution function (f_{1e}) is given by

$$f_{1e}(\omega, \mathbf{k}) = \frac{ie}{m} \frac{E_{l\parallel}(\omega, \mathbf{k})}{\omega - k_\parallel v_\parallel + i0} \frac{\partial}{\partial v_\parallel} f_{0e}, \tag{4}$$

where ω and $\mathbf{k}$ are the frequency and the wave vector of the KAW, respectively; and $i0$ is a small imaginary part of ω.

We now perturb the quasi-steady state by a high frequency test extra-ordinary mode wave fields $\mu\delta\mathbf{E}_h$ and $\mu\delta\mathbf{B}_h$ having wave vector $\mathbf{K} = (K_\perp, 0, 0)$ and frequency Ω. Here $\mu(\ll \epsilon)$ is another smallness parameter. The total perturbed electron distribution function and fields are

$$\begin{aligned} \delta f &= \mu\delta f_h + \mu\epsilon\delta f_{lh} + \mu\epsilon^2 \Delta f, \\ \delta \mathbf{E} &= \mu\delta\mathbf{E}_h + \mu\epsilon\delta\mathbf{E}_{lh}, \qquad \delta\mathbf{B} = \mu\delta\mathbf{B}_h + \mu\epsilon\delta\mathbf{B}_{lh}. \end{aligned} \tag{5}$$

We now linearize Vlasov equation (1) to the orders $\mu, \mu\epsilon$ and $\mu\epsilon^2$ to obtain the high frequency response of the electron distribution function by taking Fourier transform in space and time for the various quantities. After a straightforward algebra, we obtain from equations (2) the nonlinear dielectric function of the extra-ordinary mode in the presence of stationary kinetic Alfvén wave turbulence as

$$D_T(\Omega, \mathbf{K}) = D_0(\Omega, \mathbf{K}) + D_d(\Omega, \mathbf{K}) + D_p(\Omega, \mathbf{K}). \tag{6}$$

Here $D_0(\Omega, \mathbf{K})$ is the linear part, given by

$$D_0(\Omega, \mathbf{K}) = K_\perp^2 - \frac{\Omega^2}{c^2} + \frac{\omega_{pe}^2 \Omega}{c^2} \sum_{a,b} \int i v_y \frac{J_a(\alpha) J_b'(\alpha) \mathrm{e}^{i(a-b)\phi}}{b\Omega_e - \Omega} \frac{\partial}{\partial v_\perp} f_{0e} \mathrm{d}\mathbf{v}, \tag{7}$$

where $\alpha = K_\perp v_\perp / \Omega_e$ and J_a is the Bessel function. $D_d(\Omega, \mathbf{K})$ is the direct mode coupling term given by

$$D_d(\Omega, \mathbf{K}) = -\frac{\omega_{pe}^2}{\Omega} \left(\frac{e}{m}\right)^2 \sum_{s,t,a,b,m,n} \int v_y \frac{J_s(\alpha) J_t(\alpha) \mathrm{e}^{i(s-t)\phi}}{t\Omega_e - \Omega}$$
$$\times \left[\sum_{\mathbf{k}} |E_{l\parallel}(\mathbf{k})|^2 \left\{ \frac{\partial}{\partial v_\parallel} + \frac{t}{\alpha} \frac{k_\perp}{\omega} \left(v_\parallel \frac{\partial}{\partial v_\perp} - v_\perp \frac{\partial}{\partial v_\parallel} \right) \right\} \right.$$

$$+\sum_{k}|E_{l\perp}(\mathbf{k})E_{l\parallel}(-\mathbf{k})|\frac{t}{\alpha}\left\{\frac{\partial}{\partial v_\perp}-\frac{k_\parallel}{\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}\Bigg]$$

$$\times i\,\frac{J_m(\alpha')J_n'(\alpha')\mathrm{e}^{i(m-n)\phi}}{n\Omega_e-k_\parallel v_\parallel-(\Omega-\omega)}\frac{\partial}{\partial v_\perp}\frac{1}{\omega-k_\parallel v_\parallel-i0}\frac{\partial}{\partial v_\parallel}f_{0e}\,\mathbf{dv}, \tag{8}$$

where $\alpha'=K_\perp' v_\perp/\Omega_e$ with $K_\perp'=K_\perp-k_\perp$.

The polarization mode coupling term, $D_p(\Omega,\mathbf{K})$, can be written as

$$D_p(\Omega,\mathbf{K})=-\frac{\omega_{pe}^2\Omega}{c^2}\left(\frac{e}{m}\right)^2\frac{\omega_{pe}^2(\Omega-\omega)}{RR'[(\Omega-\omega)^2-c^2k_\parallel^2]}$$

$$\times\sum_{\mathbf{k}}|E_{l\parallel}(\mathbf{k})|^2[(A+B)\times(C+D+F)], \tag{9}$$

where

$$A=\sum_{s,t,m,n,}\int v_y\frac{J_s(\alpha)J_t(\alpha)\mathrm{e}^{i(s-t)\phi}}{i(\Omega-t\Omega_e)}\left\{\frac{\partial}{\partial v_\parallel}+\frac{t}{\alpha}\frac{k_\perp}{\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}$$

$$\times\frac{J_m(\alpha')J_n(\alpha')\mathrm{e}^{i(m-n)\phi}}{n\Omega_e-k_\parallel v_\parallel-(\Omega-\omega)}\frac{n}{\alpha'}\left\{\frac{\partial}{\partial v_\perp}+\frac{k_\parallel}{\Omega-\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}f_{0e}\mathbf{dv}, \tag{10}$$

$$B=\sum_{s,t}\int v_y\frac{J_s(\alpha)J_t(\alpha)\mathrm{e}^{i(s-t)\phi}}{i(t\Omega_e-\Omega)}\frac{t}{\alpha}\left\{\frac{\partial}{\partial v_\perp}+\frac{k_\parallel}{\Omega-\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}$$

$$\times\frac{1}{k_\parallel v_\parallel-\omega-i0}\frac{\partial}{\partial v_\parallel}f_{0e}\mathbf{dv}, \tag{11}$$

$$C=\sum_{m,n,a,b}\int v_x\frac{J_m(\alpha')J_n(\alpha')\mathrm{e}^{i(m-n)\phi}}{n\Omega_e-k_\parallel v_\parallel-(\Omega-\omega)}\left\{\frac{\partial}{\partial v_\parallel}+\frac{n}{\alpha'}\frac{k_\perp}{\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}$$

$$\times\frac{J_a(\alpha)J_b'(\alpha)\mathrm{e}^{i(a-b)\phi}}{\Omega-b\Omega_e}\frac{\partial}{\partial v_\perp}f_{0e}\mathbf{dv}, \tag{12}$$

$$D=\sum_{m,n}\int v_x\frac{J_m(\alpha')J_n'(\alpha')\mathrm{e}^{i(m-n)\phi}}{\Omega-\omega+k_\parallel v_\parallel-n\Omega_e}\frac{\partial}{\partial v_\perp}\frac{1}{\omega-k_\parallel v_\parallel-i0}\frac{\partial}{\partial v_\parallel}f_{0e}\mathbf{dv}, \tag{13}$$

$$F=X(Y+Z), \tag{14}$$

with

$$X=\sum_{m,n}\int v_x\frac{J_m(\alpha')J_n'(\alpha')\mathrm{e}^{i(m-n)\phi}}{\Omega-\omega+k_\parallel v_\parallel-n\Omega_e}\left\{\frac{\partial}{\partial v_\perp}+\frac{k_\parallel}{\Omega-\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}f_{0e}\mathbf{dv}, \tag{15}$$

$$Y=\frac{\omega_{pe}^2(\Omega-\omega)}{S[(\Omega-\omega)-c^2(\mathbf{K}-\mathbf{k})^2]}\sum_{m,n,a,b}\int iv_y\frac{J_m(\alpha')J_n'(\alpha')\mathrm{e}^{i(m-n)\phi}}{n\Omega_e-k_\parallel v_\parallel-(\Omega-\omega)}$$

$$\times\left\{\frac{\partial}{\partial v_\parallel}+\frac{n}{\alpha'}\frac{k_\perp}{\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}\frac{J_a(\alpha)J_b'(\alpha)e^{i(a-b)\phi}}{\Omega-b\Omega_e}\frac{\partial}{\partial v_\perp}f_{0e}\,\mathbf{dv}, \tag{16}$$

$$Z=\frac{\omega_{pe}^2(\Omega-\omega)}{S[(\Omega-\omega)-c^2(\mathbf{K}-\mathbf{k})^2]}\sum_{m,n}\int iv_y\frac{J_m(\alpha')J_n'(\alpha')e^{i(m-n)\phi}}{n\Omega_e-k_\parallel v_\parallel-(\Omega-\omega)}$$
$$\times\frac{\partial}{\partial v_\perp}\frac{1}{\omega-k_\parallel v_\parallel+i0}\frac{\partial}{\partial v_\parallel}f_{0e}\mathbf{dv}, \tag{17}$$

$$R=1-\frac{\omega_{pe}^2(\Omega-\omega)}{(\Omega-\omega)^2-c^2k_\parallel^2}\sum_{m,n}\int v_x\frac{J_m(\alpha')J_n(\alpha')e^{i(m-n)\phi}}{n\Omega_e-k_\parallel v_\parallel-(\Omega-\omega)}\frac{n}{\alpha'}$$
$$\times\left\{\frac{\partial}{\partial v_\perp}+\frac{k_\parallel}{\Omega-\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}f_{0e}\mathbf{dv}, \tag{18}$$

$$S=1-\frac{\omega_{pe}^2(\Omega-\omega)}{(\Omega-\omega)^2-c^2(\mathbf{K}-\mathbf{k})^2}\sum_{m,n}\int iv_y\frac{J_m(\alpha')J_n'(\alpha')e^{i(m-n)\phi}}{n\Omega_e-k_\parallel v_\parallel-(\Omega-\omega)}$$
$$\times\left\{\frac{\partial}{\partial v_\perp}+\frac{k_\parallel}{\Omega-\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}f_{0e}\mathbf{dv}, \tag{19}$$

$$R'=1-\frac{\omega_{pe}^2(\Omega-\omega)}{S[(\Omega-\omega)^2-c^2(\mathbf{K}-\mathbf{k})^2]}\sum_{m,n}\int v_y\frac{J_m(\alpha')J_n(\alpha')e^{i(m-n)\phi}}{i(n\Omega_e-k_\parallel v_\parallel-(\Omega-\omega))}$$
$$\times\frac{n}{\alpha'}\left\{\frac{\partial}{\partial v_\perp}+\frac{k_\parallel}{\Omega-\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}f_{0e}\mathbf{dv}$$
$$\times\frac{\omega_{pe}^2(\Omega-\omega)}{R[(\Omega-\omega)^2-c^2k_\parallel^2]}\int v_x\frac{J_m(\alpha')J_n'(\alpha')e^{i(m-n)\phi}}{\Omega-\omega+k_\parallel v_\parallel-n\Omega_e}$$
$$\times\left\{\frac{\partial}{\partial v_\perp}+\frac{k_\parallel}{\Omega-\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}f_{0e}\mathbf{dv}. \tag{20}$$

3. Growth rate of the extra-ordinary mode

We assume throughout this paper that KAW turbulence is driven by an electron beam. The unperturbed electron distribution function in this case is given by

$$f_{0e}=\left(\frac{m}{2\pi T_e}\right)^{3/2}\exp\left[-\frac{mv_\perp^2}{2T_e}\right]\exp\left[-\frac{m}{2T_e}(v_\parallel-v_0)^2\right]. \tag{21}$$

We also assume that the effective Alfvén velocity v_A^* [12] is less than the drift velocity v_0, i.e. $v_A^*\ll v_0$ and $v_0-v_A^*\ll v_e$, where v_e is the electron thermal velocity. Then we obtain from equation (7) for $\Omega\sim\Omega_e$,

$$\mathrm{Re}D_0(\Omega,\mathbf{K})=1-\frac{c^2K_\perp^2}{\Omega^2}+\frac{1}{2}\frac{\omega_{pe}^2}{\Omega(\Omega_e-\Omega)}, \tag{22}$$

and

$$\frac{\partial}{\partial \Omega}[\mathrm{Re}D_0(\Omega,\mathbf{K})] = \frac{\omega_{pe}^2 \Omega_e}{2\Omega^2(\Omega_e - \Omega)^2}. \tag{23}$$

The growth rate of the extra-ordinary mode due to the direct coupling term is given by

$$\gamma_d(\Omega,\mathbf{K}) = -\mathrm{Im}D_d(\Omega,\mathbf{K}) \Big/ \frac{\partial}{\partial \Omega}[\mathrm{Re}D_0(\Omega,\mathbf{K})]_{\Omega=\Omega_r}, \tag{24}$$

where Ω_r is the real part of Ω.

It is found that the first term of the direct coupling contribution (eq. (8)), coming from coupling between the parallel electric fields ($E_{l\parallel}(\mathbf{k})$) vanishes after partial integration. Next, we estimate the second term of the direct coupling contribution, which comes from the coupling between $E_{l\parallel}$ and $E_{l\perp}$ fields. In this case we use the amplitude ratio of the electric fields of the KAW [12]:

$$\frac{E_{l\parallel}(\mathbf{k})}{E_{l\perp}(\mathbf{k})} = -\frac{k_\parallel}{k_\perp}\frac{T_e}{T_i}(1 - I_0(\beta_i)\exp(-\beta_i)) = -\frac{k_\parallel}{k_\perp}Q^{-1}, \tag{25}$$

where T_e, T_i and I_0 are the electron, ion temperatures and modified Bessel function, respectively;

$$\beta_i = k_\perp^2 \rho_i^2/2, Q = (T_i/T_e)[1 - I_0(\beta_i)\exp(-\beta_i)]^{-1}$$

Here ρ_i is the ion gyro-radius.

Using equation (25) in the second term of eq. (8), we obtain the growth rate of the extra-ordinary mode due to the direct nonlinear coupling as

$$\begin{aligned} \frac{\gamma_d(\Omega,\mathbf{K})}{\Omega_e} =& \sqrt{\pi}\left(\frac{\omega_{pe}}{\Omega_e}\right)(K_\perp \rho_e)\frac{k_e}{k_\perp}\left(\frac{v_A^*}{c}\right)Q(Q+1)^{-2}W_T \\ & \times \frac{v_0 - v_A^*}{v_e}\exp\left[-\frac{m}{2T_e}(v_A^* - v_0)^2\right], \end{aligned} \tag{26}$$

where W_T is the normalized turbulent energy density of the KAW and is given by

$$W_T = \sum_{\mathbf{k}} \frac{|E_{l\parallel}(\mathbf{k})|^2}{16\pi n_0 T_e}\left(\frac{k_\perp}{k_\parallel}\right)^2\left(\frac{c}{v_A^*}\right)^2(Q+1)^2. \tag{27}$$

Next, we consider the growth rate of the extra-ordinary mode due to the polarization coupling term (eq. (9)). We have

$$\gamma_p(\Omega,\mathbf{K}) = -\mathrm{Im}D_p(\Omega,\mathbf{K}) \Big/ \frac{\partial}{\partial \Omega}[\mathrm{Re}D_0(\Omega,\mathbf{K})]_{\Omega=\Omega_r}, \tag{28}$$

where

$$\begin{aligned} \mathrm{Im}\,D_p(\Omega,\mathbf{K}) =& \frac{\omega_{pe}^2}{\Omega}\left(\frac{e}{m}\right)^2 \frac{\omega_{pe}^2(\Omega-\omega)}{RR'[(\Omega-\omega)^2 - c^2k_\parallel^2]}\sum_{\mathbf{k}}|E_{l\parallel}(\mathbf{k})|^2 \\ & \times [\mathrm{Im}\,B \times \mathrm{Re}(C+D+F) + \mathrm{Re}(A+B) \times \mathrm{Im}(D+F)]. \end{aligned} \tag{29}$$

It can be seen that

$$\begin{aligned}\mathrm{Re}(C+D+F) &= \mathrm{Re}(C+D)+\mathrm{Re}(XY)+\mathrm{Re}(XZ),\\ \mathrm{Im}(D+F) &= \mathrm{Im}D+\mathrm{Re}X\,\mathrm{Im}Z.\end{aligned}$$

From equations (10)–(20), (28) and (29) we finally obtain the growth rate due to the polarization term as

$$\begin{aligned}\frac{\gamma_p(\Omega,\mathbf{K})}{\Omega_e} = {} & \sqrt{\pi}\left(\frac{\omega_{pe}}{\Omega_e}\right)\left(\frac{v_e}{c}\right)^2\left(\frac{K_\perp}{k_\parallel}\right)^2\left(\frac{k_\parallel}{k_\perp}\right)^2\left(\frac{k_e}{k_\parallel}\right)^2\left(\frac{v_A^*}{c}\right)^2 W_T\\ & \times\ (Q+1)^{-2}\left(\frac{v_0-v_A^*}{v_e}\right)\exp\left[-\frac{m}{2T_e}(v_A^*-v_0)^2\right].\end{aligned} \tag{30}$$

In deriving the above equations we have assumed $\Omega\sim\Omega_e$ with $\Omega_e\gg\omega_{pe}$, since the electromagnetic branch of the extra-ordinary mode under consideration exists only for $cK_\perp>\omega_{pe}$ [13]. Furthermore, we take $K_\perp>k_\perp, v_0-v_A^*\ll v_e$.

4. Application and discussion

The discovery of intense non-thermal radio emission from Saturn [14] raised the number of radio planets to three, i.e. the Earth, Jupiter and Saturn. However, by the end of 1980s, after Voyager 2 spacecraft visited Uranus and Neptune, this number could be read as five. It is understood that as compared to the environments of the other planets, the inner magnetosphere of Saturn, where radiation is thought to be produced, seems to be simpler. Since the magnetic field is dipolar, with a dipole axis almost perfectly aligned with the rotation axis of the planet [15], the cold plasma distribution is dominated either by the ionosphere or by a plasma disc which has been observed by the Voyager spacecraft [16]. The superposition of these two populations leads to low electron densities or to very small values of the parameter ω_{pe}/Ω_e, which plays a significant role in interpreting the generation mechanism. Such a strongly magnetized plasma makes the Doppler shifted cyclotron maser instability a promising candidate for explaining the SKR generation. The theoretical model developed in this paper allows for the first time the derivation of the nonlinear dielectric function of the extra-ordinary mode in the presence of kinetic Alfvén wave turbulence through the plasma maser process. For a plasma for which $\Omega_e\gg\omega_{pe}$, the growth rate of the extra-ordinary mode through the plasma maser process comes from two mode coupling terms, i.e. direct and polarization coupling term. It is shown that in an open plasma system with an external magnetic field, the instability of the high frequency mode occurs through the plasma maser interaction between the kinetic Alfvén wave turbulence and electrons.

As an illustration, we apply our result to SKR. Accordingly, we take typical plasma parameters pertaining to Saturn [17–19] $\omega_{pe}/\Omega_e=0.1, k_\perp/k_\parallel=10$ (since in this case $k_\perp\sim\rho_i^{-1}\gg k_\parallel\sim L_{\mathrm{MHD}}^{-1}$, here ρ_i is the ion gyro-radius $\sim 500\,\mathrm{km}, L_{\mathrm{MHD}}$ is MHD scale length (typically the size of the inhomogeneity scale of the plasma)), $v_A^*/c=0.001$ ($v_A^*\sim 450\,\mathrm{km/s}$), $v_e/c=0.1$, $Q=1.5$, $K_\perp/k_\parallel\gg 1$, $K_\perp/k_\perp\gg 1$, $k_e/k_\parallel=10^4$ and $K_\perp\rho_e\simeq 1$. Then from equations (26) and (30), we get $\gamma_d(\Omega,\mathbf{K})/\Omega_e\sim 10^{-2}W_T$ and $\gamma_p(\Omega,\mathbf{K})/\Omega_e\sim 10^{-2}W_T$ respectively. Thus both the direct coupling and the polarization coupling give the same order of magnitude contribution to the growth rate. This growth rate is high enough to explain the radiation mechanism of SKR in extra-ordinary mode. We may conclude that the plasma maser interaction including the MHD mode has a potential importance in interpreting various anomalous radiation phenomena in cosmic plasmas.

A comparison of our results with earlier results on the plasma maser instability of ordinary mode in the presence of KAW turbulence [12] shows that ordinary mode is more unstable, because for ordinary mode $\Omega \sim \Omega_e \sim \omega_{pe}$, whereas for extra-ordinary mode considered in this paper $\Omega \sim \Omega_e \gg \omega_{pe}$ since the electromagnetic branch of the extra-ordinary mode under consideration here exists only for $cK_\perp > \omega_{pe}$. It may be noted that the growth rate of both the modes are proportional to the ratio ω_{pe}/Ω_e.

Acknowledgements

The authors would like to thank the Council of Scientific and Industrial Research and the Indian Space Research Organisation for financial support.

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