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Gravitationally induced particle creation in a q-scalar field

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Abstract. Tom and Goodison [5] have shown that for generic values of q, gravitationally induced particle creation is impossible in the ordinary vacuum state. Here we consider the evolution of a q-deformed scalar field in a curved spacetime and observe that if the field is either represented by a coherent state or a squeezed state, there is a change in the energy density of the field indicating the possibility of particle creation.

Keywords. q-scalar field; quon; coherent state; squeezed state; particle creation; stress energy tensor.

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1. Introduction

The study of quantum field theory in curved spacetime is important as it is an essential key to a knowledge of the scenario in the early universe. The behaviour of the classical scalar field near the initial singularity is best approximated quantum mechanically by constituting a complete set of coherent states for each mode of the scalar field [1]. The quantum state of the scalar field near the initial singularity is inaccessible to an observer in the present time, and Hawking [2] proposed that this inaccessible nature can be expressed by taking a random superposition of all allowed states in the inaccessible region. Berger [3] realised this by superposing coherent states in a random manner. Parker [4] studied the particle creation in an expanding universe, with gravitational metric treated as an unquantised external field. Considering the evolution of a scalar field in an expanding universe, Tom and Goodison [5] showed that if the field quanta obey the quantum statistics, then particle creation is impossible in the vacuum state.

Here we study the evolution of a scalar field whose quanta obey q-deformed statistics in the asymptotic region, which is assumed to be flat. By constructing a superposition of coherent states in both in- and out-regions, we show that there is a non-zero probability for particle creation. We calculate the expectation values of the stress energy tensor in the coherent state of each mode of the field. Section 2 contains the calculation of the different components of the stress energy tensor. In § 3, the coherent state representation of the field and coherent state expectations are discussed, and in § 4, we discuss the situation where the field is in a squeezed state. Section 5 is the concluding section.

2. Stress energy tensor of the scalar field in curved spacetime

We assume a spatially flat Robertson–Walker spacetime with the metric

$$ds^2 = -dt^2 + \sum_i R^2(t)(dx^i)^2 \tag{1}$$

where $R(t) = R_1$ for $t \leq t_1$ and $R(t) = R_2$ for $t \geq t_2$. We call the portion of spacetime with $t < t_1$ the in-region and that with $t > t_2$, the out-region. In the in-region, the scalar field is expanded as

$$\phi(x) = \sum_k \frac{1}{(2\pi)^{3/2}} \frac{1}{(2\omega_k)^{1/2}} [a_k F_k(x) + a_k^\dagger F_k^*(x)] \tag{2}$$

where ω_k is given by

$$\omega_k^2 = g\left(\sum_i \frac{k_i^2}{R^2} + m^2\right) \tag{3}$$

with

$$g = |g_{\mu\nu}|. \tag{4}$$

We take $\phi_q(x)$ as a real scalar field in the Heisenberg picture and $F_k(x)$s are assumed to form a complete set of positive frequency solutions to the Klein–Gordon equation:

$$(F_k, F_{k'}) = (2\pi)^3 (2\omega_k)\delta_{kk'}, \quad (F_k, F_{k'}^*) = 0. \tag{5}$$

In the in-region, the field statistics are assumed to be given by

$$a_k a_{k'}^\dagger - a_{k'}^\dagger a_k = q^{N_k} \delta_{kk'} \tag{6}$$

with $|q| \leq 1$ and $[N_k] = a_k^\dagger a_k$. Also operators corresponding to different modes commute:

$$[a_k, a_{k'}] = 0 = [a_k^\dagger, a_{k'}^\dagger]. \tag{7}$$

In the out-region, the field is expanded as

$$\phi(x) = \sum_k \frac{1}{(2\pi)^{3/2}} \frac{1}{(2\omega_k)^{1/2}} [b_k G_k(x) + b_k^\dagger G_k^*(x)]. \tag{8}$$

$G_k(x)$s also are assumed to form a complete set of positive frequency solutions. In general, b_k differs from a_k if there is particle creation due to the expansion of the universe. In the out-region, the statistics are assumed as

$$b_k b_{k'}^\dagger - b_{k'}^\dagger b_k = q'^{N_k} \delta_{kk'} \tag{9}$$

with $|q'| \leq 1$, and generally q', may not be equal to q. Here also

$$[b_k, b_{k'}] = 0 = [b_k^\dagger, b_{k'}^\dagger]. \tag{10}$$

Since the mode solutions in both the regions are complete, one set of solutions can be expressed in terms of the other:

$$G_k(x) = \sum_{k'} (\alpha_{kk'} F_{k'}(x) + \beta_{kk'} F^*_{k'}(x)),$$

$$F'_k(x) = \sum_{k} (\alpha_{kk'} G_k(x) - \beta_{kk'} G^*_k(x)). \tag{11}$$

The above relations are called the Bogoliubov transformations and the coefficients $\alpha_{kk'}$ and $\beta_{kk'}$ are called Bogoliubov coefficients [6, 7]. Using the Bogoliubov transformations and equating the field expansions in the in- and out-regions, we can obtain the following relation:

$$a_k = \sum_{k'} (\alpha_{kk'} F_{k'}(x) + \beta_{kk'} F^*_{k'}(x)). \tag{12}$$

For a spacetime whose metric is given by (1), the Bogoliubov coefficients are diagonal. Hence we can write

$$a_k = \alpha_k b_k + \beta_k b^\dagger_k. \tag{13}$$

Expansions given by (2) and (8) are special cases of the expansion given by Parker [4]. The stress energy tensor for the scalar field is

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \tfrac{1}{2} g_{\mu\nu} (g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + m^2 \phi^2). \tag{14}$$

If we transform the coordinates as

$$g^{1/2} \mathrm{d}\tau = \mathrm{d}t, \tag{15}$$

then

$$T_{00} = \frac{1}{2g} \left[\left(\frac{\partial \phi}{\partial \tau} \right)^2 + g \left[\sum_i \frac{1}{R^2} (\partial_i \phi)^2 + m^2 \phi^2 \right] \right], \tag{16}$$

$$T_{ii} = (\partial_i \phi)^2 + \frac{1}{2g} R^2 \left(\frac{\partial \phi}{\partial \tau} \right)^2 - \frac{1}{2} R^2 \left[\sum_j \frac{1}{R^2} (\partial_j \phi)^2 + m^2 \phi^2 \right]. \tag{17}$$

The spatial average of $T_{\mu\nu}$ is defined by [4],

$$\bar{T}_{\mu\nu} = \frac{1}{(2\pi)^3} \int \mathrm{d}^3 x T_{\mu\nu}. \tag{18}$$

Taking $a_{-k} = a^\dagger_k$ and $b_{-k} = b^\dagger_k$, the spatial averages of T_{00} and T_{ii} in the in-region are

$$(\bar{T}_{00})\,\mathrm{in} = \frac{1}{16\pi^3 g} \sum_k \omega_k^2 (a_k a^\dagger_k + a^\dagger_k a_k),$$

$$(\bar{T}_{ii})\,\mathrm{in} = \frac{R^2}{16\pi^3 g} \sum_k \frac{1}{\omega_k} \left(\frac{2k_i^2}{R^2} g - \omega_k^2 \right) (a_k a^\dagger_k + a^\dagger_k a_k). \tag{19}$$

Similarly for the out-region

$$(\bar{T}_{00})\,\mathrm{out} = \frac{1}{16\pi^3 g} \sum_k \omega_k^2 (b_k b^\dagger_k + b^\dagger_k b_k),$$

$$(\bar{T}_{ii})\,\text{out} = \frac{R}{16\pi^3 g}\sum_k \frac{1}{\omega_k}\left(\frac{2k_i^2}{R^2}g - \omega_k^2\right)(b_k b_k^\dagger + b_k^\dagger b_k). \tag{20}$$

3. Coherent state representation

Berger [3] represented the field as a superposition of coherent states because it is a natural method for relating the quasi-classical behaviour to singularity parameters. Coherent states are the eigenstates of the annihilation operator and are normalisable to unity but are not orthogonal. In a coherent state, the fluctuations in the two quadratures are equal and minimize the uncertainty product given by Heisenberg's uncertainty relation.

We represent the field as a superposition of q-coherent states, which are eigenstates of a q-annihilation operators a_k and b_k.

$$a_k|\lambda_k\rangle = \lambda_k|\lambda_k\rangle, \tag{21}$$

$$b_k|\chi_k\rangle = \chi_k|\chi_k\rangle. \tag{22}$$

The states $|\lambda_k\rangle$ and $|\chi_k\rangle$ represent coherent states for the k^{th} mode. The set $\lambda_k = 0$ or $\chi_k = 0$ for all $\mathbf{k}$ refers to vacuum in the in- and out-regions respectively.

$$\langle\lambda_k|\bar{T}_{00}^k|\lambda_k\rangle = \frac{1}{16\pi^3 g}\omega_k^2((1+q)|\lambda_k|^2 + 1),$$

$$\langle\lambda_k|\bar{T}_{ii}^k|\lambda_k\rangle = \frac{R^2}{16\pi^3 g\omega_k}\left(\frac{2k_i^2}{R^2}g - \omega_k^2\right)[(1+q)|\lambda_k|^2 + 1]. \tag{23}$$

The vacuum energy density in the in-region is

$$\rho_0(\text{in}) = -\langle 0|\bar{T}_0^0|0\rangle, \\ = \frac{1}{16\pi^3 g}\sum_k \omega_k^2. \tag{24}$$

The anisotropic pressure in this region is

$$P_{i0}(\text{in}) = \langle 0|\bar{T}_i^i|0\rangle, \\ = \frac{1}{16\pi^3 g}\sum_k \frac{1}{\omega_k}\left(\frac{2k_i^2}{R_i^2}g - \omega_k^2\right). \tag{25}$$

In the in-region, the field is represented by a coherent state $\langle\Pi_k|\lambda_k\rangle$. The expectation value of the stress energy tensor in that region

$$\rho(\text{in}) = \rho_0(\text{in}) + \rho^{\text{cl}}(\text{in}), \\ P_i(\text{in}) = P_{i0}(\text{in}) + \rho_i^{\text{cl}}(\text{in}) \tag{26}$$

where

$$\rho^{\text{cl}}(\text{in}) = \frac{1}{16\pi^3 g}\sum_k \omega_k^2(1+q)|\lambda_k|^2$$

and

$$P_i^{\rm cl}({\rm in}) = \frac{1}{16\pi^3 g} \sum_k \frac{1}{\omega_k} |\lambda_k|^2 \left(\frac{2k_i^2}{R_i^2} g - \omega_k^2 \right). \tag{27}$$

For the out-region

$$\begin{aligned} \rho({\rm out}) &= \rho_0({\rm out}) + \rho^{\rm cl}({\rm out}), \\ P_i({\rm out}) &= P_{i0}({\rm out}) + \rho_i^{\rm cl}({\rm out}) \end{aligned} \tag{28}$$

where

$$\rho^{\rm cl}({\rm out}) = \frac{1}{16\pi^3 g} \sum_k \omega_k^2 (1+q) |\chi_k|^2$$

and

$$P_i^{\rm cl}({\rm out}) = \frac{1}{16\pi^3 g} \sum_k \frac{1}{\omega_k} |\chi_k|^2 \left(\frac{2k_i^2}{R_i^2} g - \omega_k^2 \right). \tag{29}$$

Thus the vacuum expectation values of energy density and anisotropic pressure is the same for both in- and out-regions. Nevertheless, for a general coherent state, the expectation values for energy density and anisotropic pressure differ for the in- and out-regions.

The change in energy density and for each mode depends on the value of $|\chi_k|^2 - |\lambda_k|^2$. Similarly the probability for observing a non-zero number of quanta in the k^{th} mode in the in-region is

$$1 - |\langle \lambda_k | 0 \rangle|^2 = \frac{\exp_q |\lambda_k|^2 - 1}{\exp_q |\lambda_k|^2} \tag{30}$$

where the q-exponential $\exp_q z$ is defined by

$$\exp_q z = \sum_{n=0}^{\infty} \frac{z^n}{[n]!} \tag{31}$$

with $[n]! = [n] \cdot [n-1] \cdot [n-2] [2] \cdot [1]$ and $[n] = (q^n - 1/q - 1)$.

In the out-region, such a probability is given by

$$1 - |\langle \chi_k | 0 \rangle|^2 = \frac{\exp_{q'} |\chi_k|^2 - 1}{\exp_{q'} |\chi_k|^2}. \tag{32}$$

So if the Bogoliubov coefficients α_k and β_k are non-zero, there is a non-zero probability of particle creation as the field evolves from the in-region to the out-region.

It is interesting to calculate the density fluctuations of the q-deformed field. In the q-vacuum, fluctuations of energy density and anisotropic pressure vanish. For the q-CS, the variances are obtained as

$$\begin{aligned} (\Delta\rho^2)({\rm in}) &= \frac{1}{(16\pi^3 g)^2} \sum_2 \omega_k^4 (1+q)^2 |\lambda_k|^2 [1 + (q-1)|\lambda_k|^2], \\ (\Delta P_i)^2({\rm in}) &= \frac{R^4}{(16\pi^3 g)^2} \sum_k \frac{1}{\omega^2} \left(\frac{2k_i^2}{R^2} g - \omega_k^2 \right)^2 [1 + (q-1)|\lambda_k|^2]. \end{aligned} \tag{33}$$

Similar expressions hold for the out-region also. For $q \geq 1$, the variances are positive. For $q \leq 1$, the definite positivity of the variances imposes the condition $|\lambda_k|^2 \leq (1-q)^{-1}$, which is indeed statisfied by a q-CS [8]. Thus no additional restrictions are required and our formalism is applicable for any q-CS defined by [8].

4. Squeezed states

Like coherent states, squeezed states are minimum uncertainty states, but in squeezed state the variance of one of the quadrature components goes below the minimum value allowed by Heisenberg's uncertainty principle, while that of the other component goes above it, keeping the product at the minimum value. Grishchuk and Sidorov [9] introduced the language of squeezed states, a well-known concept in quantum optics, in cosmology and showed that gravitons and other primordial perturbations created from zero point quantum fluctuations in the process of cosmological evolution should now be in a strongly squeezed state. Gasperini and Giovannini [10] established the dependence of the entropy growth in the cosmological process of pair creation on the associated squeezing parameter. Albrecht *et al* [11] have found that the squeezed state formalism provides a framework for studying the amplifying process during the cosmological inflation. Making use of the squeezed state formalism, Hu *et al* [12] arrived at a systematic description of the dependence of particle creation on the initial state. Suresh *et al* [13] have calculated the expectation values of the stress energy tensor of the scalar field in curved spacetime as well as the quantum fluctuations of density and anisotropic pressure by making use of the squeezed state formalism. Motivated by these works, we construct squeezed states corresponding to the in- and out-regions and calculate the expectation values of the stress energy tensor in these states. We define two new operatcrs A_k and B_k with their adjoint operators by

$$A_k = \mu a_k + \nu a_k^\dagger, \tag{34}$$

$$B_k = \eta b_k + \theta b_k^\dagger \tag{35}$$

with

$$|\mu|^2 - |\nu|^2 = 1 \tag{36}$$

and

$$|\eta|^2 - |\theta|^2 = 1. \tag{37}$$

The eigenstates of A_k and B_k are defined as q-squeezed states.

$$A_k|\beta_k\rangle = \beta_k|\beta_k\rangle, \tag{38}$$

$$B_k|\gamma_k\rangle = \gamma_k|\gamma_k\rangle. \tag{39}$$

The squeezed vacuum energy density in the in-region defined as $\lim_{\beta_k \to 0} \sum_k \langle\beta_k|\bar{T}^k_{00}|\beta_k\rangle$ is given by

$$\rho_0(\beta_k)(\text{in}) = \frac{1}{16\pi^3 g[1-|\nu|^2(q-1)]} \sum_k \omega_k^2(|\mu|^2 + |\nu|^2). \tag{40}$$

Note that as $\mu \to 1$ and $\nu \to 0$, the above expectation value reduces to the vacuum energy density given by (23), as it should. In the out-region, squeezed vacuum energy density is given by

$$\rho_0(\gamma_k)(\text{in}) = \frac{1}{16\pi^3 g[1 - |\theta|^2(q-1)]} \sum_k \omega_k^2(|\eta|^2 + |\theta|^2) \tag{41}$$

which also reduces to (23) in the appropriate limit. Thus squeezed vacuum energy density in the in- and out-regions are different and both differ from the vacuum energy density given by (23). As the field evolves, the squeezed vacuum energy density changes depending on $|\eta|^2 + |\theta|^2 - |\mu|^2 - |\nu|^2$.

Since in general, $|\eta|^2 + |\theta|^2 \neq |\mu|^2 + |\nu|^2$, there can be particle creation in the interacting region.

5. Conclusion

Tom and Goodison [5] showed that for $q \neq 1$, particle creation is impossible in the ordinary vacuum state for the dynamical evolution of the field through a curved spacetime. We have considered the evolution of a q-deformed field in a curved spacetime and observed that the field is in a coherent state or squeezed state in both the in- and out-regions, the energy density changes, indicating the possibility of particle production. This result shows that quon statistics cannot be ruled out completely for the early universe, because if it was in a coherent state or a squeezed state then scalar quon particles would be produced.

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