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Jeans–Buneman instability in a dusty plasma

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Abstract. A self consistent formulation of the Jeans instability of a dusty plasma with proper inclusion of charge dynamics is described. It is shown that charge fluctuations significantly affect the Jeans as well as the Buneman mode. For plasma particles (electrons and ions) in local thermal equilibrium, the Jeans length λ_J is given by $\lambda_J \approx \lambda_g F(R, \epsilon, \beta/\eta)$, where λ_g is the Debye length of the charged grains, R is the square of the ratio of the Jeans to the plasma frequency of the grains, ϵ is the square of the ratio of the Debye length of the grains and the plasma particles and β/η is the ratio of the attachment to the decay frequency of the electronic charges to the grain surface. The functional form of F is given in the text. Numerical investigation of the Jeans–Buneman mode for a two and three component plasma shows that the Jeans mode dominates at $k\lambda_D \ll 1$ (where k is the wave number and λ_D is the Debye length of plasma particles), whereas at $k\lambda_D \gg 1$ only the Buneman mode operates. Charge fluctuations reduce the area of overlap of the two modes. Furthermore, in the absence of gravity, there exists a new, charge fluctuation induced unstable mode in a streaming dusty plasma. Astrophysical applications of the results are discussed.

Keywords. Jeans–Buneman instability; Jeans length; charge fluctuations.

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1. Introduction

Dust grains are present everywhere in the universe. Dark haloes in galaxies [1], infrared emission from circumstellar shells of early as well as late type stars [2, 3], spectral features of the emission from active galactic nuclei (AGN) [4], ring and spoke formation in planetary systems [5], structure of cometary tails [6], are all attributed to the presence of micron sized grains. Back on earth, grains are present in uncontrolled (e.g. in the earth's magnetosphere and ionosphere) as well as controlled (e.g. Tokamak plasmas, plasma arcs etc.) environments. The grains are generally composed of graphite, silicate and metallic compounds in cosmic environments. The size distribution of the grain has been investigated by comparing the observed interstellar extinction curve with the theoretical one. The observed extinction curve do not fit to a single size and single composition implying that the sizes, masses and electric charges of the grains vary in a wide range. For example, grain sizes may vary between macroscopic (few cm) and microscopic (10^{-8} cm) scales, masses vary between 10^{-5}–10^{-15} g and electronic charges between $(1 - 10^6)$. Charging of a grain takes place because of the radiative and plasma background in which it is immersed in most of the astrophysical situations. It is quite possible that the sizes and masses of the grains might depend on the same plasma environment, e.g. Coulombic coagulation [7] could lead to bigger and heavier grains while breaking of the massive grains due to Maxwellian stress could be the source of lighter and smaller grains. These charged grains couple with the plasma dynamics through electromagnetic (EM) forces. A dusty/grainy plasma is thus a three component plasma consisting of electrons, ions and charged grains.

Collective behaviour of a dusty plasma has been investigated in considerable detail in the recent past [5, 8–11]. As most of these studies pertain to fixed charge on the grain, waves and instabilities in such a plasma are not very different from that of three component ion-electron plasmas. It is through the inclusion of charge dynamics [12–19] or mass dynamics [20] that new collective features have emerged.

The gravito-electromagnetic coupling of an atomic plasma in local thermal equilibrium was studied in the context of star formation by Eddington [21]. He showed that self-gravity is unimportant for such a plasma as $R = \omega_J^2/\omega_{pg}^2 \sim 10^{-36}$, where $\omega_J = (4\pi G n_g m_g)^{1/2}$ is the Jeans frequency and $\omega_{pg} = (4\pi n_g Q^2/m_g)^{1/2}$ is the plasma frequency of the grain with G as the gravitational constant and n_g, m_g and Q as the number density, mass and charge of the grain respectively. Therefore, the scales at which the two forces operate are widely separated. The formation of large scale structures have entirely been attributed to gravitational condensation of matter whereas the radiation processes are attributed to electromagnetic interactions of the plasma particles. However, for typically cm sized grains ($m_g \sim 10^{-5}$ gm), $R \sim O(1)$ and thus the two scales may overlap. The broad spectrum of observed masses and charges [22, 23] suggests that the dynamics of a dusty plasma can be studied in any of the following regimes: (a) EM $\gg$ GF (gravitational force), (b) EM force $\sim$ GF and (c) EM force $\ll$ GF. The first case corresponds to the plasma processes like radiation, heating etc., the second case is thought to be the cause of Spoke formation in Saturn's ring, thickness of the Jovian rings etc., and the last case corresponds to the formation of stars, clusters etc. [24].

An important distinction should be pointed out between the $R \sim 1$ and $F_{grav} \sim F_{Em}$ (F_{grav} and F_{Em} are the gravitational and the electromagnetic force on the grains). Whereas $R \sim 1$ leads to unusual collective behaviour of a self-gravitating dusty plasma due to overlap of two scales, $F_{grav} \sim F_{Em}$ implies merely the balance of the two forces whose origin could be quite different. For example in the planetary rings, the self repulsion of the grain is balanced by the gravitational attraction of the planets for micron and submicron sized grains [25].

Recently, the Jeans instability of a dusty plasma has been studied by several authors [26–31]. Time dependent nonlinear solutions of a self-gravitating three component dusty plasma showed [27] that when self-gravity is annuled by the electric polarization of a collapsing plasma i.e., $R \sim 1$, plasma may become gravitationally unstable due to the properties of the background plasma. Therefore, merely balancing the self attraction of the grains with self repulsion is not sufficient for determining the fate of condensation. Avinash and Shukla [29] studied the Jeans instability of a dusty plasma with the inclusion of ion dynamics and found enhanced gravitational condensation of the grains. Later, Pandey and Dwivedi [30] showed that the proper inclusion of ion dynamics does not destabilize the Jeans mode any further but the collapse of the grain follows the same dynamics as the collapse of the neutral matter under gravity. This is attributed to the fact that ions are responding at the free fall time scale of the grains and as a result they shield the grains electrostatically, making them behave as 'neutral'. Mohanta *et al* [31] studied the Jeans instability of a dusty plasma in the presence of charge fluctuations and external magnetic field and found that the presence of a magnetic field alters the collapse criteria for the grains.

All the previous studies, with the exception of the last one, exclude charge dynamics from consideration. However, the grain charge is a dynamical variable and in the present work we investigate the Jeans instability with proper charge dynamics of the grains. We consider an infinite, homogeneous, unmagnetized, nonrotating plasma and, invoking 'Jeans swindle', ignore the zeroth order gravitational field. However, as has been demonstrated [27], the construction of a proper equilibrium does not change the plasma dispersion relation, as one can always consider homogeneity in a limiting sense. We first consider the dusty plasma with electrons and ions in local thermal equilibrium and find that in the presence of charge fluctuations, the Jeans length gets modified, implying that the charge dynamics has a bearing on the gravitational collapse of the matter.

Next, we consider the gravitational instability of a two component dusty plasma with streaming ions. We find that when charge fluctuations are absent, a new gravito-streaming mode may give rise to the accretion of matter, when self gravity is annulled by the thermal pressure and self repulsion of the grain. Inclusion of charge fluctuations modifies both gravitational as well as streaming modes. Also, a new charge fluctuation induced instability in the presence of streaming ions exists in such a plasma. However, the growth rate of this new mode is much smaller than that of the Buneman mode.

Lastly, the dynamics of a three component dusty plasma is analysed. In the presence of streaming ions, the picture is similar to the previous case i.e., the Jeans and the Buneman modes dominate in two different windows with a small overlap of the two. The streaming instability sets in at a smaller wavelength in the presence of charge fluctuation than in the absence of it. The growth rate of the Jeans mode is enhanced in the presence of charge dynamics of the grains. Also, a charge fluctuation induced instability exists in such a plasma with a growth rate much smaller than that of the Buneman mode.

The paper is organized in the following fashion. In §2, basic equations are given and the equilibrium is discussed. Section 3 describes the linearized equations and gives the general dispersion relation. Also, a special case of the dispersion relation is discussed when electrons and ions are in local thermal equilibrium. Section 4 describes the Jeans–Buneman mode for two and three component dusty plasma systems. In §5 a summary of the results is given and possible applications to interstellar media are indicated.

2. Basic equation

A three component dusty plasma consisting of electrons, ions and charged dust grains, is assumed to have grains of equal radius and carrying identical charge due to the collisional attachment of the electrons and ions to the grain surface. Effects of the photoemission, secondary emission etc. is ignored in the present study. It should be mentioned here that this assumption is not entirely satisfactory for the thermal grains. The charging and the discharging of the grain give rise to sink and source terms in the continuity and momentum exchange equations. Owing to the large mass difference between the plasma and the grains, we assume that the gravitational potential is determined mainly by the grains. Then the dynamics of the dusty plasma is described by the following equations:

Continuity equations:

$$\frac{\partial n_\alpha}{\partial t} + \nabla\cdot(n_\alpha \mathbf{v}_\alpha) = -\beta_\alpha(n_\alpha - n_{\alpha 0}), \tag{1}$$

$$\frac{\partial n_{\rm g}}{\partial t} + \nabla\cdot(n_{\rm g}\mathbf{v}_{\rm g}) = 0, \tag{2}$$

where $\alpha = e, i$; g = grain. The subscript 0 means equilibrium quantities and $\beta_\alpha = I_\alpha n_{\rm g}/q_\alpha n_\alpha$ is the attachment frequency of the plasma particles to the grains.

Equations of motion:

$$m_\alpha n_\alpha \frac{\mathrm{d}\mathbf{v}_\alpha}{\mathrm{d}t} = -T_\alpha \nabla n_\alpha - q_\alpha n_\alpha \nabla\phi - m_\alpha n_\alpha \nabla\psi - m_\alpha n_\alpha \beta_\alpha(\mathbf{v}_\alpha - \mathbf{v}_{\rm g}), \tag{3}$$

$$m_{\rm g} n_{\rm g} \frac{\mathrm{d}\mathbf{v}_{\rm g}}{\mathrm{d}t} = -T_{\rm g}\nabla n_{\rm g} - Q n_{\rm g}\nabla\phi - m_{\rm g} n_{\rm g}\nabla\psi. \tag{4}$$

Charge equation:

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = I_e(Q,\phi) + I_i(Q,\phi), \tag{5}$$

where the currents are given by [32, 33]

$$I_e = -\pi a^2 e\sqrt{\frac{8T_e}{\pi m_e}} n_e \exp\left[\frac{e(\phi_{\rm g} - \phi)}{T_e}\right], \tag{6}$$

$$I_i = \pi a^2 e\sqrt{\frac{8T_i}{\pi m_i}} n_i \left[1 - \frac{e(\phi_{\rm g} - \phi)}{T_i}\right] \tag{7}$$

and the two Poisson's equation are

$$\nabla^2\phi = 4\pi e[n_e - n_i] - 4\pi Q n_{\rm g}, \tag{8}$$

$$\nabla^2\psi = 4\pi G n_{\rm g} m_{\rm g}, \tag{9}$$

where $\mathrm{d}/\mathrm{d}t = \partial/\partial t + \mathbf{v}\cdot\nabla$ and m, n, v, T, ϕ, ψ denote the mass, number density, velocity, temperature, electrostatic and gravitational potential respectively. The quantity $\phi_{\rm g} - \phi$ is the potential difference between the grain surface potential and the background plasma potential so that the unperturbed charge Q_0 is given by $Q_0 = C(\phi_{\rm g} - \phi)$ where C is the capacitance of the grain and is given by [34], $C = a\exp(-a/\lambda_{\rm D})$ with a as the radius of the grain. The exponential factor reflects the screening of the grain charge by the plasma particles. In equilibrium, electron and ion currents equal each other and thus the grain is at the 'floating potential'. Note that for $n_{e0}/n_{i0} \approx 1$ (a situation relevant to most of the planetary system [10] and interstellar media [9]), $\beta_e = \beta_i$ i.e., electrons and ions get attached to the grain at the same rate after the initial build up of a negative charge on the grain similar to the behaviour of plasma particles near the sheath [35]. The above set of equations (1)–(9) form the basic set of equations for the linear analysis.

3. Linear analysis

We consider an infinite, homogeneous plasma and neglect the zeroth order electrostatic and gravitational potential. Whereas the neglect of zeroth order electrostatic potential is guaranteed by the existence of opposite charges, there is no such thing as opposite mass and thus, there is no way to make the gravitational potential disappear in zeroth order. Therefore, the problem of a self gravitating system is in general an eigenvalue problem. However, important physical insight can be gained by invoking 'Jeans Swindle' i.e ignoring the zeroth order potential field of gravity. To reiterate, we consider the linear gravitational instability of an infinite, homogeneous plasma characterized by

$$n_{e0} = n_{i0} + Zn_{g0}, \quad \psi_0 = \phi_0 = 0, \quad \mathbf{v}_{i0} = V_0\hat{x}, \quad \mathbf{v}_{e0} = \mathbf{v}_{g0} = 0,$$

where V_0 is the ion streaming velocity and $Z = Q/e$. The linearized set of equations are

$$\frac{\partial n_{\alpha 1}}{\partial t} + \nabla\cdot(n_{\alpha 1}\mathbf{v}_0 + n_{\alpha 0}\mathbf{v}_{\alpha 1}) = -\beta_{\alpha 0} n_{\alpha 1}, \tag{10}$$

$$\frac{\partial n_{g1}}{\partial t} + n_{g0}\nabla\cdot\mathbf{v}_{g1} = 0, \tag{11}$$

where the subscript 1 means perturbed quantities.

$$n_{\alpha 0}\left(\frac{\partial \mathbf{v}_{\alpha 1}}{\partial t} + \mathbf{v}_{\alpha 0}\cdot\nabla\mathbf{v}_{\alpha 1}\right) = -C_\alpha^2\nabla n_{\alpha 1} - \frac{q_\alpha n_{\alpha 0}}{m_\alpha}\nabla\phi_1 - n_{\alpha 0}\nabla\psi_1$$
$$- \beta_{\alpha 0} n_{\alpha 0}(\mathbf{v}_{\alpha 1} - \mathbf{v}_{g1}) - \beta_{\alpha 1} n_{\alpha 0}\mathbf{v}_{\alpha 0} - \beta_{\alpha 0} n_{\alpha 1}\mathbf{v}_{\alpha 0}, \tag{12}$$

$$\frac{\partial \mathbf{v}_{g1}}{\partial t} = -C_g^2\frac{\nabla n_{g1}}{n_{g0}} - \frac{Q_0}{m_g}\nabla\phi_1 - \nabla\psi_1, \tag{13}$$

$$\frac{\partial Q_1}{\partial t} + \eta Q_1 = |I_{e0}|\left(\frac{n_{i1}}{n_{i0}} - \frac{n_{e1}}{n_{e0}}\right), \tag{14}$$

where $\eta = (e|I_{e0}|/C)(1/T_e + 1/w_0)$, $w_0 = T_i - e\phi_{f0}$ with T_i as the ion temperature and ϕ_{f0} as the equilibrium floating potential of the grains and $C_\alpha = (T_\alpha/m_\alpha)^{1/2}$ is the thermal velocity of the αth species and $\beta_{\alpha 1}$ is obtained from the definition of β_α by perturbing it around $\beta_{\alpha 0}$. Taking the spatial and temporal dependence of all perturbed quantities as $\exp[-i(\omega t - kx)]$, we get the following relation between fluctuation density and potentials:

$$[\omega^2 + \omega_J^2 - k^2C_g^2]\left(\frac{n_{g1}}{n_{g0}}\right) = -k^2\phi_1\left(\frac{Q}{m_g}\right), \tag{15}$$

$$[(\omega + i\beta_e)^2 - k^2C_e^2]\frac{n_{e1}}{n_{e0}} = -k^2\phi_1\left[\left(\frac{e}{m_e}\right) + \frac{\omega_J^2 - i\beta_e\omega}{\omega^2 + \omega_J^2 - k^2C_g^2}\left(\frac{Q}{m_g}\right)\right] \tag{16}$$

and

$$\left[(\bar{\omega} + i\beta_i)^2 - k^2C_i^2 + i\beta_i kV_0 + \chi\beta_i^2\frac{n_i}{Zn_g}\left(\frac{kV_0}{\omega + i\eta}\right)\right]\frac{n_{i1}}{n_{i0}} = k^2\phi_1$$
$$\times\left[\left(\frac{e}{m_i}\right) - \frac{\omega_J^2 - i\beta_i\bar{\omega}}{\omega^2 + \omega_J^2 - k^2C_g^2}\left(\frac{Q}{m_g}\right) - \chi\beta_i^2\frac{n_i}{Zn_g}\left(\frac{kV_0}{\omega + i\eta}\right)\frac{\left[\left(\frac{e}{m_e}\right) + \frac{\omega_J^2 - i\beta_e\omega}{\omega^2 + \omega_J^2 - k^2C_g^2}\left(\frac{Q}{m_g}\right)\right]}{(\omega + i\beta_e)^2 - k^2C_e^2}\right], \tag{17}$$

where use has been made of $\beta_{i1} = \beta_i((n_{g1}/n_{g0}) - \chi(Q_1/Q))$, $\beta_{e1} = \beta_e((n_{g1}/n_{g0}) + \chi(Q_1/Q))$ with $\chi = 1/((T_i/e\phi_{f0}) - 1)$ [14]. Then plugging equations (15)–(17) in the Poisson's equation leads to the following dispersion relation:

$$1 = \left(1 + \frac{i\beta_e}{\omega + i\eta}\right)A + \left(1 + \frac{i\beta_i}{\omega + i\eta}\right)B + \frac{\omega_{pg}^2}{\omega^2 + \omega_J^2 - k^2C_g^2}, \tag{18}$$

where

$$A = \frac{\omega_{pe}^2 + \frac{n_e}{Zn_g}\frac{\omega_{pg}^2}{\omega^2+\omega_J^2-k^2C_g^2}[\omega_J^2 - i\beta_e\omega]}{(\omega + i\beta_e)^2 - k^2C_e^2}, \tag{19}$$

$$B = \frac{\omega_{pi}^2 - \frac{n_i}{Zn_g}\frac{\omega_{pg}^2}{\omega^2+\omega_J^2-k^2C_g^2}[\omega_J^2 + i\beta_i\bar{\omega}] - \chi\beta_i^2\frac{n_i}{Zn_g}\left(\frac{kV_0}{\omega+i\eta}\right)\left[\delta\omega_{pe}^2 + \frac{n_i}{Zn_g}\frac{\omega_{pg}^2}{\omega^2+\omega_J^2-k^2C_g^2}(\omega_J^2 - i\beta_e\omega)\right]}{\left[(\bar{\omega} + i\beta_i)^2 - k^2C_i^2 + i\beta_i kV_0 + \chi\beta_i^2\frac{n_i}{Zn_g}\left(\frac{kV_0}{\omega+i\eta}\right)\right]}, \tag{20}$$

where $\bar{\omega} = \omega - kV_0$ is the Dopler shifted frequency, $\omega_{p\alpha}^2 = 4\pi n_\alpha q_\alpha{}^2/m_\alpha$ is the plasma frequency of the αth species and $\delta = n_{e0}/n_{i0}$. Subscript 0 has been dropped from the equilibrium quantities. The above dispersion relation in the limit $\omega_{pg} \to 0$, $k\lambda_e \ll 1$, reduces to equation (18) of Bhatt and Pandey [14]. First we analyse the gravitational instability of a dusty plasma when the plasma particles are in thermal equilibrium. For this case the thermal response of the plasma particles is much faster than the grain's response to the perturbations i.e., $kC_g \ll \omega \ll kC_i \le kC_e$ where k is the wave number. Assuming $\beta_e = \beta_i = \beta$ we get the following dispersion relation:

$$\omega^2 = \frac{k^2\lambda_D^2\omega_{pg}^2}{1 + k^2\lambda_D^2 + \frac{i\beta}{\omega+i\eta}} - \omega_J^2 + k^2C_g^2, \tag{21}$$

where $\lambda_D = \lambda_e\lambda_i/[\lambda_e^2 + \lambda_i^2]^{1/2}$ is the effective plasma Debye length with $\lambda_{e,i} = (T_{e,i}/4\pi n_{e,i}e^2)^{1/2}$. The dispersion relation (21) coincides with that of Mohanta *et al* [31] in the absence of an external magnetic field for a cold dust grain. Before undertaking a complete numerical analysis of the above dispersion relation, the effect of the charge fluctuation can be studied by solving equation (21) perturbatively by treating $\eta \sim \beta < \omega$. Then the interesting root is

$$\frac{\omega}{\omega_{pg}} = i\left[R - \left(k^2\lambda_g^2 + \frac{k^2\lambda_D^2}{1 + k^2\lambda_D^2}\right)\right]^{1/2} + \frac{i\beta}{2}\frac{k^2\lambda_D^2}{(1 + k^2\lambda_D^2)^2}\frac{1}{\left[R - \left(k^2\lambda_g^2 + \frac{k^2\lambda_D^2}{1+k^2\lambda_D^2}\right)\right]}, \tag{22}$$

where $\lambda_g^2 = T_g/4\pi n_g Q_0^2$ is the Debye length of the grains. From the above expression it is clear that the attachment of the grains causes enhancement of the gravitational condensation for some $0 < k < k_J$, where k_J is the Jeans wave number at which the right hand side of equation (21) goes to zero in the absence of charge fluctuations, i.e., $\beta = 0$. As the wave number crosses the Jeans wave number, the first term in equation (22)

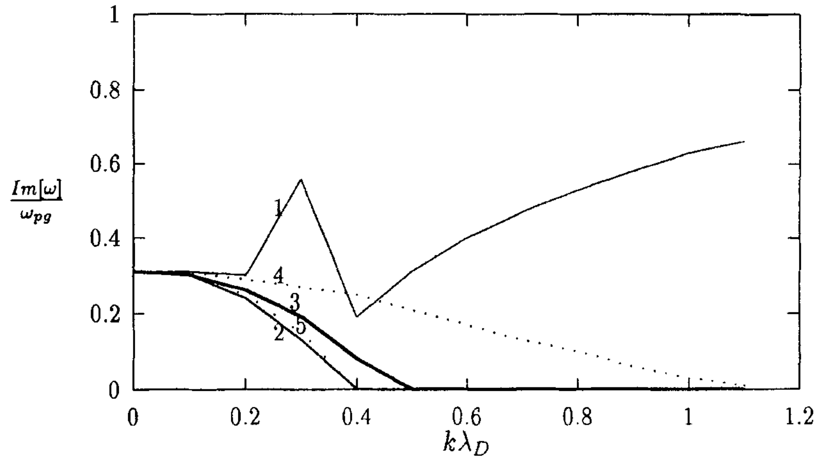

Figure 1. Curve 1 corresponds to the perturbative solution for $\beta/\eta \sim 1$ and $x2 = \beta/\omega_{pg} = 0.1$. Curves 2, 3, 4 and 5 shows the Jeans mode for $x1 = \eta/\omega_{pg} = 0$, $x2 = 0$; $x1 = 0.1$, $x2 = 0.1$ and $x1 = 0.1$, $x2 = 1$ and $x1 = 1$, $x2 = 0.1$ respectively. Here $\epsilon = 10^{-6}$, $R = 0.1$.

becomes real and thus it describes the usual dust acoustic wave [36]. Around $k = k_{\rm J}$ the perturbative analysis breaks down (see curve 1 of figure 1; the parameters for which the plot is drawn are given below). Qualitatively one can explain these features as follows: Charge fluctuations damp the dust acoustic waves. The rate of damping is maximum at certain values of k and reduces to zero for both higher and lower k values. This will cause the shift in the balance of force in favour of gravity for some range of k. As a result, gravitational instability becomes robust and the Jeans length reduces compared to the fixed charge case.

Next, we analyse equation (21) numerically. We solve equation (21) for $\epsilon = \lambda_{\rm g}^2/\lambda_{\rm D}^2 = 10^{-6}, \eta/\omega_{pg} = (0 - 0.1)$ and $\beta/\omega_{pg} \sim (0 - 1), R = 0.1$, numbers representative of the interstellar medium [10]. From figure 1 (curves 2 and 3) we see that in the absence of charge fluctuation, the growth rate goes to zero at smaller $k\lambda_{\rm D}$ than in the presence of charge fluctuation, implying that the condensation of the grain is facilitated by charge fluctuations. An increase in the attachment frequency ($\beta \gg \eta$) leads to the enhancement of the growth rate (figure 1, curve 4), since attachment of the plasma particles favourably reduces the repulsive potential well of the grain, facilitating the condensation. Similarly, an increase in η implies that more plasma particles are seen in the vicinity of a collapsing grain, repelling the incoming grain in the process (curve 5). Thus, the growth rate decreases. With the increase of R the growth rate will increase, as it is the measure of relative strength of the attraction to the repulsion.

Finally we calculate the Jeans length from equation (21). Jeans length is the critical length at which the instability vanishes. Then solving equation (21) for the marginal stability condition, we get

$$\lambda_{\rm J} = \lambda_{\rm g} F\left(R, \epsilon, \frac{\beta}{\eta}\right) \tag{23}$$

and

$$F\left(R,\epsilon,\frac{\beta}{\eta}\right)=\left[\frac{2}{-\epsilon\left(1+\frac{\beta}{\eta}\right)+(R-1)+\left(\left(\epsilon\left(1+\frac{\beta}{\eta}\right)-(R-1)\right)^2+4\epsilon R\left(1+\frac{\beta}{\eta}\right)\right)^{1/2}}\right]^{1/2}, \tag{24}$$

where $\epsilon = \lambda_g^2/\lambda_D^2$. For $R \sim 1$,

$$F\left(R,\epsilon,\frac{\beta}{\eta}\right)\approx\begin{cases}1 & \epsilon\sim 1\\ \left[\frac{1}{\epsilon\left(1+\frac{\beta}{\eta}\right)}\right]^{1/4} & \epsilon\ll 1.\end{cases} \tag{25}$$

From the above limiting expression for F it is clear that when $\epsilon \sim 1$, grain charge fluctuation plays no role in the gravitational collapse and $\lambda_J = \lambda_g$. Therefore, the gravitational condensation of the grain can be halted only by the repulsive electrostatic field for a fixed charge. However when $\epsilon \ll 1$ Jeans length is considerably modified by the charge dynamics of the grain. Depending upon the value of β/η the Jeans length can decrease or increase. For $\epsilon = 0.1$ we see from figure 2 that λ_J/λ_g initially decreases (in comparison with the value of λ_J/λ_g in the absence of charge fluctuations) and then after $\beta/\eta = 5.5$ it shoots up. Therefore, for grainy plasma whose Debye length is much smaller than the Debye length of the plasma particle, charge fluctuations can facilitate or oppose the collapse of the grain by setting up the electrostatic field in phase or out of phase with the gravitational field. Above limiting cases can be applicable to different regions in interplanetary and interstellar media. For example, it is known that dark molecular clouds (DMC) contain grains of different charges depending upon the ambient plasma temperature [37]. If the ambient temperature is between 0–10 K then the grain may have

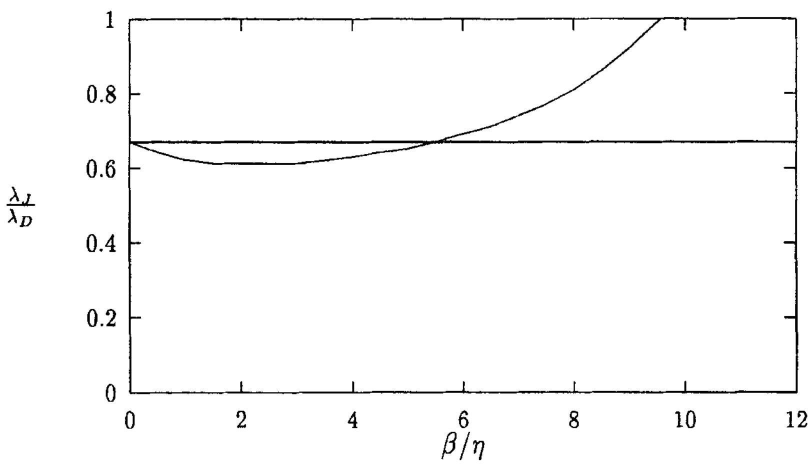

Figure 2. Normalized Jeans length is plotted against β/η for $\epsilon = 0.1$ and $R = 1$. The horizontal curve is the value of normalized Jeans length in the absence of charge fluctuation.

$0, +1, -1$ charge whereas for temperature around 100 K a grain may have 10 electronic charges. As ϵ may become $O(1)$ in DMC [9] the first of the two cases may be relevent. The above analysis indicates that for grains charge flip-flopping between $-1, 0, +1$, i.e., $\beta \sim \eta$ or for grains containing 10 or more charges $\beta > \eta$, the charge fluctuation is of no consequence to the gravitational condensation of the grains. In a planetary environment $\epsilon \ll 1$ and typically $Q \sim (10^3 - 10^6)e$ [5, 10]. Then the charge fluctuations can cause decrease as well as increase in the Jeans length. As a result charge fluctuations can facilitate or inhibit the condensation of the grains depending on the value of β/η. Therefore we see that for $\lambda_g \ll \lambda_D$ gravitational condensation of the grain can be facilitated or halted by an electrostatic field much beyond the length suggested by the analysis of Pandey *et al* [27]; Pandey and Dwivedi [30] for a fixed charge. The preliminary nature of the above results must be noted as they have been carried out for very idealized situations. For a final application of these results to relevant physical problems (viz. star formation, formation of the planetary system etc.), a model calculation must be done.

4. Jeans–Buneman mode

First, we shall consider a two component dusty plasma with streaming ions. This is an ideal situation where all the electrons are attached to the grains. This type of plasma may exist in some of the regions of dense interstellar clouds, where Zn_g may become comparable to n_i [38]. Dynamics of such a plasma has been considered [39] in the context of a tearing instability in the Saturnian rings. Then the dispersion relation equation (18) reduces to

$$\left[1 - \frac{\omega_{pg}^2}{\omega^2 + \omega_J^2 - k^2C_g^2}\right] D = \left[1 + \frac{i\beta_i}{\omega + i\eta}\right]\left[\omega_{pi}^2 - \frac{\omega_J^2\omega_{pg}^2}{\omega^2 + \omega_J^2 - k^2C_g^2} - \frac{i\beta_i\bar{\omega}\omega_{pg}^2}{\omega^2 + \omega_J^2 - k^2C_g^2}\right], \quad (26)$$

where

$$D = [(\bar{\omega}^2 + 2i\beta_i\bar{\omega}) - k^2C_i^2 - i\beta_i kV_0].$$

Before numerically solving the above 5th order polynomial, we analyse equation (26) in the absence of charge fluctuations. Then the dispersion relation becomes

$$[\omega^2 + \omega_J^2 - k^2C_g^2 - \omega_{pg}^2][(\bar{\omega}^2 - k^2C_i^2 - \omega_{pi}^2)] = \omega_{pg}^2(\omega_{pi}^2 - \omega_J^2), \quad (27)$$

and in the absence of gravity we have the usual Buneman mode [40]. Here we look for a mode in the vicinity of $kV_0 \approx (k^2C_i^2 + \omega_{pi}^2)^{1/2}$ in the presence of gravity. The unstable Jeans–Buneman root is

$$\omega \approx \left(\frac{1 + i(3)^{1/2}}{2}\right)\left(\frac{\omega_J^2\omega_{pi}}{2}\right)^{1/3} \quad (28)$$

provided $\omega_J^2 \approx \omega_{pg}^2$. The significance of this mode underlies the fact that even when self attraction is completely annulled by self repulsion, the Jeans–Buneman mode may trigger

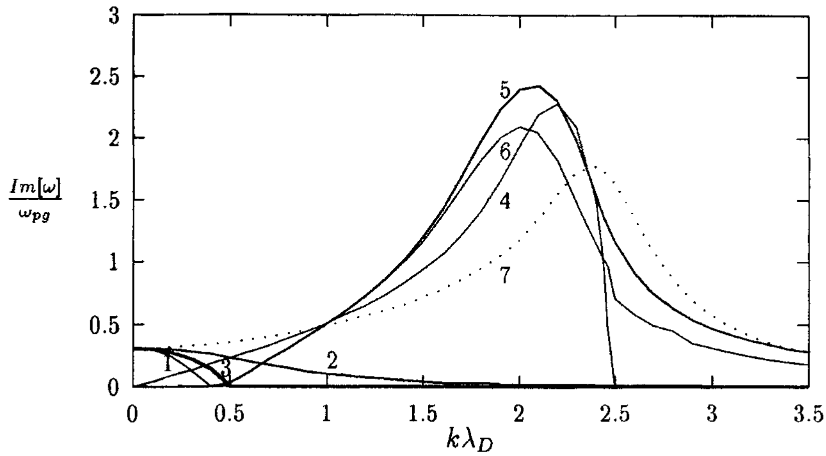

Figure 3. Curves 1, 2 and 3 are the Jeans mode with $x1 = 0, x2 = 0$; $x1 = 0, x2 = 1$ and $x1 = 1, x2 = 1$ respectively for $R = 0.1$. Curves 4, 5 and 6 shows Buneman mode for $\alpha = V_0/C_i = 1.1$, $\epsilon = 10^{-4}$, $R = 0$ with $x1 = 0$, $x2 = 0$; $x1 = 0$, $x2 = 1$ and $x1 = 1, x2 = 1$ respectively and curve 7 shows the Jeans–Buneman mode for $x1 = 1$, $x2 = 1$ with $R = 0.1$, $\alpha = 1.1$.

off a large scale condensation of the grains. The growth rate is given by

$$\Gamma \sim 2.10^{-5}\left[\frac{m_g n_g n_i^{1/2}}{m_i^{1/2}}\right]^{1/3} \text{Hz}, \tag{29}$$

where, n, m are measured in cgs units. The importance of the above mode can be gauged from the following example. It is known that the self-gravitating clouds of masses $10^2 - 10^6 M_\odot$, where $M_\odot$ is the solar mass, contain many dark cores with number densities in excess of $10^4\,\text{cm}^{-3}$ and masses $\leq 10^2 M_0$ [9]. In these dark cores, nearly one per cent of the mass is in dust form. In such cores, typically $n_g \approx 4 \times 10^{-12}(a/10^{-5})n_H$ which for hydrogen density n_H ranging between $(10^3\,\text{cm}^{-3} - 10^7\,\text{cm}^{-3})$ and cm sized grain becomes $(1\,\text{cm}^{-3} - 10^{-4}\,\text{cm}^{-3})$. Calculating ion density from $n_i/n_H \sim 10^{-8}$ $(n_H/10^5\,\text{cm}^{-3})^{-1/2}$, we get $n_i \sim (10^{-2}\,\text{cm}^{-3} - 10^{-4})\,\text{cm}^{-3}$. Assuming ions to be ionized hydrogen we get $\Gamma \sim 10^{-3}$. Thus we see that the Jeans–Buneman mode may play an important role in the formation of large scale structures.

Next, we solve the dispersion relation (26) numerically. We see from figure 3 that at large scale lengths only the Jeans mode operates, whereas at small scale lengths the Buneman mode is excited. There is an overlap of the two modes. The growth rate of the Jeans mode is smaller than that of the streaming mode. Charge fluctuations affect both gravitational and Buneman modes. In the presence of charge fluctuation, the growth rate of the Jeans mode increases and drops to zero at large $k\lambda_D$. The Buneman mode, on the other hand, starts operating at much shorter wavelengths in the presence of charge fluctuations. The area of overlap of the Jeans and Buneman modes shrinks in the presence of charge dynamics. At $k\lambda_D \ll 1$ the gravitational instability dominates over the streaming instability. In the other limit, i.e., $k\lambda_D \gg 1$, it is the streaming instability which destabilizes the grain plasma system. In the very narrow window $k\lambda_D \sim O(1)$, the two

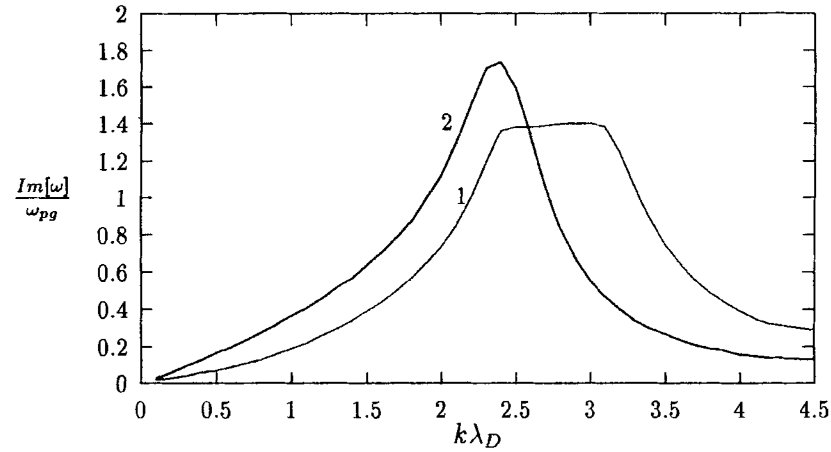

Figure 4. Curves 1, 2 show the charge fluctuation induced unstable mode for $R = 0$, $\alpha = 1.1, \epsilon = 10^{-6}$ with $x1 = 0, x2 = 1$ and $x1 = 1, x2 = 1$ respectively.

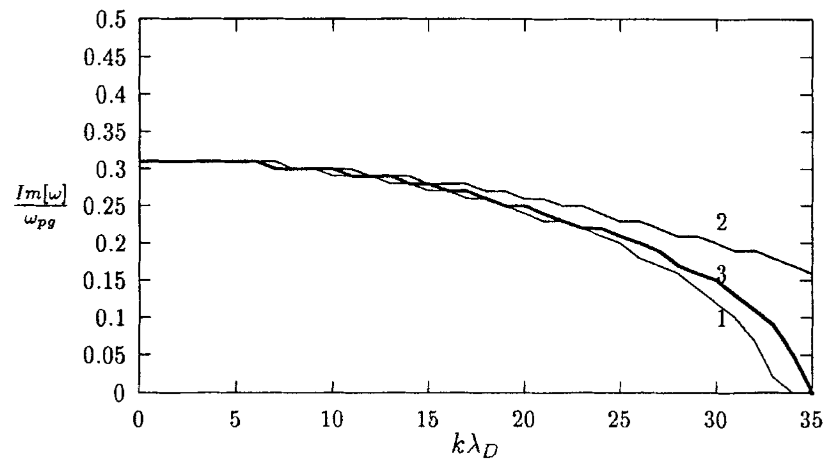

Figure 5. Curves 1, 2 and 3 shows Jeans mode for a three component plasma with $x1 = 0$, $x2 = 0$; $x1 = 0$, $x2 = 1$ and $x1 = 1$, $x2 = 1$ respectively for $R = 0.1$. Here $\epsilon = 10^{-6}$, $\delta = 0.01$, $n_i/Zn_g = 0.99$, $\lambda_i^2/\lambda_e^2 = 10^{-4}$, $\omega_{pe}^2/\omega_{pg}^2 = 10^3$, $\omega_{pi}^2/\omega_{pg}^2 = 10^2$.

modes overlap. In the absence of gravity, in addition to the Buneman mode, we find a new charge fluctuation induced unstable mode existing in a streaming plasma as shown in figure 4. The growth rate of this mode is smaller than that of the Buneman mode. This mode appears analogous to the negative energy mode reported earlier by Jana *et al* [13]. Physically as $\alpha = V_0/C_i > 1$, the ion wave can be viewed as a negative energy wave from the ion frame, so that dissipative effects associated with nonstreaming grains can give rise to its amplification.

Next, we study the stability of a three component plasma. We solve the dispersion relation (18) numerically for Boltzmannian as well as non-Boltzmannian electrons. The inclusion of electron dynamics does not affect the result significantly and the change in

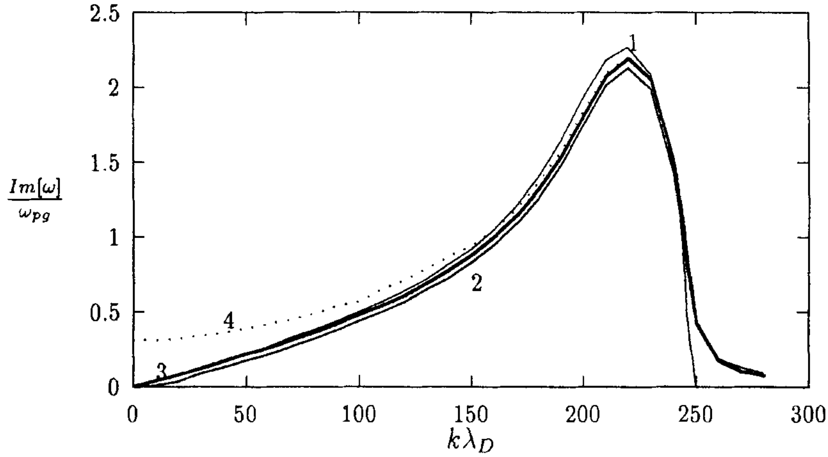

Figure 6. Curves 1, 2 and 3 shows the Buneman mode for $R = 0$ with $x1 = 0, x2 = 0$; $x1 = 0, x2 = 1$ and $x1 = 1, x2 = 1$ respectively. Here $\epsilon = 10^{-6}$, $\delta = 0.01$, $n_i/Zn_g = 0.99$, $\chi = 10^{-6}$, $\lambda_i^2/\lambda_e^2 = 10^{-4}$, $\omega_{pe}^2/\omega_{pg}^2 = 10^3$, $\omega_{pi}^2/\omega_{pg}^2 = 10^2$. Curve 4 depicts Jeans–Buneman mode for $R = 0.1$, $\alpha = 1.1$.

the result is only 1%. In the parameter regime studied by us, last term in the numerator of equation (18) is unimportant. However, there exists the parameter window in which contribution of this term is comparable with the first two terms of the numerator.

In figure 5 we plot the Jeans mode with and without charge fluctuations. As is evident from the figure, the modification due to charge fluctuations in the Jeans mode for a three component plasma is similar to a two component plasma. Figure 6 depicts the Buneman mode and also the Jeans–Buneman mode. The qualitative features are similar to the previous mode. We find that charge fluctuation in the presence of streaming ions also gives rise to negative energy mode. The behaviour of this mode is similar to that described in figure 4 for a two component plasma. We mention here that this result is at variance with the result of Bhatt and Pandey [14] who found perturbatively that the parameter regime for negative energy mode shrinks to zero in a three component plasma when $\delta \approx 1$.

5. Conclusion

We have analysed the Jeans instability of a dusty plasma self consistently by properly considering the charge dynamics. It has been shown that charge fluctuations cause a modification of the Jeans length. The charge fluctuation can facilitate or hinder the condensation of the grains depending upon the value of β/η. Implication of this result on Spoke formation in the Saturn's ring or on the thickness of the Jovian rings may be profound. For example, as the charge fluctuations modifiy the Jeans mode, the lifetime of self-levitation may be finite before the grains finally succumb to their self-gravity. In the absence of charge fluctuations the levitated grains will forever be hanging above the ring plane of the planets. In the interstellar media, even highly charged grains (e.g. $\sim 100e$ in the HII region) will undergo an enhanced condensation due to the dynamic nature of the grain charge. Therefore, our analysis indicates that charge fluctuations might help the gravitational condensation of the charged grains in different cosmic environment.

Analysis of two and three component dusty plasma do not qualitatively show different behaviour. The Buneman and the Jeans modes both act in quite distinct zone with a very narrow overlap region in the presence of charge fluctuations. For a two component plasma, we find an analytical expression for the Jeans–Buneman mode. The growth rate of streaming instability is always higher than the Jeans instability. Therefore, the existence of a mixed Jeans–Buneman mode will cause faster condensation of the grainy plasma than the purely gravitational instability. A new charge fluctuation induced unstable mode is found from the numerical solution of the dispersion relation. It might be possible that when the Buneman mode saturates, the charge fluctuation induced mode will be of significance.

Based on the above analysis one can draw a general picture of the collapse of plasma matter. It might be possible that the collapse of the plasma matter may undergo several stages: Far away from the gravitating centre, gravitational attraction may trigger off a collapse. The collapse may continue up to a certain scale length (say) L_1 at which stage plasma particles (namely ions) may develop a considerable flow ($\alpha > 1$) which results in the enhancement of the collapse rate. In the final stage which will be reached after a certain scale length $L, L \ll L_1$ streaming ions go on accreting to the grains solely due to their flow. Thus, very close to the gravitating centre, its a purely Buneman mode which facilitates the condensation of the matter.

It is known that massive objects surrounded by a charged plasma cloud (e.g. HII region) will tend to accrete matter. The resultant flow of the plasma can be of importance for stars in close binary systems [32], for compact objects (white dwarfs, neutron stars, black hole etc.) which have been proposed as the possible energy source for the radiation coming out from a very localized region of the sky [41]. The accretion rate is calculated purely by the gravitational field. However, as one sees from the above analysis, the accretion rate will become a function of the scale length. Very close to the gravitating centre the Jeans mode might be superseded by a Buneman mode and one gets an enhanced growth rate. The presence of a stellar wind in the interstellar medium supports such a picture. However, our analysis is for a very simplified model and thus the results are suggestive rather than compelling.

As a variety of plasmas is found in interstellar media, we believe that our results have bearing on star formation, cluster formation etc. Charge fluctuations affect the condensation of the grains and thus, it might be possible that the rate of star formation in the galaxies are different than the presently predicted rates. Further work in this direction is required.

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References

[1] E R Wollman, *Astrophys. J.* **392**, 80 (1992)
[2] V B Il'in and N V Voschennikov, *Astron. Rep.* **37**, 362 (1993)
[3] B G Anandrao, S R Pottasch and D B Vaidya, *Astron. Astrophys.* **273**, 570 (1993)

[4] H Netzer, *Active Galactic Nuclei*, Saas-Fee Advanced course 20, Lecture Note 1990, Swiss Society for Astrophysics and Astronomy edited by T J-L Courvoisier and M Mayor (Springer-Verlag, Berlin)
[5] C K Goertz, *Rev. Geophys.* **27**, 271 (1989)
[6] O Havnes, *Astron. Astrophys.* **193**, 309 (1988)
[7] M Horyani and C K Goertz, *Astrophys. J.* **361**, 155 (1989)
[8] U de Angelis, *Phys. Scr.* **45**, 465 (1992)
[9] T W Hartquist, O Havnes and G E Morfill, *Fund. Cosmic Plasma* **15**, 167 (1992)
[10] D A Mendis and M Rosenberg, *Ann. Rev. Astron. Astrophys.* **32**, 419 (1994)
[11] F Verheest, *Space Sci. Rev.* (1996) (in press)
[12] R K Varma, P K Shukla and V Krishan, *Phys. Rev.* **E50**, 1422 (1994)
[13] M R Jana, P K Kaw and A Sen, *Phys. Rev.* **E48**, 3930 (1994)
[14] J R Bhatt and B P Pandey, *Phys. Rev.* **E50**, 3980 (1994)
[15] S V Vladmirov, *Phys. Plasmas* **1**, 2762 (1994); *Phys. Rev.* **E50**, 1422 (1994)
[16] F Verheest, *Planet. Space Sci.* **42**, 883 (1994)
[17] R V Reddy, G S Lakhina, F Verheest and P Meuris, *Planet Space Sci.* **44**, 129 (1996)
[18] C B Dwivedi and B P Pandey, *Phys. Plasmas* **2**, 4134 (1995)
[19] F Verheest and P Meuris, *Phys. Lett.* **A198**, 228 (1995)
[20] B P Pandey and C B Dwivedi, *Pramana – J. Phys.* **45**, 255 (1995)
[21] A S Eddington, *Internal Constitution of the Stars* (Cambridge University Press, Cambridge, England, 1926)
[22] T Mukai, *Evolution of interstellar dust and related topics* in Proceedinge of the International School of Physica "Enrico Fermi", Course CI, Verenna, 1986 edited by A Bonetti, J M Greenberg and S Aiellu (North Holland, Amsterdam, 1989) p. 357
[23] S Barbash and R Lundin, *IEEE Trans. Plasma Sci.* **22**, 173 (1994)
[24] J Binney and S Tremaine, *Galactic Dynamics* (Princeton University Press, Princeton, NJ, 1988)
[25] H Alfven and A D Mendis, *Adv. Space Res.* **3**, 95 (1983)
[26] G Gisler, Q R Ahmad and E R Wollman, *IEEE Trans. Plasma Sci.* **20**, 922 (1992)
[27] B P Pandey, K Avinash and C B Dwivedi, *Phys. Rev.* **E49**, 5599 (1994)
[28] R Chhajalani and A Parihar, *Astrophys. J.* **422**, 746 (1994)
[29] K Avinash and P K Shukla, *Phys. Lett.* **A189**, 470 (1994)
[30] B P Pandey and C B Dwivedi, *J. Plasma Phys.* **55**, 395 (1996)
[31] L Mohanta, B J Saikia, B P Pandey and S Bujarbarua, *J. Plasma Phys.* **55**, 401 (1996)
[32] L Spitzer, *Physical Processes in the Interstellar Medium* (John Wiley, New York, 1978)
[33] B T Draine and E E Salpeter, *Astrophys. J.* **15**, 77 (1979)
[34] E C Whipple, T G Northrop and D A Mendis, *J. Geophys. Res.* **90**, 7405 (1985)
[35] V E Golant, A P Zhilinsky and I E Shakharov, *Fundamentals of Plasma Physics* (John Wiley, New York, 1980)
[36] F Melandso, T Aslaksen and O Havnes, *J. Geophys. Res.* **98**, 13315 (1993)
[37] B G Elmgreen, *Astrophys. J.* **232**, 729 (1979)
[38] T Umebayashi and T Nakano, *Publ. Astron. Soc. Jpn.* **32**, 405 (1980)
[39] H L F Houpis and D A Mendis, *The Moon and Planets* **29**, 39 (1983)
[40] O Buneman, *Phys. Rev.* **115**, 503 (1959)
[41] J Frank, A R King and D J Raine, *Accretion power in astrophysics* (Cambridge University Press, Cambridge, 1985)