PRAMANA
—journal of
physics

© Printed in India

Vol. 50, No. 2,
February 1998
pp. 163–171

Triviality bound on lightest Higgs mass in next to minimal supersymmetric model

S R CHOUDHURY, MAMTA and SUKANTA DUTTA
Department of Physics and Astrophysics, University of Delhi, Delhi 110 007, India
E-mail: src@ducos.ernet.in; mamta@ducos.ernet.in; sukanta@ducos.ernet.in

MS received 1 April 1997; revised 18 October 1997

Abstract. We study the implication of triviality on Higgs sector in next to minimal supersymmetric model (NMSSM) using variational field theory. It is shown that the mass of the lightest Higgs boson in NMSSM has an upper bound $\sim 10\, M_{\rm W}$ which is of the same order as that in the standard model.

Keywords. Higgs; NMSSM; triviality; variational.

PACS Nos 14.80; 11.15; 12.60; 11.30

1. Introduction

It is now widely believed that the ϕ^4-scalar theory in four space-time dimensions is trivial. Consider the Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{4}m_0^2\phi^2 - \frac{1}{4!}\phi^4 \tag{1}$$

relevant to the scalar sector of standard model (SM). The evolution of the renormalized coupling constant $\lambda(M)$ at the scale M for large M, in the one loop approximation [1] is given by the equation

$$\frac{1}{\lambda(M)} = \frac{1}{\lambda(m)} - \frac{3}{16\pi^2}\ln\left(\frac{M}{m}\right), \tag{2}$$

where m is some low energy reference scale. From (2), it is clear that $\lambda(M)$ diverges at the scale $M = \Lambda_{\rm L}$, where

$$\Lambda_{\rm L} = m\exp(16\pi^2/3\lambda(m)). \tag{3}$$

This is the 'Landau catastrophe'. The occurence of Landau catastrophe at any finite energy can thus be avoided only if $\lambda(m) = 0$, which is the standard triviality scenario. One can however regard the Lagrangian (1) as an effective one; for a given $\lambda(m)$, the Lagrangian (1) then is a good description of the theory at most up to the energy scale $\Lambda_{\rm L}$ as given by (3). The coupling constant $\lambda(m)$ can be related to Higgs mass $M_{\rm H}$. It is reasonable to expect that for a consistent effective theory $M_{\rm H}$ should not exceed the scale

Λ_L. This leads to bounds on Higgs mass. This method has been used by several authors [2, 6].

An alternate and in our opinion more transparent approach to the 'triviality' problem in ϕ^4-theories has been provided by Stevenson using variational methods for calculating the effective potential [3]. In Stevenson's approach, the relation between the bare and renormalized coupling constant λ_{REN} (defined as the fourth derivative of the effective potential at the origin and thus different from renormalized coupling mentioned above) is

$$\lambda_{\text{REN}} = \lambda_0 \frac{[1 - 12\lambda_0 I_{-1}(m_R)]}{[1 + 6\lambda_0 I_{-1}(m_R)]}, \tag{4}$$

where m_R^2 is the second derivative of the effective potential at the origin and

$$I_{-1}(m) = \int \frac{\mathrm{d}^3 k}{2(2\pi)^3(\sqrt{k^2 + M^2})^3}. \tag{5}$$

Stevenson showed that in the limit of infinite cut-off, the ground state is unstable (i.e. the energy is unbounded from below) for any positive finite λ_0. This result goes against our naive expectation since the tree level effective potential is bounded from below. However, eq. (4) clearly shows that for infinite cut-off $\lambda_{\text{REN}} = -2\lambda_0$, and this is the reflection of the energy of the ground state being unbounded from below (see eq. (4.24) of ref. [3]). Then for a stable ground state, λ_0 and λ_{REN} both tend to zero for infinite cut-off, which is just a trivial theory; we thus arrive at the conclusion identical to the one referred to earlier. Once again, if we regard the ϕ^4-theory as an effective one, λ_0 can have value up to some maximum below which a stable ground state is possible. For a given coupling constant the Higgs masses can be calculated and for the effective theory to be physically meaningful we should have all these masses sufficiently below cut-off Λ_c, say $\Lambda_c/5$ [5]. With such a restriction, one can establish that the Higgs quartic coupling cannot be arbitrarily large, i.e., it has a maximum allowed value, which translates into an upper bound for M_H/M_W.

Triviality bounds in scalar theories have been studied by various non-perturbative methods. These include RGE [5], improved perturbative [6], and lattice theoretic approach [7]. Results are similar, namely that M_H cannot be heavier than a value in the range 800 GeV–1 TeV [4]. Recently we have used a variational approach [8] to arrive at a similar result. This last approach is extremely simple and admits easy generalization to situations more complicated than SM.

Supersymmetric (SUSY) generalization of the SM have been studied in recent times [9]. The most economic SUSY-extension of the SM is a minimal (MSSM) one [9]. In this version, the quartic couplings are restricted by the gauge coupling with the result that the Higgs cannot be arbitrarily heavy. At tree level, one has the relationship

$$M_H^2 \leq M_Z^2 \cos^2 2\beta \leq M_Z^2, \tag{6}$$

where $\tan\beta$ is the ratio of the vacuum expectation values (VEV's) of the neutral components of the scalar fields H_2 and H_1 that the MSSM involves. Going beyond the tree approximation does not change (6) qualitatively. Thus, Quiros [10] gives the bound

$$M_H \leq 125\,\text{GeV} \tag{7}$$

for $m_t = 174\,\text{GeV}$ and a cut-off $\Lambda_c \approx 10^{19}\,\text{GeV}$.

An alternative supersymmetric model proposed is the next to minimal supersymmetric model (NMSSM) which has two $SU(2)\otimes U(1)$ Higgs doublets and one Higgs singlet [9]. The inclusion of a Higgs singlet is suggested in many superstring models and grand unified models. The NMSSM has more coupling parameters than the MSSM and hence it is an interesting theoretical question to enquire into the upper bounds on the Higgs spectrum, particularly the lightest of them. We expect this to be much higher than the one given in (7) and this is the subject matter of our investigation.

The method we follow here is a variational one. Starting with Hamiltonian of the NMSSM we use a gaussian trial wave functional for the ground state and obtain estimates of mass spectra in terms of the bare parameters of the theory. The strategy then is to vary bare parameters over their entire range, impose restrictions that the masses cannot get very close to the cut-off (say less than $\Lambda_c/5$) and obtain the highest mass of the lightest Higgs particle. The parameter space is however very large, and we will be making specific choices of parameters in the hope that our results will be typical of the model itself.

In the context of NMSSM, recently Wu [11] has analysed the problem (triviality bound of Higgs mass) on the basis of one-loop β function following the work of Dashen and Neuberger [2], imposing the restriction that the heaviest Higgs mass in the theory be smaller than the Landau pole position Λ_L. However his estimate of the Landau pole is based on one-loop result of the β function which is questionable since the coupling constant becomes very large in the neighbourhood of the Landau pole. Further the scalar masses have all been calculated on the basis of 'tree approximation' which again becomes uncertain for large coupling. Our variational method avoids both these problems, since the concept of triviality that we use following Stevenson [3] does not directly depend on the position of Landau pole. Further we determine all masses by variational rather than perturbative methods, which *a priori* does not restrict the coupling to be small.

2. The model

In the NMSSM, the potential of the Higgs sector [11] is

$$\begin{aligned} V =& |hN|^2(\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2)+|h\Phi_1^\dagger\Phi_2+\lambda N^2|^2+\tfrac{1}{8}g_1^2(\Phi_1^\dagger\Phi_1-\Phi_2^\dagger\Phi_2)^2 \\ &+\tfrac{1}{8}g_2^2[(\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2)^2-4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)]+m_1^2\Phi_1^\dagger\Phi_1+m_2^2\Phi_2^\dagger\Phi_2 \\ &-(m_3^2\Phi_1^\dagger\Phi_2+\text{h.c.})+m_4^2N^*N+m_5^2(N^2+N^{*2}). \end{aligned} \tag{8}$$

Here $\Phi_1=(\phi_1^\dagger,\phi_1^0)$ and $\Phi_2=(\phi_2^\dagger,\phi_2^0)$ are two $SU(2)\otimes U(1)$ doublets, N a complex singlet, m's are mass parameters, g's are gauge couplings and h, λ are Higgs couplings. The last five terms represent SUSY-breaking. Equation (8) has two coupling constants h and λ; we will study the strong coupling behaviour when h is very large and hence for simplicity we set $\lambda=0$. Also we take m_3 to be real and $m_1=m_2=m$ for simplicity.

It is more convenient to work with the fields defined by

$$\chi_{1,2}=\frac{1}{\sqrt{2}}(\Phi_1\pm\Phi_2). \tag{9}$$

Now the Higgs potential reduces to

$$V = h^2|N|^2(\chi_1^\dagger\chi_1 + \chi_2^\dagger\chi_2) + \tfrac{1}{4}h^2|\chi_1^\dagger\chi_1 - \chi_2^\dagger\chi_2 + \chi_2^\dagger\chi_1 - \chi_1^\dagger\chi_2|^2 + (m^2 - m_3^2)\chi_1^\dagger\chi_1 + (m^2 + m_3^2)\chi_2^\dagger\chi_2 + m_4^2 N^* N + m_5^2(N^2 + N^{*2}). \quad (10)$$

We now assume that the fields $\chi_{1,2}$, where

$$\chi_k \equiv \frac{1}{\sqrt{2}}\begin{pmatrix}\chi_{kR}^c + i\chi_{kI}^c \\ \chi_{kR}^0 + i\chi_{kI}^0\end{pmatrix}, \quad (k = 1,2) \quad (11)$$

(the superscripts c and 0 denoting charged and neutral components), break the $SU(2)\otimes U(1)$ symmetry by assuming a non-zero vacuum expectation value (VEV)

$$\langle\chi_1\rangle = \begin{pmatrix}0\\ v\end{pmatrix}, \quad \langle\chi_2\rangle = \begin{pmatrix}0\\ 0\end{pmatrix}. \quad (12)$$

All other fields in (10) are assumed to have zero VEV's. Writing $N = 1/\sqrt{2}(N_1 + iN_2)$, there are ten real fields in (10): $\chi_{1R}^c, \chi_{1I}^c, \chi_{1R}^0, \chi_{1I}^0, \chi_{2R}^c, \chi_{2I}^c, \chi_{2R}^0, \chi_{2I}^0, N_1$ and N_2, which we denote by η_i ($i = 1$ to 10) respectively and their tree level masses by M_i. Even after χ_1 develops a non-zero VEV, the Higgs potential has a residual symmetry. To see this explicitly we define:

$$\mathbf{G} \equiv (\chi_{1I}^c, -\chi_{1R}^c, \chi_{1I}^0) \equiv (\eta_2, -\eta_1, \eta_4),$$
$$\mathbf{H} \equiv (\chi_{2R}^c, \chi_{2I}^c, \chi_{2R}^0) \equiv (\eta_5, \eta_6, \eta_7),$$
$$S_1 = \chi_{1R}^0 \equiv \eta_3 \quad \text{and} \quad S_2 = \chi_{2I}^0 \equiv \eta_8.$$

Now shifting the fields by their VEV's, Higgs potential can be written as

$$\begin{aligned} V =& \tfrac{1}{4}h^2(N_1^2 + N_2^2)[\mathbf{G}^2 + (S_1 + v)^2 + \mathbf{H}^2 + S_2^2] \\ &+ \tfrac{1}{16}h^2[\mathbf{G}^2 + (S_1 + v)^2 - \mathbf{H}^2 - S_2^2]^2 + \tfrac{1}{4}h^2[\mathbf{G}\cdot\mathbf{H} - (S_1 + v)S_2]^2 \\ &+ \tfrac{1}{2}(m^2 - m_3^2)[\mathbf{G}^2 + (S_1 + v)^2] + \tfrac{1}{2}(m^2 + m_3^2)(\mathbf{H}^2 + S_2^2) \\ &+ \tfrac{1}{2}(m_4^2 + 2m_5^2)N_1^2 + \tfrac{1}{2}(m_4^2 - 2m_5^2)N_2^2. \end{aligned}$$

Under a $SU(2)$ rotation wherein $\mathbf{G}$ and $\mathbf{H}$ are triplets, and (S_1, S_2, N_1, N_2) all singlets, V is invariant. We expect then $M_1 = M_2 = M_4$ and $M_5 = M_6 = M_7$ on account of this symmetry. We note that this symmetry is present in a two-doublet model for the special choice of our parameters. Thus in the two-doublet model discussed in § 2 of ref. [12], setting $v_1 = v_2 = \xi = 0$ and $\lambda_1 = \lambda_4$ makes H_1^0, H_2^0 and H_4^0 degenerate. We will build in this residual symmetry in our variational approach.

In terms of ten real fields the Higgs potential

$$\begin{aligned} V =& (m^2 - m_3^2)v^2 + \frac{1}{4}h^2 v^4 + \frac{1}{2}(m^2 - m_3^2 + \frac{1}{2}h^2 v^2)(\eta_1^2 + \eta_2^2 + \eta_4^2) \\ &+ \frac{1}{2}(m^2 - m_3^2 + \frac{3}{2}h^2 v^2)\eta_3^2 + \frac{1}{2}(m^2 + m_3^2 - \frac{1}{2}h^2 v^2)(\eta_5^2 + \eta_6^2 + \eta_7^2) \\ &+ \frac{1}{2}(m^2 + m_3^2 + \frac{1}{2}h^2 v^2)\eta_8^2 + \frac{1}{2}(m_4^2 + 2m_5^2 + h^2 v^2)\eta_9^2 \end{aligned}$$

$$+\frac{1}{2}(m_4^2 - 2m_5^2 + h^2v^2)\eta_{10}^2 + \frac{1}{16}h^2\sum_{i=1}^{8}\eta_i^4 + \frac{1}{4}h^2(\eta_9^2 + \eta_{10}^2)\sum_{i=1}^{8}\eta_i^2$$
$$+\frac{1}{8}h^2[\eta_1^2(\eta_2^2 + \eta_3^2 + \eta_4^2 - \eta_5^2 + \eta_6^2 - \eta_7^2 - \eta_8^2)$$
$$+\eta_2^2(\eta_3^2 + \eta_4^2 + \eta_5^2 - \eta_6^2 - \eta_7^2 - \eta_8^2) + \eta_3^2(\eta_4^2 - \eta_5^2 - \eta_6^2 - \eta_7^2 + \eta_8^2)$$
$$+\eta_4^2(-\eta_5^2 - \eta_6^2 + \eta_7^2 - \eta_8^2) + \eta_5^2(\eta_6^2 + \eta_7^2 + \eta_8^2)$$
$$+\eta_6^2(\eta_7^2 + \eta_8^2) + \eta_7^2\eta_8^2] + V_{\text{linear}} + V_{\text{cubic}}, \tag{13}$$

where V_{linear} and V_{cubic} respectively represent the terms linear and cubic in fields. The tree level minima condition is equivalent to equating the term linear in η_3 (i.e. χ_{1R}^0) in (13) to zero,

$$m^2 - m_3^2 + \tfrac{1}{2}h^2v^2 = 0. \tag{14}$$

Inspection of the quadratic terms in (13) together with (14) immediately tells us that

$$M_1 = M_2 = M_4 = 0, \tag{15}$$

indicating that η_1, η_2 and η_4 are the Goldstones. Furthermore, we have for the charged Higgs triplet

$$M_5^2 = M_6^2 = M_7^2 = m^2 + m_3^2 - \tfrac{1}{2}h^2v^2, \tag{16}$$

where the degeneracy is as expected. Lastly,

$$M_3^2 = m^2 - m_3^2 + \tfrac{3}{2}h^2v^2, \tag{17}$$
$$M_8^2 = m^2 + m_3^2 + \tfrac{1}{2}h^2v^2, \tag{18}$$
$$M_9^2 = m_4^2 + 2m_5^2 + h^2v^2 \tag{19a}$$

and

$$M_{10}^2 = m_4^2 - 2m_5^2 + h^2v^2 \tag{19b}$$

are the masses of neutral Higgs. For simplicity we also take $m_5 = 0$ so that

$$M_9^2 = M_{10}^2. \tag{20}$$

3. Gaussian trial wave-functional

In order to obtain information on masses beyond the tree level, we follow a variational method with a Gaussian trial wave-functional. Most generally, this wave functional would be the vacuum state of a set of free fields of masses $\Omega_1, \ldots, \Omega_{10}$ with the Ω's (and v in eq. (12)) representing the variational parameters. However, taking a variational ground state that respects the residual symmetry stated in the last section, we set $\Omega_1 = \Omega_2 = \Omega_4$ and $\Omega_5 = \Omega_6 = \Omega_7$. Further imposing the symmetry in (20) we also put $\Omega_9 = \Omega_{10} = \Omega_N$. We thus have five independent masses Ω_1, Ω_3, Ω_5, Ω_8 and Ω_N, and of course v, as variational parameters.

Following standard techniques [13], the expectation value of Hamiltonian density $\mathcal{H}$ in our trial vacuum state wave functional is

$$\begin{aligned} V_{\mathrm{G}} = & (m^2 - m_3^2)v^2 + \frac{1}{4}h^2v^4 + 2\left[\frac{1}{2}(M_N^2 - \Omega_N^2)I_0(\Omega_N^2) + I_1(\Omega_N^2)\right] \\ & + \sum_{i=1}^{8}\left[\frac{1}{2}(M_i^2 - \Omega_i^2)I_0(\Omega_i^2) + I_1(\Omega_i^2)\right] + \frac{1}{2}h^2 I_0(\Omega_N^2)\sum_{i=1}^{8} I_0(\Omega_i^2) \\ & + \frac{3}{16}h^2[5I_0^2(\Omega_1^2) + I_0^2(\Omega_3^2) + 5I_0^2(\Omega_5^2) + I_0^2(\Omega_8^2)] \\ & + \frac{1}{8}h^2[3I_0(\Omega_1^2)(I_0(\Omega_3^2) - I_0(\Omega_5^2) - I_0(\Omega_8^2)) \\ & + 3I_0(\Omega_5^2)(I_0(\Omega_8^2) - I_0(\Omega_3^2)) + I_0(\Omega_3^2)I_0(\Omega_8^2)]. \end{aligned} \tag{21}$$

Here

$$I_1(\Omega) = \frac{1}{2(2\pi)^4}\int \mathrm{d}^4k_E \ln(k_E^2 + \Omega^2) + \text{constant} \tag{22a}$$

and

$$I_0(\Omega) = \frac{1}{(2\pi)^4}\int \mathrm{d}^4k_E \frac{1}{(k_E^2 + \Omega^2)}. \tag{22b}$$

Differentiating (21) with respect to Ω_N^2, Ω_1^2, Ω_3^2, Ω_5^2 and Ω_8^2 gives us five mass equations:

$$\Omega_N^2 = M_N^2 + \tfrac{1}{2}h^2[3I_0(\Omega_1^2) + 3I_0(\Omega_5^2) + I_0(\Omega_3^2) + I_0(\Omega_8^2)], \tag{23a}$$

$$\Omega_1^2 = M_1^2 + \tfrac{1}{4}h^2[4I_0(\Omega_N^2) + 5I_0(\Omega_1^2) + I_0(\Omega_3^2) - I_0(\Omega_5^2) - I_0(\Omega_8^2)], \tag{23b}$$

$$\Omega_3^2 = M_3^2 + \tfrac{1}{4}h^2[4I_0(\Omega_N^2) + 3I_0(\Omega_1^2) + 3I_0(\Omega_3^2) - 3I_0(\Omega_5^2) + I_0(\Omega_8^2)], \tag{23c}$$

$$\Omega_5^2 = M_5^2 + \tfrac{1}{4}h^2[4I_0(\Omega_N^2) - I_0(\Omega_1^2) - I_0(\Omega_3^2) + 5I_0(\Omega_5^2) + I_0(\Omega_8^2)], \tag{23d}$$

$$\Omega_8^2 = M_8^2 + \tfrac{1}{4}h^2[4I_0(\Omega_N^2) - 3I_0(\Omega_1^2) + I_0(\Omega_3^2) + 3I_0(\Omega_5^2) + 3I_0(\Omega_8^2)]. \tag{23e}$$

Using equations (23), V_{G} reduces to

$$\begin{aligned} V_{\mathrm{G}} = & (m^2 - m_3^2)v^2 + \tfrac{1}{4}h^2v^4 \\ & + [2I_1(\Omega_N^2) + 3I_1(\Omega_1^2) + I_1(\Omega_3^2) + 3I_1(\Omega_5^2) + I_1(\Omega_8^2)] \\ & - \tfrac{3}{16}h^2[5I_0^2(\Omega_1^2) + I_0^2(\Omega_3^2) + 5I_0^2(\Omega_5^2) + I_0^2(\Omega_8^2)] \\ & - \tfrac{1}{2}h^2 I_0(\Omega_N^2)[3I_0(\Omega_1^2) + I_0(\Omega_3^2) + 3I_0(\Omega_5^2) + I_0(\Omega_8^2)] \\ & - \tfrac{1}{8}h^2[3I_0(\Omega_1^2)(I_0(\Omega_3^2) - I_0(\Omega_5^2) - I_0(\Omega_8^2)) \\ & + 3I_0(\Omega_5^2)(I_0(\Omega_8^2) - I_0(\Omega_3^2)) + I_0(\Omega_3^2)I_0(\Omega_8^2)]. \end{aligned} \tag{24}$$

Here Ω_i's are to be understood as depending on v through (23). Differentiating V_{G} with respect to v^2, we get

$$\begin{aligned} \frac{\mathrm{d}V_{\mathrm{G}}}{\mathrm{d}v^2} = & m^2 - m_3^2 + \frac{1}{2}h^2v^2 + \frac{1}{4}h^2 \\ & \times [4I_0(\Omega_N^2) + 3I_0(\Omega_1^2) + 3I_0(\Omega_3^2) - 3I_0(\Omega_5^2) + I_0(\Omega_8^2)]. \end{aligned} \tag{25}$$

Setting dV_G/dv^2 to zero and using (23c), we get

$$\Omega_3^2 = h^2 v^2. \tag{26}$$

It is clear from (25), that increasing h would increase Ω_3^2 without limits. However, the limit would be set by demanding that stationary solution (25) be stable, i.e. the stability matrix $(\partial^2 V_G)$ be positive definite. Stability condition is obtained by considering $(\partial^2 V_G)$ to be a function of six variables — five mass parameters (Ω_1^2, Ω_3^2, Ω_5^2, Ω_8^2 and Ω_N^2) and v^2, and demanding all its eigenvalues to be positive. As in the case of standard model [8], we expect that as h increases, this condition would no longer be satisfied beyond a certain maximum value of h.

Our query regarding triviality bounds does not involve the complete numerical solution. Of five independent masses $\Omega_1, \Omega_3, \Omega_5, \Omega_8$ and Ω_N, Ω_1 is the mass of the Goldstone bosons. This in an exact calculation is expected to be zero but in variational methods (see ref. [13] for elaboration), we can get a small but non-vanishing mass. Of the remaining four Higgs masses, we wish to find out whether they can be made arbitrarily heavy relative to v^2 (or M_W^2). The only condition we would impose is the same as laid down by Hasenfratz and Nager, namely that for a cut-off theory to make any physical sense, each one of the masses Ω_i must not be close to or greater than the cut-off; we put an upper limit of $\Lambda_c/5$ for definiteness, for all Ω_i's (see ref. [5]).

Our task is then to set Ω_i's at their maximum possible values and determine the value of h for which the eigenvalues of stability matrix go from positive to negative. Since there are only three mass input parameters m, m_3 and m_4, all the five Ω_i's cannot be assigned arbitrary values by suitably choosing m's. Furthermore, only Ω_N^2 involves m_4^2, so that we can set Ω_N as the highest acceptable mass namely $\Lambda_c/5$ right away. Of the remaining four masses, we immediately have the sum rule,

$$\begin{aligned}\Omega_8^2 - \Omega_5^2 &= \Omega_3^2 - \Omega_1^2 \\ &= h^2 v^2 + \frac{h^2}{2}[I_0(\Omega_3^2) - I_0(\Omega_1^2) + I_0(\Omega_8^2) - I_0(\Omega_5^2)].\end{aligned} \tag{27}$$

Also since $I_0(\Omega^2)$ is a decreasing function of Ω^2, we get from (23) and (27)

$$\Omega_8^2 > \Omega_5^2 \quad \text{and} \quad \Omega_3^2 > \Omega_1^2.$$

The first possibility is to set

$$\Omega_8 = \Omega_3 = \Lambda_c/5;$$

Ω_1 is the Goldstone and Ω_5 is then the lightest Higgs mass. However in this case $\Omega_5 = \Omega_1$, and the maximum value of Ω_5 turns out to be lower than in the next sequence of masses.

Next we assume $\Omega_8 = \Lambda_c/5$ and the sequence

$$\Omega_8 > \Omega_5 > \Omega_3 > \Omega_1.$$

In this Ω_3 is the lightest Higgs. From (27), pushing Ω_5 towards Ω_8 will make Ω_3 go towards Ω_1, i.e. will make Ω_3 lighter. We then expect some kind of optimal situation to arise if $\Omega_5 = \Omega_3$. With the values of Ω_3 (obtained from (23)) corresponding for various h, we can calculate the eigenvalues of stability matrix. As expected one eigenvalue crosses over from positive to negative at a value of Higgs coupling,

$$h \equiv h_{\max} = 4.52. \tag{28}$$

Using this value, then the upper bound on mass of the lightest Higgs $M_{\mathrm{LH}} = \Omega_3 = \Omega_5$ is

$$\frac{M_{\mathrm{LH}}|_{\mathrm{max}}}{M_{\mathrm{W}}} = 10.1. \tag{29}$$

The Goldstone mass Ω_1 for this choice, as we stated before, is not zero but smaller than other masses in the spectrum.

4. Conclusion

We have shown that in a certain range of parameters of the NMSSM, the triviality upper bound on the mass of the lightest Higgs particle is $\sim 10\, M_{\mathrm{W}}$. Relaxing restrictions on the parameters can lead to even larger masses for the lightest Higgs. We have however not attempted to determine the absolute bound taking into account the full range of variation of parameters in the NMSSM (including SUSY breaking parameters). This is because our main aim was to show that in the non-minimal version of supersymmetric model, one need not be constrained by the rather strict limits on the Higgs mass that one obtains in the minimal model.

We would also like to comment on the upper bound of about 140 GeV for the lightest Higgs boson mass obtained by several authors [14]. This arises by the requirement that the SUSY theory remain perturbative up to some scale $M_{\mathrm{GUT}} \sim 10^{15}$ GeV. This clearly forces the quartic coupling constant to remain small. From a phenomenological point of view where the couplings of the NMSSM model have to be realistic over a huge energy range, such restrictions are eminently reasonable and hence in that respect the bounds obtained by reference [14] are phenomenologically more relevant. We, in our analysis, have not constrained the coupling constant from any such requirement and thus the limits obtained are much higher. Comparing our result with that of Wu [11] who obtained a much lower bound by a factor of four, we find that the criterion used by him to define triviality bound (as explained in the introduction) is different from ours. Also the method of calculating the scalar mass particles is based on tree approximation unlike our non-perturbative variational method which goes beyond the tree approximation. The difference in our numerical bounds we believe can be attributed to these differences in our approaches. Also limits on the Higgs mass above 1 TeV are of little interest. There is no possibility in any near future to detect any signals for such a heavy Higgs. Moreover Higgs particle with masses above 1 TeV with widths comparable to masses will make the mass parameter rather meaningless from an experimental point of view. Further, it is interesting to note that the bound obtained for the lightest Higgs ($\sim 10\, M_{\mathrm{W}}$) is of the same order as the bounds in the SM suggesting that in a supersymmetric theory, the Higgs mass bounds have the same features as the regular SM.

References

[1] T P Cheng and L F Li, *Gauge theory of elementary particle physics* (Clarendon Press, Oxford, 1986) pp. 84
[2] R Dashen and H Neuberger, *Phys. Rev. Lett.* **50**, 1897 (1983)
[3] P M Stevenson, *Phys. Rev.* **D32**, 1389 (1985)
[4] For a summary, see K Huang, MIT preprint, CTP # 1631 (1988)

[5] P Hasenfratz and J Nager, *Z. Phys.* **C37**, 477 (1988)
[6] M Luscher and P Weisz, *Nucl. Phys.* **B290**, 25 (1987); **B295**, 65 (1988)
[7] A Hasenfratz *et al*, *Phys. Lett.* **B199**, 531 (1987)
J Kuti, L Lin and Y Shen, *Phys. Rev. Lett.* **61**, 678 (1988)
[8] S R Choudhury and Mamta, *Int. J. Mod. Phys.* **A12**, 1847 (1997)
[9] See e.g. J F Gunion, H E Haber, G Kane and S Dawson, *The Higgs Hunter's Guide* (Addison-Wesley, 1990)
[10] M Quiros, CERN preprint, CERN-TH. 7507/94 (November, 1994)
[11] Y Y Wu, *Phys. Rev.* **D51**, 5276 (1995)
[12] J F Gunion and H E Haber, *Nucl. Phys.* **B272**, 1 (1986)
[13] P M Stevenson, B Allès and R Tarrach, *Phys. Rev.* **D35**, 2407 (1987)
[14] B Ananthanarayan and P Pandita, *Phys. Lett.* **B353**, 566 (1996); *Phys. Lett.* **B371**, 245 (1996)