Ratio of the color suppressed decays $\Lambda_b \rightarrow \Lambda J/\psi$ and $B^0 \rightarrow K J/\psi$

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Abstract. Based on the factorization approximation, we have estimated the ratio of the branching fractions for the color suppressed nonleptonic decays $\Lambda_b \rightarrow \Lambda J/\psi$ and $B^0 \rightarrow K J/\psi$. Treating the s-quark as heavy, we have used the HQET to calculate the hadronic matrix elements. The mesonic Isgur-Wise function is calculated in the quark model whereas for the baryonic IW function we have employed the bound state soliton picture. The results obtained agree very well with the recent CDF experimental data.

Keywords. Factorization approximation; heavy quark effective theory; Isgur-Wise function.

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1. Introduction

Recently the decay mode $\Lambda_b \rightarrow \Lambda J/\psi$ has been observed at the Fermilab [1] and the production cross section times branching fraction for the decay $\Lambda_b \rightarrow \Lambda J/\psi$ relative to that of $B^0 \rightarrow K J/\psi$ has been measured to be $0.27 \pm 0.12 \pm 0.05$. Based on the factorization approach, we have shown that the CDF data [1] for the ratio $\text{Br}(\Lambda_b \rightarrow \Lambda J/\psi)/\text{Br}(B^0 \rightarrow K J/\psi)$ can be well accounted for in the framework of heavy quark effective theory (HQET) [2–5]. Much of the success of HQET has been in the treatment of decays from one heavy flavor to another, namely $b \rightarrow c$ transitions. However, one approximation used recently in the literature has been to treat the strange quark as heavy, so that the decays we are interested here can be considered as heavy to heavy transitions. In fact the previous challenging studies [6–9] suggest that the heavy quark effective theory is still effective for $s$-system, if a constituent quark mass is employed for $m_s$ and $1/m_s$ corrections are taken into account properly. Therefore it is interesting to examine the applicability of the heavy flavor symmetry in the $s$-quark system even if the $s$ quark mass is not large enough compared to the QCD scale $\Lambda_{QCD}$. There is a lot of altercation in the literature regarding the fact, ‘whether HQET could be applied to $s$ quark system or not’. In a recent letter Chakraverty et al [10] have shown that heavy quark symmetry is not reliable for the $s$ quark system. On the other hand applying HQET to $B \rightarrow K^{(*)}$ transitions, Robert and Ledroit [11] suggested that the heavy $s$ limit may give an acceptable description of unpolarized
data but does not give reliable results when applied to polarization observables \((\Gamma_L/\Gamma)\). In this paper we assume the validity of HQET for \(s\)-quark system and examine the ratio of the branching fractions for the decays \(\Lambda_b \rightarrow \Lambda J/\psi\) relative to \(B^0 \rightarrow K J/\psi\), with the factorization approximation.

Factorization in two body non-leptonic decays of heavy pseudoscalar mesons [12] was resurrected a few years ago as a means of estimating their decay rates using the existing calculation of the semileptonic decay form factors. It relates the complicated non-leptonic decay amplitudes to products of meson decay constant and hadronic matrix elements of current operators similar to the ones encountered in semileptonic decays. This approximation appears to work well phenomenologically in the \(B\) decays, where it has been tested very well [13].

Nonleptonic decays of heavy baryons in general receive contributions from \(W\) decay and \(W\) exchange diagrams. The calculation of \(W\) exchange amplitudes has to rely on phenomenological models and introduces large theoretical uncertainties [14]. The decay mode \(\Lambda_b \rightarrow \Lambda J/\psi\) is however free from such contributions and proceeds through the \(W\) decay diagram. This allows to make rather clean theoretical prediction for the decay rate. Assuming factorization, the matrix element of the effective Hamiltonian is given by the product of the baryonic decay transition and the meson decay constant, i.e.,

\[
\langle J/\psi | \mathcal{H}_{\text{eff}} | \Lambda_b \rangle \approx \langle J/\psi | \bar{c} \gamma^\mu (1 - \gamma_5) c | 0 \rangle \langle \Lambda | \bar{s} \gamma_\mu (1 - \gamma_5) b | \Lambda_b \rangle.
\]  

To evaluate the hadronic matrix elements we use the results of HQET, treating \(s\)-quark as heavy. Since we are dealing with the evaluation of the ratio of the branching fractions for \(B^0 \rightarrow K J/\psi\) and \(\Lambda_b \rightarrow \Lambda J/\psi\) decay processes, we have not included \(1/m_s\) corrections in our calculations. As these small corrections appear both in numerator and in denominator, it is assumed to be cancelled without effecting much the leading order result. Therefore at the leading order these matrix elements can be described by the single universal form factor, the well known Isgur–Wise (IW) function. HQET predicts that the Isgur–Wise function is related to the overlap of the light degrees of freedom wavefunctions of the heavy particles. In general the light component is very complicated, but in the valence quark approximation the light component of a heavy meson is a light antiquark whereas the light component of the heavy baryon is a light diquark. This implies the fact that the mesonic and baryonic Isgur–Wise functions are quite different from each other and need to be evaluated separately.

The IW functions are not calculable from perturbative QCD. On the contrary, heavy quark symmetry tells us only the normalization of the Isgur–Wise function at the zero recoil point i.e., when the initial and the final heavy quarks have the same velocity. Since apart from the zero recoil point HQET does not predict the shape of the IW function, it makes sense to calculate it in a model, which gives the same result in the appropriate limit as predicted by the HQET. Here we therefore use the quark model of Ali et al [8] to evaluate the mesonic Isgur–Wise function and the bound state soliton picture [15] for the baryonic function.

The outline of the paper is as follows. In \(\S\ 2\) we present the formalism for the description of nonleptonic decays of heavy mesons and baryons using the factorization approximation. The Isgur–Wise functions are calculated in \(\S\ 3\). Section 4 contains result and discussions.
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2. Formalism

Neglecting the penguin contribution, the effective Hamiltonian describing the decays is given as [11]

\[ \mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{c*}^* [C_1(m_b)(\bar{c}\gamma_\mu(1 - \gamma_5)b)(\bar{s}\gamma^\mu(1 - \gamma_5)c) ] + C_2(m_b)(\bar{s}\gamma_\mu(1 - \gamma_5)b)(\bar{c}\gamma^\mu(1 - \gamma_5)c)], \]

(2)

where \( G_F \) is the Fermi coupling constant; \( C_1 \) and \( C_2 \) are the Wilson coefficients which contain the short distance QCD corrections. In the leading logarithmic approximations they are given to be [16]

\[ C_1(m_b) = 1.12 \quad \text{and} \quad C_2(m_b) = -0.26. \]

(3)

The effective Hamiltonian describes two classes of nonleptonic decays. The first class is of the type: \( \Lambda_b \to \Lambda_c D_s \) and \( B^0 \to D D_s \), while the second class represent \( \Lambda_b \to \Lambda J/\psi \) and \( B^0 \to K J/\psi \); \( J/\psi \) is the charmonium state.

To evaluate the matrix element of the effective Hamiltonian, we employ factorization assumption. By Fierz rearrangement, we rewrite the effective Hamiltonian in a form suitable for the use of this assumption. Upon Fierz rearrangement, the factorized Hamiltonian for the decays is given as

\[ \mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{c*}^* a_2(\bar{s}\gamma_\mu(1 - \gamma_5)b)(\bar{c}\gamma^\mu(1 - \gamma_5)c), \]

(4)

where \( a_2 = C_2 + C_1/N_c \), with \( N_c \) is the number of colors. One may expect \( N_c = 3 \), but the decay rate obtained from factorization is too small for \( N_c = 3 \), while the limit \( N_c \to \infty \) gives a remarkable value. Since in this paper we are dealing with the ratio of the branching fractions, the dependence on \( N_c \) is unimportant.

As pointed out in the previous section we evaluate the matrix elements of the effective Hamiltonian using the factorization approximation. For \( B^0 \to K J/\psi \) transition, we therefore write the transition amplitude as

\[ A(B^0(v_1) \to K(v'_1)J/\psi(p_\psi, e_\psi)) = \frac{G_F}{\sqrt{2}} V_{cb} V_{c*}^* a_2 J/\psi(p_\psi, e_\psi)|\bar{c}\gamma^\mu(1 - \gamma_5)c|0 \times \langle K(v'_1)|\bar{s}\gamma_\mu(1 - \gamma_5)b|B^0(v_1)\rangle. \]

(5)

The hadronic matrix element \( \langle K(v'_1)|\bar{s}\gamma_\mu(1 - \gamma_5)b|B^0(v_1)\rangle \) is extracted from the corresponding semileptonic decays of \( B \) mesons into the Kaon state. In the heavy \( s \)-limit it is given in the HQET [4] as

\[ \langle K(v'_1)|\bar{s}\gamma_\mu(1 - \gamma_5)b|B^0(v_1)\rangle = \sqrt{m_b m_s} \xi(\omega_1)(v_1 + v'_1)_{\mu}, \]

(6)

where \( \xi(\omega_1) \) is the mesonic Isgur–Wise function with \( \omega_1 = (v_1 \cdot v'_1) \). The remaining matrix element can be written as

\[ \langle J/\psi(p_\psi, e_\psi)|\bar{c}\gamma^\mu(1 - \gamma_5)|0 \rangle = M_\psi f_\psi e_\psi^\mu, \]

(7)

where \( f_\psi \) is the charmonium decay constant and \( e_\psi^\mu \) denotes its polarization.
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Using eqs (5-7) we obtain the partial decay width of \( B^0 \rightarrow KJ/\psi \) decay mode in the rest frame of initial \( B^0 \) meson as

\[
\Gamma(B^0(v_1) \rightarrow K(v'_1)J/\psi(p_\psi, \epsilon_\psi)) = \frac{G_F^2}{16\pi} |p_1| |V_{cb} V_{cs}^*|^2 a_2 f_\psi^2 \xi_1^2(\omega_1) \\
\times m_b m_\psi (\omega_1^2 - 1) \left( 1 + \frac{M_K}{M_B} \right)^2,
\]

(8)

where \( |p_1| \) is the c.o.m momentum of the emitted particles and is given as

\[
|p_1| = \frac{1}{2M_B} \sqrt{[(M_B^2 - M_K^2 - M_\psi^2)^2 - 4M_\psi^2 M_\Lambda^2]}^{1/2}.
\]

(9)

Now similarly for the \( \Lambda_b \rightarrow \Lambda J/\psi \) transition, the hadronic matrix element of the factorized amplitude from eq. (1) can be written as

\[
\langle \Lambda(v'_2, s') | \gamma \mu (1 - \gamma_5) b | \Lambda_b(v_2, s) \rangle = \eta(\omega_2) \bar{u}_\Lambda(v'_2, s') \gamma_\mu (1 - \gamma_5) u_{\Lambda_b}(v_2, s),
\]

(10)

where \( \eta(\omega_2) \) is the baryonic Isgur–Wise function with \( \omega_2 = v_2 \cdot v'_2 \). The spinors present in the above equation \( u_\Lambda(v'_2, s') \) and \( u_{\Lambda_b}(v_2, s) \) are the usual Dirac spinors for the \( \Lambda \) and \( \Lambda_b \) states, with spin sum given as

\[
\sum_s u(v, s) \bar{u}(v, s) = \frac{\gamma + 1}{2}.
\]

(11)

With eqs (1), (7) and (10) we obtain the partial decay width for \( \Lambda_b \rightarrow \Lambda J/\psi \) decay as

\[
\Gamma(\Lambda(v_2, s) \rightarrow \Lambda(v'_2, s')J/\psi(p_\psi, \epsilon_\psi)) = \frac{G_F^2}{8\pi M_\Lambda^2} |p_2| |V_{cb} V_{cs}^*|^2 a_2 f_\psi^2 \eta_2^2(\omega_2) \\
\times [(M_\Lambda^2 - M_\psi)^2 + M_\psi^2 (M_\Lambda^2 + M_\Lambda^2 - 2M_\psi^2)],
\]

(12)

where \( |p_2| \) is the c.o.m momentum of the emitted particles in the rest frame of \( \Lambda_b \) baryon.

After evaluating the partial decay widths for the two types of decays, we now obtain the ratio of their branching fractions as

\[
\frac{Br(\Lambda_b \rightarrow \Lambda J/\psi)}{Br(B^0 \rightarrow KJ/\psi)} = \frac{T_{\Lambda_b} \Gamma(\Lambda_b \rightarrow \Lambda J/\psi)}{T_{B^0} \Gamma(B^0 \rightarrow KJ/\psi)} = \frac{\tau_{\Lambda_b} |p_2| \eta_2(\omega_2) 2M_B^2 \left[(M_\Lambda^2 - M_\Lambda^2)^2 + M_\psi^2 (M_\Lambda^2 + M_\Lambda^2 - 2M_\psi^2)\right]}{\tau_{B^0} |p_1| \xi_1^2(\omega_1) M_\Lambda^2 \left[(\omega_1^2 - 1)m_b m_\psi (M_K + M_B)^2\right]}.
\]

(13)

Thus one can see from eq. (13) that the ratio of the branching fractions is independent of any model dependent parameters. The only unknown quantities present in the expression are the Isgur–Wise functions \( \xi(\omega_1) \) and \( \eta(\omega_2) \). These universal functions are evaluated in a simplified manner as presented in the following section.

3. Evaluation of the Isgur–Wise functions

The Isgur–Wise function which represents the nonperturbative QCD effects, are a measure of the light cloud (spectator quarks) rearrangement around the heavy quark
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during the transition. This function is normalized to unity at the point of zero velocity recoil. We calculate it in the quark model of Ali et al [8] for the $B^0 \rightarrow K^0 J/\psi$ transition in the subsection 3.1 and in the bound state soliton picture [15] for the $\Lambda_b \rightarrow \Lambda J/\psi$ case in subsection 3.2.

3.1 $\xi(\omega_1)$ for $B^0 \rightarrow K^0 J/\psi$

The Isgur–Wise function calculated in the quark model, predicts the decay widths for rare $B \rightarrow K^* \gamma$ [8] and the branching ratios of nonleptonic $B$ mesons [7, 17] very well. In this model the function $\xi(\omega_1)$ can be obtained from the overlap integral as

$$\xi(\omega_1) = \int r^2 dr \phi_F^*(r)\phi_I(r) j_0(\Lambda r \sqrt{\omega_1^2 - 1}),$$  

where $\phi_I$ and $\phi_F$ refer to the radial wave functions of the initial and final mesons respectively. $j_0$ is the spherical Bessel function of the zeroth order. The ‘inertia parameter $\Lambda$’ is taken to be [18]

$$\Lambda = \frac{M_K \cdot m_d}{m_s + m_d} = \frac{3}{8} M_K,$$

for the constituent quark mass values $m_d = 330$ MeV and $m_s = 550$ MeV. To calculate the Isgur–Wise function (14) we use the wave functions of harmonic oscillator for $\phi_I$ and $\phi_F$ in the form

$$\phi(r) = \left(\frac{4\beta^3}{\sqrt{\pi}}\right)^{1/2} \exp(-\beta^2 r^2/2),$$

with oscillator strength $\beta$. The values of $\beta_K$ and $\beta_B$ fitted in ref. [19] ($\beta_K = 0.34$ GeV and $\beta_B = 0.41$ GeV) are not equal. However we have assumed $\beta$ to be same, for the initial and final mesons and obtain

$$\xi(\omega_1) = \exp\left(\frac{-\Lambda^2(\omega_1^2 - 1)}{4\beta^2}\right).$$

We have taken $\beta = 0.295$ GeV, which is extracted from the best fit in ref. [7].

The value of $\omega_1$ is determined by considering the kinematics of the system. Since we are dealing with the two body decays $B^0(v_I) \rightarrow K(v'_I)J/\psi(p_\psi)$, momentum conservation gives

$$M_B v_I^\mu = M_K v'_I^\mu + p_\psi^\mu,$$

which gives

$$\omega_1 = v_I \cdot v'_I = \frac{M_B^2 + M_K^2 - M_\psi^2}{2M_B M_K}.$$

Thus with eqs (17) and (19) we have obtained the mesonic Isgur–Wise function $\xi(\omega_1)$ to be

$$\xi(\omega_1) = 0.317.$$

3.2 $\eta(\omega_2)$ for $\Lambda_b \to \Lambda J/\psi$

Here we have presented the evaluation of the Isgur-Wise function in the same manner as suggested in ref. [15]. The Isgur-Wise function calculated in this manner explains very well the two body non-leptonic decays $\Lambda_b \to \Lambda P(V)$ [20] and the radiative rare decay $\Lambda_b \to \Lambda \gamma$ [21]. In this model the heavy baryons are treated as the bound state of the soliton with the heavy meson. The Isgur-Wise function is given as

$$\eta(\omega_2) = \int \, d^3q\Phi^*(q)\Phi(q + m_B(v_2 - v')) \tag{21}$$

where $m_B$ represents the mass of the soliton which is taken as the mass of nucleon, for the ground state $\Lambda_Q$ baryons.

The binding potential between the heavy meson and the chiral soliton is simple harmonic [22] and hence the wave function is taken as

$$\Phi(q) = \left(\frac{m_B}{\sqrt{2}\kappa}\right)^{3/8} e^{-\frac{\sqrt{2}m_B}{\kappa}} \tag{22}$$

$\kappa$ is the spring constant and its value is taken to be $(440 \text{ MeV})^3$ [23]. In the rest frame of the initial state, $v_2 = (1, 0)$ and $v'_2$ directed along $z$-axis we obtain the Isgur-Wise function (21) using (22) for non-relativistic recoils i.e., $|v'_2|^2 \approx 2(\omega_2 - 1)$, as

$$\eta(\omega_2) = e^{-\frac{(\omega_2 - 1)}{2} \sqrt{\frac{m_B^3}{\kappa}}} \tag{23}$$

Again $\omega_2$ is obtained by considering the kinematics of the system, given as

$$\omega_2 = v_2 \cdot v'_2 = \frac{M_{\Lambda_b}^2 + M_{\Lambda}^2 - M_{\Lambda_j}^2}{2M_{\Lambda_b}M_{\Lambda}} \tag{24}$$

Now with eq. (24) the baryonic Isgur-Wise function (23) is found to be

$$\eta(\omega_2) = 0.263. \tag{25}$$

4. Results and discussion

In order to obtain the value of the ratio of the branching fractions $\frac{Br(\Lambda_b \to \Lambda J/\psi)}{Br(B^0 \to K J/\psi)}$ with eq. (13) we use the following values. The quark masses are taken as $m_b = 4.5 \text{ GeV}$ and $m_s = 550 \text{ MeV}$. The masses of $\Lambda_b$ and $B^0$ particles are taken from ref. [1] as $M_{\Lambda_b} = 5621\text{ MeV}$ and $M_{B^0} = 5281\text{ MeV}$. The other particle masses and the life times of $\Lambda_b$ and of $B^0$ are taken from ref. [24]. With these values we obtain the ratio of the branching fractions to be

$$\frac{Br(\Lambda_b \to \Lambda J/\psi)}{Br(B^0 \to K J/\psi)} = 1.017. \tag{26}$$

Using the value $\frac{\sigma_{\Lambda_b}}{\sigma_{B^0}} = 0.1/0.375$ from ref. [1], we obtain the production cross section times branching fraction for the decay $\Lambda_b \to \Lambda J/\psi$ relative to that of the decay
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\[ B^0 \rightarrow KJ/\psi \text{ as} \]

\[ \frac{\sigma_{\Lambda_b} \text{Br}(\Lambda_b \rightarrow \Lambda J/\psi)}{\sigma_{B^0} \text{Br}(B^0 \rightarrow KJ/\psi)} = 0.271, \]  

(27)

which is in excellent agreement with the CDF data \( 0.27 \pm 0.12 \pm 0.05 \) [1]. The overall agreement of our result with the experimental data justifies the fact that factorization approximation works well for the description of color suppressed nonleptonic decays (decays proportional to \( a_2 \)) of the heavy hadrons.

In this paper we have estimated the ratio of the branching fractions of the color suppressed nonleptonic decays \( \Lambda_b \rightarrow \Lambda J/\psi \) and \( B^0 \rightarrow KJ/\psi \), based on the factorization approximation. Treating the \( s \)-quark as heavy, we have used HQET for the evaluation of the hadronic matrix elements. Since HQET does not predict the shape of the Isgur-Wise function and of course visualizing the fact that the IW functions are different for the baryonic and mesonic sectors, due to the different configuration in their light degrees of freedom, we employ the well established models for their evaluation. The Isgur–Wise function is evaluated in the quark model [8] for \( B^0 \rightarrow KJ/\psi \) decay and in the bound state soliton picture [15] for \( \Lambda_b \rightarrow \Lambda J/\psi \) decay mode. Since the evaluation of the Isgur–Wise function is of course model dependent, there are some theoretical uncertainties introduced as two different models are followed for the estimation of mesonic and baryonic IW functions. The ISGW quark model for the mesonic case is quite well established [6–8, 17]. In fact if the baryonic IW function could have been evaluated in the quark model then hopefully the uncertainties would have been small. However in the absence of such a treatment we have followed a different model for it, so it is necessary to test the reliability of the model.

For the sake of comparison we therefore calculate the branching ratio for \( B^0 \rightarrow KJ/\psi \) process using eqs (8) and (20) and thus from eq. (26) we obtain the branching ratio for \( \Lambda_b \rightarrow \Lambda J/\psi \) decay process. Now comparing this predicted value with the quark model calculation [25] (without using HQET) the validity of the bound state soliton picture for the baryonic IW function can be seen easily.

To calculate the branching fraction for \( B^0 \rightarrow KJ/\psi \), we use \( |V_{cs}| = 0.9738, |V_{cb}| = 0.038, a_2 = 0.23 \). The decay constant \( f_\psi \) can be obtained from the leptonic width of the charmonium state [11] as \( f_\psi = 382 \text{ MeV} \). Thus we obtain \( \text{Br}(B^0 \rightarrow KJ/\psi) \) to be

\[ \text{Br}(B^0 \rightarrow KJ/\psi) = 3.917 \times 10^{-4}, \]

(28)

and with eq. (26) we obtain

\[ \text{Br}(\Lambda_b \rightarrow \Lambda J/\psi) = 3.984 \times 10^{-4}. \]

(29)

Using \( 1/m_Q \) corrections to the baryonic form factors in the quark model Cheng et al [25] obtained

\[ \text{Br}(\Lambda_b \rightarrow \Lambda J/\psi) = 2.1 \times 10^{-4}. \]

(30)

Our present calculation is somewhat greater than their predictions. This decay mode is now observed experimentally by CDF [1]. The branching ratio of \( \text{Br}(\Lambda_b \rightarrow \Lambda J/\psi) = (3.7 \pm 1.7 \pm 0.4) \times 10^{-4} \) assuming \( \text{Br}(B^0 \rightarrow KJ/\psi) = 3.7 \times 10^{-4} \). It is seen from eq. (30) that the quark model [25] prediction does not agree well with the experiment. In contrast, our calculation is very simple and free from any theoretical uncertainties besides in the
sector of Isgur–Wise functions. It is interesting to note that our theoretical predictions (28) and (29) agree reasonably well with these experimental values. Therefore it should be inferred from the above observation that the evaluation of the IW function using two different models do not introduce much theoretical uncertainties. Our results indicate that the heavy quark symmetry does work in the first approximation to calculate the exclusive decays involving $b \rightarrow s$ transitions.

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