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Finite temperature Cornwall–Jackiw–Tomboulis formalism of Φ^6 theory

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Abstract. The finite temperature effective potential for a scalar field with Φ^6 interaction is calculated by extending the CJT formalism for composite operators. It is found that unrenormalized terms appear in the effective potential due to the presence of an unrenormalized mass term. Nonzero turning points are obtained both for positive and negative λ. High temperature expansion is performed and the results are analysed numerically. Graphical analysis indicates symmetry restoration when $T \to 0$.

Keywords. Composite operator; double Legendre transform; CJT formalism; effective potential; finite temperature; symmetry restoration; high temperature expansion.

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1. Introduction

Study of symmetry changing phase transitions of a quantum field in the presence of a surrounding thermal bath is very important in the study of the evolution of the universe and in the analysis of very high energy collisions where very high matter and radiation density exist. Detailed study of these phase transitions has been done by various authors [1–4]. The effective potential method is very useful in studying spontaneous symmetry breaking (SSB) at zero temperature [5, 6]. Estimation of the critical temperature of phase transitions can be done by extending this approach to finite temperature. These studies mainly using loop expansion techniques have played a pivotal role in framing our understanding about the early universe, unified theories, quark gluon plasma etc.

Recently there has been a revival of interest in finite temperature quantum field theory, apparently caused by the recognition of the importance of symmetry breaking phase transitions and the problem of precise determination of critical temperature. An accurate analysis of phase transitions (both analytical and numerical) becomes necessary because most of the cosmological models critically depend on it. For example baryon asymmetry may be generated at the electro-weak level if the phase transition is of first order [7–10]. For calculation of critical temperature various perturbative and non-perturbative techniques have been suggested [11–19].

Effective potential defined as single Legendre transform provides an efficient way to obtain quantum corrections to the classical potential. But this popular method suffers from a serious shortcoming. Since it is defined as a single Legendre transform, it is always a convex function. This forbids the double well shape for the exact effective potential and implies the absence of local maximum at the symmetric origin. But in a theory possessing SSB classical analysis predicts a maximum at the origin. Various procedures to avoid this difficulty have been suggested earlier [3, 20, 21]. One of the most efficient of these approaches is to define an effective action by including a source $K(x,y)$ coupled to a term which is quadratic in the field variable. By this procedure an effective potential with a proper loop expansion to each order which is not convex, is obtained. This idea was first put forward by Hawking and Moss in the context of quantum field theory in the early universe [22]. A self-consistent improvement for the finite temperature effective potential has also been suggested [23]. In this formalism it is possible to sum a large class of ordinary perturbation theory diagrams that contribute to the effective action and the gap equation which determines the form of the propagator, is obtained by variational method.

All the above improvements developed for conventional finite temperature effective potential are based on an important contribution by Cornwall, Jackiw and Tomboulis (CJT). They defined the effective action for composite fields in flat space and zero temperature as a double Legendre transform with two sources $J(x)$ and $K(x,y)$. These two sources are coupled respectively to $\phi(x)$ and $\phi(y)$ [24]. The CJT formalism is considered to be best suited for studying phase transitions because it uses a generalized effective action in which not only the mean field but also the correlation functions appear as independent variables. A simple series expansion has been developed for this improved effective action [24].

The CJT formalism has recently been used to resolve various difficulties in quantum field theory [25, 26]. For example, it has been applied to the triviality problem in Φ^4 theory [27]. An improved effective potential based on this formalism has also been developed [28]. The self-consistent improvement of finite temperature effective potential involves the summation of daisy and superdaisy diagrams, for which a novel resummation procedure has been recently proposed [29].

Recently, there has been considerable interest in the application of functional techniques in field theories with dimensions less than $(3+1)$, mainly because some of the problems afflicting 4-dimensional theories are absent there [30–32]. According to Coleman's theorem, spontaneous symmetry breaking can occur only when the dimensions are higher than $(1+1)$. In $(2+1)$ dimensions the most general renormalizable theory is for a Φ^6 model. This model has been studied earlier by using various methods and it has been shown that it possesses an ultraviolet fixed point in $1/N$ expansion and Gaussian approximation [33–35]. Finite temperature field theory has also been analysed [17, 18]. It is well known that in finite temperature CJT analysis of Φ^4 theory, the effective potential shows a cut-off dependence due to the presence of a $(\lambda\phi^4/12)$ term. It is natural to think that in lower dimensions where coupling constant renormalization is not required this difficulty will not be there. Even though this is found to be true for Φ^4 theory in $(2+1)$ dimensions, an unrenormalized mass term appears in the expression for effective potential for Φ^6 theory. Thus the diffiiculty persists in a disguised form, in $(2+1)$ dimensional Φ^6 theory.

2. CJT formalism

The CJT method provides a generalization of the ordinary effective action $\Gamma(\phi)$ (the generating functional for single particle irreducible n-point functions). This generalized effective action $\Gamma(\phi, G)$ depends both on $\phi(x)$ the expectation value of quantum field $\Phi(x)$, and $G(x,y)$ the expectation value of time ordered product $T\langle\Phi(x)\Phi(y)\rangle$. Physical (on-shell) solutions require the following variational equations:

$$\frac{\delta\Gamma(\phi, G)}{\delta\phi(x)} = 0, \tag{1}$$

$$\frac{\delta\Gamma(\phi, G)}{\delta G(x,y)} = 0. \tag{2}$$

Consider the vacuum persistence amplitude $Z(J,K)$ in the presence of two source terms $J(x)\phi(x)$ and $\frac{1}{2}\phi(x)\phi(y)K(x,y)$:

$$Z(J,K) \equiv N\int \mathcal{D}\Phi \exp\left[i\int \mathrm{d}^4x\left[\mathcal{L}[\Phi(x)] + J(x)\Phi(x) + \frac{1}{2}\Phi(x)K(x,y)\Phi(y)\right]\right]. \tag{3}$$

$W(J,K)$, the generating functional for connected diagrams is defined as

$$Z(J,K) \equiv \exp[iW(J,K)]. \tag{4}$$

The classical action $I(\Phi) = \int \mathrm{d}^4x\mathcal{L}(x)$ may be written as

$$I(\phi) = \int \mathrm{d}^4x\mathrm{d}^4y\phi(x)D_0^{-1}(x-y)\phi(y) + I_{\mathrm{int}}(\phi) \tag{5}$$

$$I_{\mathrm{int}}(\phi) = \int \mathrm{d}^4x\mathcal{L}_{\mathrm{int}}(x). \tag{6}$$

$D_0(x-y)$ is the free propagator that satisfies

$$D_0^{-1}(x-y) = -(\Box + 2)\delta^4(x-y). \tag{7}$$

The generalized effective action $\Gamma(\phi, G)$ is the double Legendre transform of $W(J,K)$,

$$\begin{aligned}\Gamma(\Phi, G) =& W(J,K) - \int \mathrm{d}^4(x)\Phi(x)J(x) - \frac{1}{2}\int \mathrm{d}^4x\mathrm{d}^4y[\Phi(x)K(x,y)\Phi(y)]\\ &-\frac{1}{2}\int \mathrm{d}^4x\mathrm{d}^4yG(x,y)K(x,y),\end{aligned} \tag{8}$$

where $J(x)$ and $K(x,y)$ are determined by

$$\frac{\delta W(J,K)}{\delta J(x)} = \phi(x), \tag{9}$$

$$\frac{\delta W(J,K)}{\delta K(x,y)} = \frac{1}{2}[\phi(x)\phi(y) + G(x,y)]. \tag{10}$$

By actually performing functional differentiation on (8) we find

$$\frac{\delta\Gamma(\phi,G)}{\delta\phi(x)} = -J(x) - \int d^4yK(x,y)\phi(y), \tag{11}$$

$$\frac{\delta\Gamma(\phi,G)}{\delta G(x,y)} = -\frac{1}{2}K(x,y). \tag{12}$$

In the absence of sources, (1) and (2) are regained which permit a variational solution. The conventional effective action $\Gamma(\phi) = \Gamma(\phi, G_0)$ where G_0 is the solution of (2). Generalized effective action $\Gamma(\phi, G)$ is the generating functional for the two particle irreducible (2PI) Greens functions expressed in terms of the full propagator. The series expansion for $\Gamma(\phi, G)$ is shown to be [24]

$$\Gamma(\phi,G) = I_{\text{class}}.(\phi) + \tfrac{1}{2}\text{tr}\ln D_0G^{-1} + \tfrac{1}{2}\text{tr}[D^{-1}G - 1] + \Gamma^{(2)}(\phi,G), \tag{13}$$

where 'tr' is the functional trace 'ln' is the functional logarithm and $D^{-1}G$ is the functional product. The inverse propagator is defined as the functional second derivative of action:

$$D^{-1}(\phi;x,y) = \frac{\delta^2 I(\phi)}{\delta\phi(x)\delta\phi(y)} = D^{-1}(x-y) + \frac{\delta^2 I_{\text{int}}(\phi)}{\delta\phi(x)\delta\phi(y)}. \tag{14}$$

Computation of the quantity $\Gamma^{(2)}(\phi, G)$ is done as follows. In the classical action $I(\Phi)$, the field Φ is shifted by $\phi(x)$. The shifted action $I(\Phi + \phi)$, possesses terms cubic and higher in Φ. These define as interaction part I_{int} with vertices depending on $\phi(x)$. $\Gamma^{(2)}(\phi, G)$ is then given by all the 2PI vacuum graphs and the propagator is set equal to $G(x, y)$. The theory in its full generality is not translationally invariant since vertices depend on $\phi(x)$ and G is not a function of $(x - y)$ alone. The propagator of the theory is determined by finding the gap equation for G using the variation equations [eqs (1) and (2)].

3. Effective potential

Translationally invariant solutions are obtained by imposing the following conditions (homogeneous states):

$$\phi(x) = \text{Constant}, \tag{15}$$

$$G(x,y) = G(x-y), \tag{16}$$

$$\Gamma(\phi,G) = -E(\phi,G)\int dt, \tag{17}$$

where $E(\phi, G)$ is the minimum of the energy when varying over all the normalized states with constraints:

$$\langle\Phi(x)\rangle = \phi(x), \tag{18}$$

$$\langle\Phi(x)\Phi(y)\rangle = \phi(x)\phi(y) + G(x,y), \tag{19}$$

$$E(\phi,G) = V(\phi,G)\int d^{\nu-1}x, \tag{20}$$

where ν is the space-time dimension of the theory. Thus the effective potential is given by

$$V(\phi, G) = -\frac{\Gamma(\phi, G)}{\int \mathrm{d}^\nu x}. \tag{21}$$

A series expansion for the effective potential is obtained by defining the following Fourier transformed propagators

$$G(k) = \int \mathrm{d}^\nu x \mathrm{e}^{ik(x-y)} G(x-y), \tag{22}$$

$$D(\phi, k) = \int \mathrm{d}^\nu x \mathrm{e}^{ik(x-y)} D(\phi; x-y), \tag{23}$$

$$D_0(k) = \int \mathrm{d}^\nu x \mathrm{e}^{ik(x-y)} D_0(x-y), \tag{24}$$

$$\begin{aligned} V(\phi, G) = U(\phi) &+ \frac{1}{2}\int \frac{\mathrm{d}^\nu x}{2\pi} \ln \det D_0(k) G^{-1}(k) \\ &+ \frac{1}{2}\int \frac{\mathrm{d}^\nu k}{(2\pi)^\nu} \operatorname{tr}[D^{-1}(\phi, k) G^{-1} k - 1] + V_{(2)}(\phi, G), \end{aligned} \tag{25}$$

where $U(\phi)$ is the classical potential, $V_{(2)}(\phi, G)$ is the sum of 2PI vacuum graphs, with vertices given by $I_{\mathrm{int}}(\phi, \Phi)$ and the propagator is set equal to $G(k)$. The field ϕ on which vertices depend is now a constant parameter. Trace and logarithms apply to component fields and determinants are no more functional.

To describe the theory at finite temperature we use the Euclidean time τ satisfying the boundary conditions $0 \le \tau \le \beta \equiv 1/T$. All the Feynman diagrams (2PI diagrams) developed at zero temperature are valid here also. The Feynman rules for writing the algebra of the diagrams are different at finite temperature [21]. They are

$$\omega_n = \frac{2\pi n}{\beta}, \tag{26}$$

$$\text{loop integral} \to \frac{1}{\beta}\sum_n \int \frac{\mathrm{d}^{\nu-1} k}{(2\pi)^{\nu-1}}, \tag{27}$$

$$\text{vertex delta function} \to \beta (2\pi)^{\nu-1} \delta \sum_{\omega_n} \delta^{\nu-1}\left(\sum_i k_i\right). \tag{28}$$

Field Φ satisfies the periodic boundary conditions

$$\Phi\left(-\frac{\beta}{2}, x\right) = \Phi\left(\frac{\beta}{2}, x\right). \tag{29}$$

With these modifications we can write the series expansion for finite temperature CJT effective potential (analog of Gibb's potential) with time integration suppressed and a summation performed

$$\int \frac{\mathrm{d}^\nu k}{(2\pi)^\nu} \to \frac{1}{\beta}\sum_n \int \frac{\mathrm{d}^{\nu-1} k}{(2\pi)^{\nu-1}}. \tag{30}$$

4. Φ^6 Theory

The classical potential of the theory is given by

$$U(\Phi) = \frac{1}{2} m_{\mathrm{B}}^2 \Phi^2 + \frac{\lambda_{\mathrm{B}}}{4!} \Phi^4 + \frac{\xi_{\mathrm{B}}}{6!} \Phi^6, \tag{31}$$

where λ and ξ represent coupling constants for Φ^4 and Φ^6 interactions respectively and the subscript B indicates bare parameters. The functional operator D^{-1} is given by

$$D^{-1}(\Phi; (x,y)) = -\left[\Box + m_{\mathrm{B}}^2 + \frac{\lambda_{\mathrm{B}}}{2} \Phi^2 + \frac{\xi_{\mathrm{B}}}{24}\right] \delta^{\nu}(x-y). \tag{32}$$

After shifting the field Φ to $(\Phi + \phi)$, the interaction Lagrangian takes the form

$$L_{\mathrm{int}}(\Phi, \phi) = \left[\frac{\lambda_{\mathrm{B}}}{6} \phi \Phi^3 + \frac{\lambda_{\mathrm{B}}}{4!} \Phi^6 + \frac{\xi_{\mathrm{B}}}{6!} \Phi^6 + \frac{\xi_{\mathrm{B}}}{5!} \phi \Phi^5 + \frac{\xi_{\mathrm{B}}}{48} \phi^2 \Phi^4 + \frac{\xi_{\mathrm{B}}}{36} \phi^3 \Phi^5\right]. \tag{33}$$

A few of the 2PI vacuum graphs up to three loops are shown in figure 1. We select only those graphs with vertices depending on first order in the coupling constant. This approximation (Hartree–Fock) corresponds to a systematic variational procedure and is superior to commonly used one-loop approximation. No graphs with internal lines appear. Daisy and superdaisy graphs shown in figure 3 are of this type. Thus only the graphs shown in figure 2 need be summed. The sum of the relevant 2PI graphs takes the form

$$\Gamma_{\beta}^{(2)}(\phi, G) = \frac{3}{4!} \lambda_{\mathrm{B}} \int \mathrm{d}^{\nu}x G(x,x) G(x,x) + \frac{10}{6!} \xi_{\mathrm{B}} \int \mathrm{d}^{\nu}x G(x,x) G(x,x) G(x,x). \tag{34}$$

The expression for the finite temperature nonlocal composite operator effective action in Hartree–Fock approximation becomes

$$\begin{aligned} \Gamma_{\beta}(\phi, G) = & I_{\mathrm{class}} + \frac{1}{2} \operatorname{tr} \ln D_0 G^{-1} + \frac{1}{2} \operatorname{tr}(D^{-1} G - 1) \\ & + \frac{3}{4!} \lambda_{\mathrm{B}} \int \mathrm{d}^{\nu}x G(x,x) G(x,x) + \frac{10}{6!} \xi_{\mathrm{B}} \int \mathrm{d}^{\nu}x G(x,x) G(x,x) G(x,x). \end{aligned} \tag{35}$$

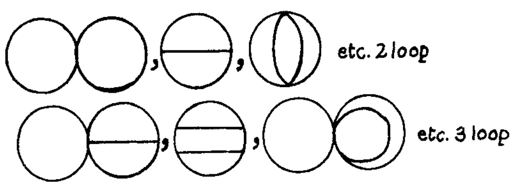

Figure 1. 2PI graphs for Φ^4 and Φ^6.

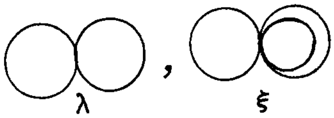

Figure 2. 2PI graphs chosen according to Hartree–Fock approximation.

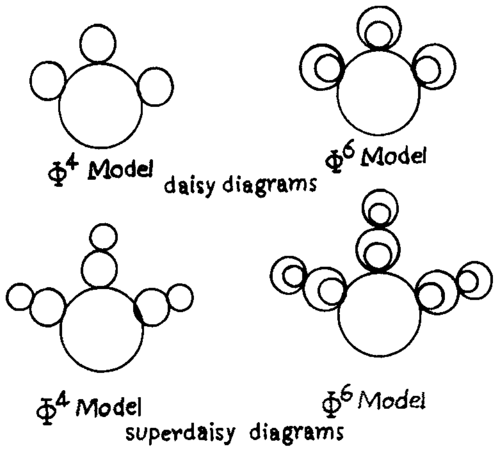

Figure 3. Daisy and superdaisy graphs for Φ^4 and Φ^6.

By performing variation with respect to G the modified gap equation is obtained as

$$G^{-1}(x,y) = D^{-1}(x,y) + \left[\frac{\lambda_B}{2}G(x,x) + \frac{\xi_B}{12}G(x,x)G(x,x)\right]\delta^3(x-y). \tag{36}$$

By iteration it can be seen that G generates all daisy and superdaisy graphs of figure 3, which reveals the fact that we have achieved a definite improvement in the resummation of diagrams.

Since we are interested only in translation invariant theories we fix an ansatz for G and define the Fourier transformed propagators [eqs (22), (23) and (24)]

$$G(k) \equiv \int \frac{d^\nu x}{(2\pi)^\nu} G(x-y)e^{ik(x-y)} = \frac{1}{k^2+M^2}, \tag{37}$$

$$D(k) \equiv \int \frac{d^\nu x}{(2\pi)^\nu} D(x-y)e^{ik(x-y)} = \frac{1}{k^2+m_B^2+\frac{\lambda_B}{2}\phi^2+\frac{\xi_B}{24}\phi^4}. \tag{38}$$

Here the propagator is chosen in terms of an effective mass M which acts as a variational parameter. Effective potential in terms of M^2 and ϕ can be written using static configuration and constant background field

$$\begin{aligned} V_\beta(\phi,M) = & \left[\frac{1}{2}m_B^2\phi^2 + \frac{\lambda_B}{4!}\phi^4 + \frac{\xi_B}{6!}\phi^6\right] + \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\ln(k^2+M^2) \\ & -\frac{1}{2}\left[M^2 - m_B^2 - \frac{\lambda_B}{2}\phi^2 - \frac{\xi_B}{24}\phi^4\right]G(x,x) + \frac{\lambda_B}{8}G(x,x)G(x,x) \\ & + \frac{\xi_B}{48}G(x,x)G(x,x)G(x,x). \end{aligned} \tag{39}$$

Putting $\xi_B = 0$ we get Φ^4 theory in (2+1) dimensions. Comparing the expression obtained with the Gaussian effective potential studies [16] we see that both are formally similar. But as far as Φ^6 theory is concerned dissimilarities emerge because of the term $\xi\phi_0^2$. It is interesting to note that this factor is not contributed by the daisy or superdaisy diagrams, but by a graph with vertex not proportional to ξ. This graph is not considered in Hartree–Fock approximation.

Since the effective potential is an ordinary function (not a functional) stationary requirements with respect to ϕ and M^2 is obtained by ordinary differentiation

$$\frac{\partial V}{\partial \phi} = \phi\left[m_{\rm B}^2 + \frac{\lambda_{\rm B}}{6}\phi^2 + \frac{\xi_{\rm B}}{120}\phi^4 + \frac{\lambda_{\rm B}}{2}G(x,x) + \frac{\xi_{\rm B}}{12}\Phi^2 G(x,x)\right] = 0, \tag{40}$$

$$\frac{\partial V}{\partial M^2} = -\frac{1}{2}\left[M^2 - m_{\rm B}^2 - \frac{\lambda_{\rm B}}{2}\phi^2 - \frac{\xi_{\rm B}}{24}\phi^4 - \frac{\lambda_{\rm B}}{2}G(x,x) \right.$$
$$\left. - \frac{\xi_{\rm B}}{8}G(x,x)G(x,x)\right]\frac{\partial G(x,x)}{\partial M^2} = 0. \tag{41}$$

Conventional effective potential is defined at the solution of (41). The effective mass is given by

$$M^2(\phi) = \left[m_{\rm B}^2 + \frac{\lambda_{\rm B}}{2}\phi^2 + \frac{\xi_{\rm B}}{24}\phi^4 + \frac{\lambda_{\rm B}}{2}G(x,x) + \frac{\xi_{\rm B}}{8}G(x,x)G(x,x)\right]. \tag{42}$$

Required expression for the effective potential is obtained by replacing the effective mass M by $M(\phi)$ in (39). Equation (39) shows certain very important peculiarities of Φ^6 and Φ^4 theories relevant at zero temperature

$$V'(\phi) = \phi\left[M^2(\phi) - \left(\frac{\lambda_{\rm B}}{3}\phi^2\right)\right]. \tag{43}$$

For Φ^4 theory $M^2(\phi)$ is intrinsically positive. Hence if $\lambda_{\rm B} < 0$ only solution to the above equation is $\phi = 0$ (or potential is unbounded from below). That is for negative $\lambda_{\rm B}$ non-zero turning points do not exist. In the case of Φ^6 theory

$$V'(\phi) = \phi\left[M^2(\phi) - \left(\frac{\lambda_{\rm B}}{3}\phi^2 + \frac{\xi_{\rm B}}{30}\phi^4\right)\right] - \frac{\xi}{4}\left[1 - \frac{\phi^2}{3}\right]G^2, \tag{44}$$

non-zero turning points are possible also for $\lambda_{\rm B} < 0$. Φ^6 theory in Hartee–Fock approximation requires up to three loops for obtaining the effects of ξ coupling. We have four parts for the effective potential

$$V_\beta(\phi, M(\phi)) = V^0 + V^1 + V^2 + V^3, \tag{45}$$

$$V^0 = \left[\frac{1}{2}m_{\rm B}^2\phi^2 + \frac{\lambda_{\rm B}}{4!}\phi^4 + \frac{\xi_{\rm B}}{6!}\phi^6\right], \tag{46}$$

$$V^1 = \frac{1}{2}\int \frac{{\rm d}^3k}{(2\pi)^3}\ln[k^2 + M^2(\phi)], \tag{47}$$

$$V^2 = -\frac{\lambda_{\rm B}}{8}G(x,x)G(x,x), \tag{48}$$

$$V^3 = -\frac{\xi_{\rm B}}{24}G(x,x)G(x,x)G(x,x), \tag{49}$$

which are obtained by substituting $M(\phi)$. Effective potential for both Φ^4 and Φ^6 theories can be obtained from this equation.

5. Renormalization

The effective mass term $M(\phi)$ defining the effective potential is divergent mainly due to the presence of $G(x,x)$. Following renormalization prescription is employed in $(2+1)$ dimensions to regularize $M(\phi)$ [6, 23]. Define

$$G(M(\phi)) \equiv -\frac{M(\phi)}{4\pi} + \frac{1}{\beta}\int \frac{d^2k}{(2\pi)^2}\frac{1}{E(e^{\beta E}-1)} \tag{50}$$

with

$$E \equiv [k^2 + M^2(\phi)]^{1/2}. \tag{51}$$

In $(2+1)$ dimensions coupling constant renormalization is not required. Define

$$m_r^2 \equiv m_B^2 + \frac{1}{2}\lambda I_1 + \frac{\xi'}{4}I_1 + \frac{\xi}{8}I_1^2, \tag{52}$$

$$I_1 \equiv \int \frac{d^2k}{(2\pi)^2}\frac{1}{2k} = \lim_{\Lambda\to\infty}\left(\frac{\Lambda}{4\pi}\right), \tag{53}$$

$\xi' \equiv \xi G(M(\phi))$. Using the summation procedure developed by Dolan and Jackiw [2, 23] the summation in time co-ordinate can be performed

$$G(x,x) = \int \frac{d^2k}{(2\pi)^2}\frac{1}{2E} + \frac{1}{\beta}\int \frac{d^2k}{(2\pi)^2}\frac{1}{E(e^{\beta E}-1)}. \tag{54}$$

By actual evaluation, introducing a cut-off parameter Λ

$$G(x,x) = G(M(\phi)) + I_1. \tag{55}$$

Equation (54) shows that $G(M\phi))$ is the finite part of the vacuum propagator. A finite expression for $M(\Phi)$ is obtained by expressing it in terms of the renormalized parameters

$$M^2(\phi) = -m_r^2 + \frac{\lambda}{2}\phi^2 + \frac{\xi}{24}\phi^4 + \frac{\lambda}{2}G(M(\phi)) + \frac{\xi}{8}G(M(\phi))G(M(\phi)). \tag{56}$$

Second derivative of the tree level potential is defined as m_{tree} (tree level mass)

$$M^2(\phi) = m_{\text{tree}}^2(\phi) + \frac{\lambda}{2}G(M(\phi)) + \frac{\xi}{8}G(M(\phi))G(M(\phi)), \tag{57}$$

$$V_\beta^1(M(\phi)) = -\frac{M^3}{6\pi} + \frac{1}{\beta}\int \frac{d^2k}{(2\pi)^2}\ln(1-e^{\beta E}) + \frac{\Lambda^3}{6\pi}. \tag{58}$$

At zero temperature the second term vanishes and the last term is cut-off dependent. Cancellation of this divergence is obtained by combining V^0, V^2 and V^3:

$$V_\beta(M) = -\frac{M^3}{6\pi} + \frac{1}{\beta}\int \frac{d^2k}{(2\pi)^2}\ln(1-e^{\beta E}) + \frac{M^4}{2\lambda}$$
$$-\frac{1}{2}M^2G(M) - \frac{M^2}{24\eta}G^2(M) - F(\phi), \tag{59}$$

with

$$\eta^{-1} = \frac{\xi}{\lambda} \text{ and } F(\phi) = \left[\frac{m^2}{24\eta} - \frac{\lambda}{12} - \frac{29}{6!}\xi\phi^2\right]\phi^4. \tag{60}$$

$\xi = 0$ reproduces the result of Camellia and Pi. Using unrenormalized gap equation we combine V^0, V^2 and V^3 and writing them in terms of renormalized parameters

$$V^0 + V^2 + V^3 = \frac{\lambda_r}{8}\left[\phi^2 - \frac{2m_r^2}{\lambda}\right]^2 - \frac{\eta}{24}m_r^2\phi^4 - \frac{\lambda_r}{8}G^2(M) - \frac{\xi_r}{48}G^3(M) - F(\phi). \tag{61}$$

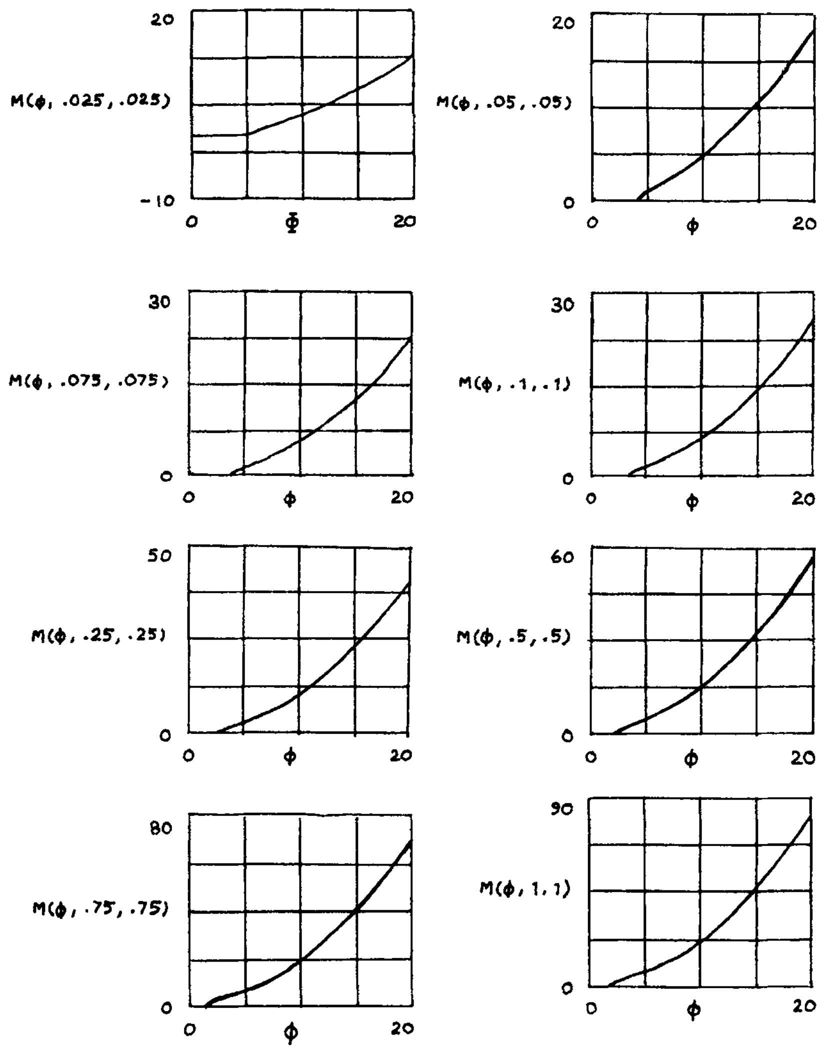

Figure 4. Effective mass for various couplings-Φ^6 theory.

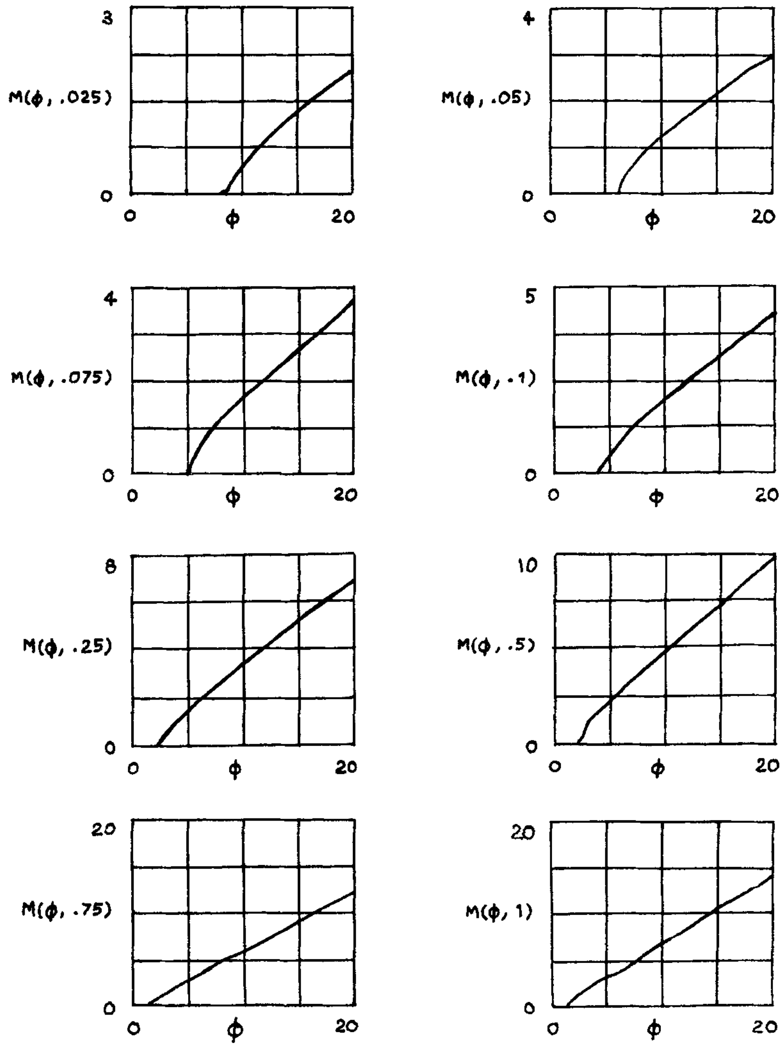

Figure 5. Effective mass for various couplings-Φ^4 theory.

In the case of $(2+1)$ dimensional Φ^4 theory $F(\phi) = \lambda/12$ which is finite. Thus unlike $(3+1)$ dimensional Φ^4 theory the effective potential does not contain any unrenormalized parameters. But in the case of Φ^6 theory $F(\phi)$ contains m which is an unrenormalized mass parameter. But here we can make $F(\phi) = 0$ by adjusting the parameters suitably and make the unrenormalized parameters vanish.

6. High temperature expansion

Evaluation of the effective potential at high temperature up to one loop level has been done earlier in $(3+1)$ and $(1+1)$ dimensions [2, 35]. Additional terms appearing in the

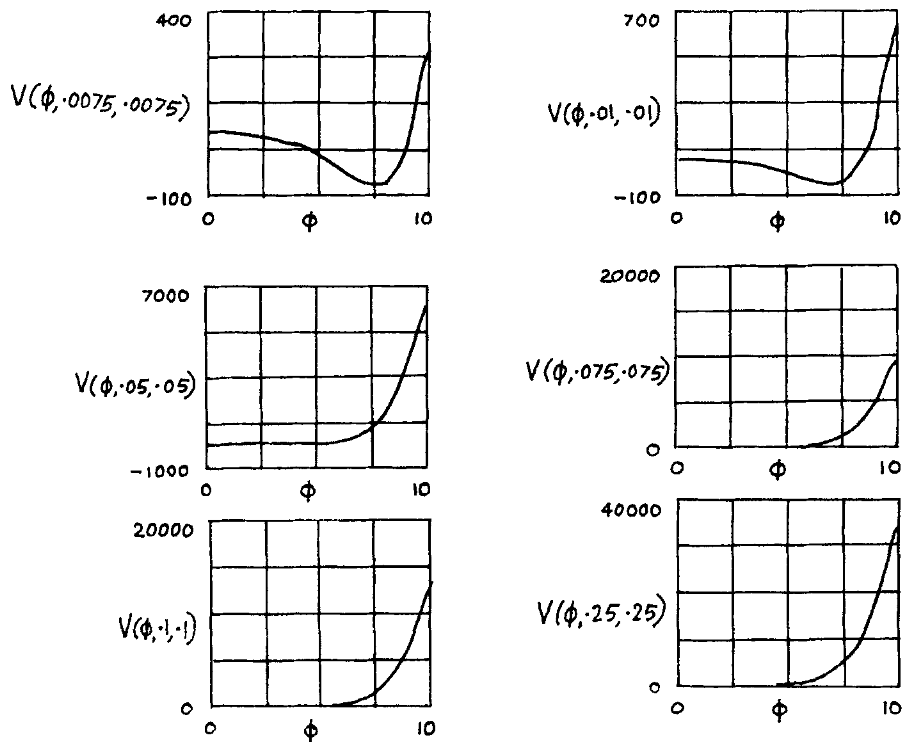

Figure 6. Effective potential for various couplings-Φ^6 theory.

expression for the effective potential can be obtained by evaluating $G(M(\phi))$, for $M(\phi)/T \ll 1$. The relevant integral (eq. (50)) is of the form

$$h_n(y) = \frac{1}{\Gamma(n)} \int_0^\infty \frac{x^{n-1}\mathrm{d}x}{(x^2+y^2)^{(1/2)}} \frac{1}{\mathrm{e}^{(x^2+y^2)^{(1/2)}} - 1}, \tag{62}$$

$$y = \frac{m}{T}. \tag{63}$$

These integrals satisfy the differential equation

$$\frac{\mathrm{d}h_{n+1}}{\mathrm{d}y} = -\frac{y h_{n-1}}{n}. \tag{64}$$

High temperature expansion for the integral is obtained by using the identity

$$\frac{1}{\mathrm{e}^z - 1} = \frac{1}{z} - \frac{1}{2} + 2\sum_{l=1}^{\infty} \frac{z}{z^2 + (2\pi l)^2}. \tag{65}$$

Multiplying the integrand by a factor $x^{-\epsilon}$ for convergence, performing term by term integration and letting $\epsilon \to 0$ at the end we get

$$h_1(y) = \frac{\pi}{2y} + \frac{1}{2}\ln\frac{y}{4\pi} + \frac{1}{2}\gamma + O(y^2). \tag{66}$$

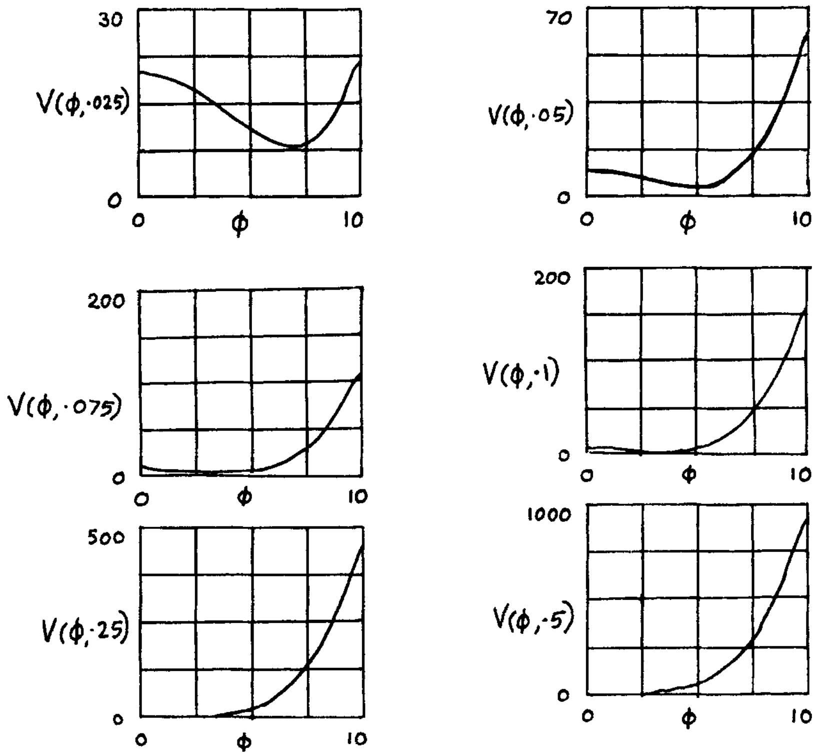

Figure 7. Effective potential for various couplings-Φ^4 theory.

$\gamma = 0.5772\ldots$ which is the Euler constant.

$$h_2(y) = -\ln(1 - e^{-\nu}). \tag{67}$$

Other values of h can be found by using the differential equation (64). We have

$$\frac{1}{\beta}\int \frac{d^2k}{(2\pi)^2}\frac{1}{E(e^{\beta E}-1)} = \frac{1}{\pi}\int \frac{kdk}{(2\pi)^2}\frac{1}{E(e^{\beta E}-1)}. \tag{68}$$

By using the formula (67) we get

$$G(M(\phi)) = -\frac{M(\phi)}{4\pi} + \frac{1}{\pi}\ln(1 - e^{-M/T}) \tag{69}$$

which can be used to evaluate V_{eff} at high temperature using (60). High temperature expansion for the effective mass can be obtained from (56).

7. Conclusions

A self-consistent improvement for the finite temperature Φ^6 theory is obtained as an extension of CJT formalism. Certain peculiarities of the Φ^6 field theory in $(2+1)$ dimensions are analysed. Φ^4 theory in $(2+1)$ dimensions does not contain any

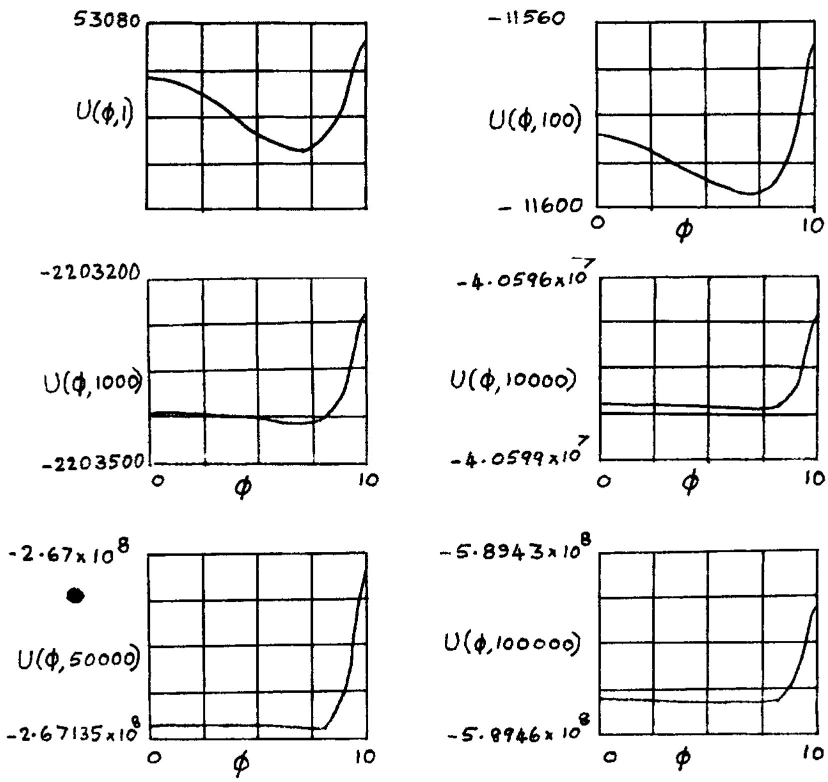

Figure 8. Effective potential at various temperatures-Φ^6 theory.

unrenormalized terms unlike its $(3+1)$ dimensional counterpart. Φ^6 theory in $(2+1)$ dimensions contains divergent terms in the form of an unrenormalized mass parameter in the expression for effective potential, but can be made to vanish. In this model physically meaningful stable theory is possible both for positive and negative λ, indicating the possibility of bound states. High temperature expansion for the effective potential is obtained.

Behavior of the effective mass can be clearly understood by numerically solving the equation for effective mass. Figures 4 and 5 show these graphs for certain relevant values at zero temperature for Φ^4 and Φ^6 theories. The straight region parallel to the ϕ axis indicates imaginary values of the effective mass. It is clear that this region indicates a broken symmetry phase. Comparison between Φ^4 and Φ^6 theories shows that they are identical in shape except for higher numerical values for Φ^6 model. A study of the behaviour of effective potential (figures 6 and 7) for different couplings is also given (in weak coupling range since the reliability of the approximation in the strong coupling range is not well established). Shape of the graph is same for a reasonable range of couplings with a notable difference in the stretch of the straight region.

Graph for effective potential at finite temperature (figures 8 and 9) shows that as temperature increases the minimum gradually dissappears indicating an approach to symmetry restoration (critical temperature). At sufficiently high temperature symmetry is found to be restored.

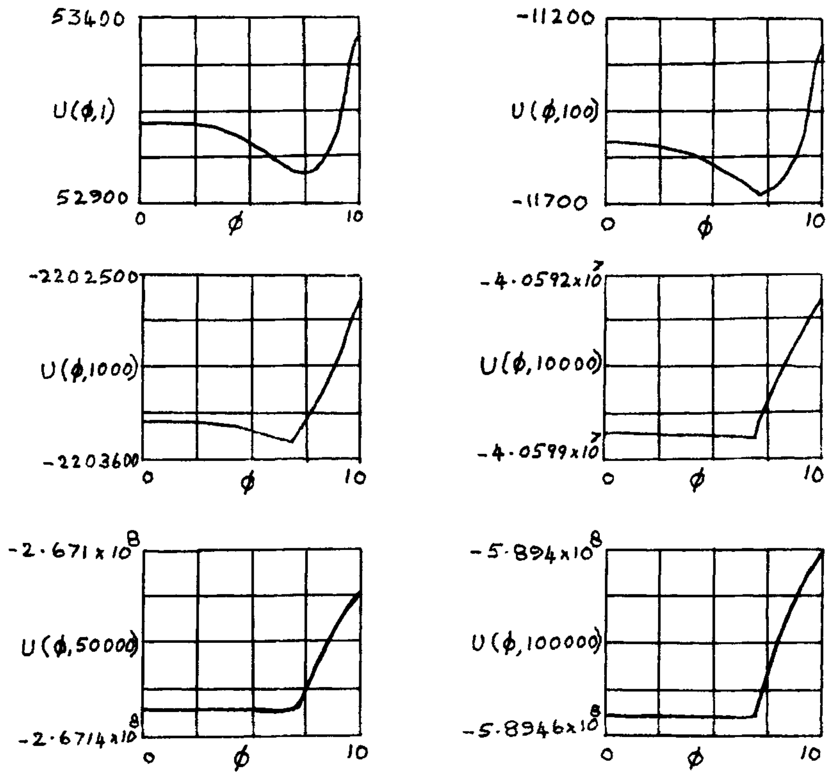

Figure 9. Effective potential at various temperatures-Φ^6 theory high temperature expansion.

Using the numerically obtained value of the effective mass, effective potential can be calculated using high temperature expansion (figure 9). The graphs clearly indicate broken symmetry phase at zero temperature. Comparison with the graphs obtained earlier [23] shows that behaviours of Φ^6 and Φ^4 theories are similar apart from considerable differences in numerical values.

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