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Causal dissipation in Robertson–Walker cosmological models

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Abstract. Some of the Robertson–Walker cosmological models filled with a fluid with bulk viscosity have been derived which are consistent with causal thermodynamics. The models are discussed briefly.

Keywords. FRW models; viscous dissipation; cosmology; extended irreversible thermodynamics.

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1. Introduction

Introduction of viscosity into the material content of model universes is gaining importance [1]. This is in the hope of having realistic description of the real universe mainly during the early stages of its evolution. Several workers have considered viscous fluid but only a few of them have tried to make it thermodynamically consistent. Thermodynamical consistency of the viscous fluid is undoubtedly essential as it leads to more realistic description. Moreover, if this consistency occurs in a way which allows the dissipation of viscosity causally, the reality of description is further enhanced.

Initial efforts to obtain the above consistency were based on inadequate theories given by Eckart [2], and Landau and Lifshitz [3] – inadequate in the sense that these (theories) allowed viscous dissipation with super luminal speeds [4, 5]. However, this drawback was later on overcome by resorting to extended irreversible thermodynamics (EIT) which provided the relativistic causality in a natural way [6, 7]. Thus the viscosity obeying EIT may be a reasonable resort.

In the frame work of standard (Robertson–Walker) cosmology, models have been derived which are causally well behaved as regards their viscous-fluid contents. Hiscock and Salmonsan [8] have considered the flat case with bulk viscosity. Zakari and Jou [9] and Maartens [10], while discussing it, have investigated the possibility of exponential inflation. Banerjee and Beesham [11] have obtained a power law solution as well. In the non-flat case we have, of course, the solution given by Mindez and Pavon [12], but in the framework of Eckart's theory.

In the present paper we resort to EIT and investigate both the flat and non-flat cases. We also discuss the solutions obtained briefly.

2. Derivations of the models

We consider the Robertson–Walker space-time given by

$$ds^2 = -dt^2 + R^2(t)[(1 - kr^2)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)], \tag{1}$$

where $k = 0, \pm 1$ and the energy momentum tensor T_{ij} in the Einstein's field equation

$$G_{ij} = T_{ij} \quad (\text{for } 8\pi G = 1 = c) \tag{2}$$

of the form

$$T_{ij} = (\rho + p_{\text{eff}})u_i u_j + p_{\text{eff}}\, g_{ij}, \tag{3}$$

where $p_{\text{eff}} = p + \Pi$; ρ, p, Π and u_i being respectively the energy density, equilibrium pressure, bulk viscous stress and the unit flow vector of the fluid co-moving orthogonally to the surface of homogeneity. We also consider the laws of

1. *Energy conservation*

$$T^{ij}_{;j} = 0. \tag{4}$$

2. *Number conservation*

$$N^i_{;i} = 0, \quad N^i = nu^i. \tag{5}$$

alongwith

3. *The H-theorem*

$$S^i_{;i} \geq 0. \tag{6}$$

4. *The Gibb's equation*

$$\begin{aligned} S^i &= sN^i - (\tau\Pi^2/2\xi T)u^i, \\ Tds &= d(\rho/n) + pd(1/n), \end{aligned} \tag{7}$$

where s, n, $T \geq 0$, $\xi \geq 0$ and $\tau \geq 0$ are respectively the specific entropy, number density, temperature, bulk viscosity coefficient and the relaxation coefficient for bulk viscous effects. A semicolon in the above indicates covariant differentiation.

Equations (3)–(7) give rise to evolution equation [10, 11] for the bulk viscosity

$$\Pi + \tau\dot{\Pi} = -3\xi H - (\epsilon/2)\tau\Pi(3H + \dot{\tau}/\tau - \dot{\xi}/\xi - \dot{T}/T), \tag{8}$$

where

$$H = (1/3)\theta = (1/3)u^i_{;i} = \dot{R}/R,$$

θ being the expansion scalar and an overhead dot standing for differentiation with respect to time t. Equations (1)–(3) lead to

$$(\dot{R}/R)^2 + k/R^2 = \rho/3 \tag{9}$$

and

$$\ddot{R}/R = -(1/6)[\rho + 3(p + \Pi)]. \tag{10}$$

Thus we get only three independent equations (8)–(10) in seven unknowns $\rho, p, \Pi, \xi, \tau, T$ and R. So, as usual, we assume the following ad-hoc relations

$$p = (\gamma - 1)\rho, \quad 1 \leq \gamma \leq 2, \tag{11}$$

$$\xi = \alpha\rho^q, \quad \alpha \geq 0, \quad q \geq 0, \tag{12}$$

$$\tau = \xi/\rho, \tag{13}$$

and

$$T = \beta\rho^r, \quad \beta > 0, \quad r \geq 0. \tag{14}$$

Now with the help of (9), (10) and (11), eq. (8) can be written in the form

$$\begin{aligned} &\tau\ddot{H} + (3/2)\tau(\epsilon + 2\gamma)H\dot{H} + \dot{H} + (1/2)\epsilon\tau(\dot{\tau}/\tau - \dot{\xi}/\xi - \dot{T}/T)\dot{H} \\ &\quad + (9/4)\epsilon\gamma\tau H^3 + (3/4)\epsilon\gamma\tau(\dot{\tau}/\tau - \dot{\xi}/\xi - \dot{T}/T)H^2 + (3/2)\gamma H^2 \\ &\quad - (3/2)\xi H + \{(3\gamma - 2)/2R^2\}\cdot k\tau \\ &\quad \times [1/\tau + (3/2)\epsilon H - 2\dot{R}/R + (\epsilon/2)(\dot{\tau}/\tau - \dot{\xi}/\xi - \dot{T}/T)] = 0. \end{aligned} \tag{15}$$

Using (12)–(14), eq. (15) becomes

$$\begin{aligned} &\ddot{H} + (3/2)(2\gamma + \epsilon)\dot{H}H + \alpha^{-1}[3(H^2 + k/R^2)]^{1-q} \\ &\quad \times [\dot{H} + 3\gamma H^2/2 + (k/2R^2)(3\gamma - 2)] + (9/4)\epsilon\gamma H^3 \\ &\quad + [(k/2R^2)(3\gamma - 2)(3\epsilon/2 - 2) - (9/2)(H^2 + k/R^2)]H \\ &\quad - (r + 1)\epsilon H\cdot(\dot{H} - k/R^2)/(H^2 + k/R^2) \\ &\quad \times [\dot{H} + (3/2)\gamma H^2 + (k/2R^2)(3\gamma - 2)] = 0. \end{aligned} \tag{16}$$

A general solution of eq. (16) is difficult to obtain. However, for the case $k = 0$, Maartens [10] has obtained an exponential solution whereas Banerjee and Beesham [11] have derived a power law solution. We notice in the present paper that $R = t$ with $q = 1/2$ is also a solution provided:

(i) *For* $k = 1$:

$$\alpha = \begin{cases} \sqrt{6}(3\gamma - 2)/(5 + 6\gamma) > 0 & \text{when } \epsilon = 0 \\ 2\sqrt{6}(3\gamma - 2)/4(5 + r) - 3\gamma(1 + 2r) > 0, & \\ \quad 0 < r < (20 - 3\gamma)/2(3\gamma - 2) & \text{when } \epsilon = 1. \end{cases}$$

(ii) *For* $k = 0$:

$$\alpha = \begin{cases} \sqrt{3}(3\gamma - 2)/(5 + 6\gamma) > 0 & \text{when } \epsilon = 0 \\ 2\sqrt{3}(3\gamma - 2)/4(5 + r) - 3\gamma(1 + 2r) > 0, & \\ \quad 0 < r < (20 - 3\gamma)/2(3\gamma - 2) & \text{when } \epsilon = 1. \end{cases}$$

3. Discussion

For $k = -1$, the solution $R = t$, holds identically, though eq. (16) is not the proper way to go through in this case. Actually, in this case, $\rho = 0 = p$. Thus no matter exists and the universe is empty. Thus $R = t$ is inconsistent when $k = -1$.

When $k = 1$, we obtain for the solutions corresponding to both the truncated ($\epsilon = 0$) and full ($\epsilon = 1$) theories the following expressions for concerned quantities:

$$\rho = 6/t^2, \quad p = 6(\gamma - 1)/t^2, \quad \Pi = 2(2 - 3\gamma)/t^2,$$
$$\xi = \alpha\sqrt{6}/t, \quad \tau = (\alpha/\sqrt{6})t, \quad T = 6^r \beta t^{-2r}.$$

Clearly, for $t > 0$, each of the quantities ρ, p, ξ, τ and T is positive, while Π is negative. Moreover, both ρ and p tend to ∞ as $t \to 0$, and they tend to 0 as $t \to \infty$. Also $\theta = 3/t \to \infty$ and 0 respectively when $t \to 0$ and ∞. Thus the model starts with a big bang at $t = 0$ and its rate of expansion vanishes asymptotically. The temperature decreases gradually to zero when the expansion stops. We observe that $-\Pi = p + \rho/3$ which restricts the model to be inflationary (actually, the condition for inflation $\ddot{R} > 0$ requires, by virtue of (10), that $-p_{\rm eff} > \rho/3$). The behaviour of the models are similar to that corresponding to the solutions for $k = 0$, in which case the expressions for various quantities differ slightly by some positive constants only. The solutions for $k = 0$, form the particular cases of that obtained by Banerjee and Beesham [11].

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