Two body non-leptonic Λ_b decays in the large N_c limit

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Abstract. The two body non-leptonic Λ_b decays are analysed in the HQET with factorization approximation and large N_c limit. In this limit, Λ_b and Λ_c baryons can be treated as the bound states of chiral soliton and heavy meson, and consequently the Isgur–Wise function comes out in a straight forward manner. The results obtained remain well below their previously predicted upper limit.

Keywords. Heavy quark effective theory; factorization approximation.

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1. Introduction

In the last few years considerable progress has been achieved in the understanding of the weak decays of heavy hadrons due to the development of the heavy quark effective theory (HQET) [1–5]. Contrary to the significant progress made in the studies of the meson decays, advancement in the arena of heavy baryons has been very slow. In particular the non-leptonic weak decays of heavy baryons have not been understood clearly till now. At present, there are not many experimental results available for heavy baryons. But in the future we may expect more and more data coming from the colliders. Hence the study of heavy baryons is of great interest in the near future. In fact some phenomenological approaches such as pole model, current algebra etc. have been employed to analyse these decay processes. The well known factorization hypothesis [6–12] which has been applied successfully to the heavy meson decays can also be applied for heavy baryon cases for large N_c limit [13].

In this article we intend to study the two body non-leptonic Λ_b decays in the heavy quark effective theory [1–5] considering factorization approximation. The HQET provides a convenient and simplified framework to analyse the weak decays of heavy hadrons, composed of one heavy quark and any number of light quarks. Of particular importance are the semileptonic decays of heavy mesons, where in the limit of infinite quark masses, all the hadronic form factors can be expressed in terms of a single universal function $\xi(v \cdot v')$, the Isgur–Wise function [1]. The function depends only on the four velocities of the heavy particles involved, and is normalized at the point of zero recoil. Similarly in case of weak decays of heavy baryons one can also write the form factors in terms of another Isgur–Wise function [3, 5].

We therefore consider it worthwhile to investigate the two-body nonleptonic decays $\Lambda_b \rightarrow \Lambda_c^+ P^-$ and $\Lambda_b \rightarrow \Lambda_c^+ V^-$, where P and V denote pseudoscalar and vector mesons

respectively, using the HQET in conjunction with factorization approximation. In fact factorization method works well for the description of non-leptonic decays of heavy baryons, in the large N_c limit [13]. The use of HQET implies that the expression for the decay widths should contain the universal Isgur–Wise function, which is normalized at the point of zero recoil. However other than at the point of zero recoil, HQET does not predict the shape of the Isgur–Wise (IW) function. Therefore to know the value of Isgur–Wise function at the particular kinematic point of interest, we evaluate it in the large N_c limit [14, 15], where factorization approximation is valid. Earlier these decays have been studied by Mannel *et al* [13]. They have parametrized the Isgur–Wise form factor $G_1(v\cdot v')$ in three different forms as

$$G_1(v\cdot v') = 1 + \frac{1}{4}a(v - v')^2(v + v')^2, \quad G_1(v\cdot v') = \frac{1}{1 - (v - v')^2/\omega_0^2},$$

$$\text{and} \quad G_1(v\cdot v') = \exp[b(v - v')^2], \tag{1}$$

and they have taken the values of the parameters a, b, and ω_0, from the work on B meson decays [16]. They have also predicted the upper limits for the branching ratios for these decay processes, considering the normalized value of the IW function.

The report is organized as follows. In §2 we present the general framework for the study of the nonleptonic decays in the factorization method. The Isgur–Wise function is evaluated in the large N_c limit in §3. Section 4 contains results and discussions.

2. General framework

Neglecting the penguin contribution, the four fermion effective Hamiltonian relevant to the $\Lambda_b \rightarrow \Lambda_c^+ P^-$ and $\Lambda_b \rightarrow \Lambda_c^+ V^-$ decays is given as [13]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{UD}^* V_{cb}[C_1(m_b)\mathbf{O}_1 + C_2(m_b)\mathbf{O}_2], \tag{2}$$

with

$$\mathbf{O}_1 = (\bar{D}U)^\mu(\bar{c}b)_\mu \quad \text{and} \quad \mathbf{O}_2 = (\bar{c}U)^\mu(\bar{D}b)_\mu, \tag{3}$$

where G_F is the Fermi coupling constant and the quark current $(\bar{q}'q)_\mu$ is a short hand for $\bar{q}'_\alpha\gamma_\mu(1 - \gamma_5)q_\alpha$; α is the color index. U and D are either c, s or u, d quarks. Thus for U, D = u, d we have π^- and ρ^- in the final state as P and V while for U, $D = c$, s the final P/V states are D_s/D_s^* mesons. The values of the Wilson coefficients $C_{1,2}$ can be calculated using the leading logarithmic approximation (LLA) [17] and are given as

$$C_1(m_b) = 1.11 \quad \text{and} \quad C_2(m_b) = -0.26. \tag{4}$$

The decays $\Lambda_b \rightarrow \Lambda_c^+ P^-$ and $\Lambda_b \rightarrow \Lambda_c^+ V^-$ can occur by the operator $\mathbf{O}_1$ where it is assumed that the $\Lambda_b \rightarrow \Lambda_c$ transition is caused by the current operator $(\bar{c}b)$ and that $P^-(V^-)$ are created by the current operator $(\bar{D}U)$. In the factorization approximation it is assumed that the $\Lambda_b \rightarrow \Lambda_c$ transition and the $P^-(V^-)$ creation are independent of each other, and hence the amplitude can be written as

$$\langle\Lambda_c^+ P^-(V^-)|(\bar{D}U)(\bar{c}b)|\Lambda_b\rangle = \langle P^-(V^-)|(\bar{D}U)|0\rangle\langle\Lambda_c^+|(\bar{c}b)|\Lambda_b\rangle. \tag{5}$$

In the large N_c limit, where factorization approximation is valid, the contribution of O_2 to these decays is suppressed. Therefore one can write the amplitude for the decays $\Lambda_b \to \Lambda_c^+ P^-$ and $\Lambda_b \to \Lambda_c^+ V^-$ as

$$\mathcal{M}(\Lambda_b \to \Lambda_c^+ P^-(V^-)) = \frac{G_F}{\sqrt{2}} V_{UD}^* V_{cb} C_1(m_b) \langle P^-(V^-)|(\bar{D}U)^\mu|0\rangle \langle \Lambda_c^+|(\bar{c}b)_\mu|\Lambda_b\rangle.$$

(6)

To evaluate the factorized amplitudes we use the following matrix elements:

$$\langle P(p)|(\bar{D}U)^\mu|0\rangle = -if_P p^\mu,$$

(7)

and

$$\langle V(p,\epsilon)|(\bar{D}U)^\mu|0\rangle = f_V M_V \epsilon^\mu,$$

(8)

where f_P and f_V are the pseudoscalar and vector meson decay constants respectively. The matrix element $\langle \Lambda_c|(\bar{c}b)_\mu|\Lambda_b\rangle$ is given in the HQET [3,4] as

$$\langle \Lambda_c^+(v',s')|\bar{c}\gamma_\mu(1-\gamma_5)b|\Lambda_b(v,s)\rangle = \eta(v\cdot v')\bar{u}_c(v',s')\gamma_\mu(1-\gamma_5)u_b(v,s),$$

(9)

where $\eta(v\cdot v')$ is the baryonic Isgur–Wise function, $u_c(v',s')$ and $u_b(v,s)$ are the spinors of the Λ_c and Λ_b baryons. Thus with eqs (6–9) we obtain the decay widths for the decay processes $\Lambda_b \to \Lambda_c^+ P^-$ and $\Lambda_b \to \Lambda_c^+ V^-$, given as

$$\Gamma(\Lambda_b(v) \to \Lambda_c^+(v')P^-(p)) = \frac{G_F^2}{8\pi M_{\Lambda_b}^2} |V_{UD}^* V_{cb}|^2 C_1^2(m_b) f_P^2 \eta^2(v\cdot v')|\mathbf{p}|$$

$$\times [(M_{\Lambda_b}^2 - M_{\Lambda_c}^2)^2 - M_P^2(M_{\Lambda_b}^2 + M_{\Lambda_c}^2)],$$

(10)

and

$$\Gamma(\Lambda_b(v) \to \Lambda_c^+(v')V^-(p)) = \frac{G_F^2}{8\pi M_{\Lambda_b}^2} |V_{UD}^* V_{cb}|^2 C_1^2(m_b) f_V^2 \eta^2(v\cdot v')|\mathbf{p}|$$

$$\times [(M_{\Lambda_b}^2 - M_{\Lambda_c}^2)^2 + M_V^2(M_{\Lambda_b}^2 + M_{\Lambda_c}^2 - 2M_V^2)],$$

(11)

where $|\mathbf{p}|$ is the c.o.m momentum of the emitted particles in the rest frame of initial Λ_b baryon and M's are the corresponding pseudoscalar, vector meson and baryon masses. The above expressions for the decay widths contain besides the known quantities, the unknown Isgur–Wise function, which can be calculated in the large N_c limit in a simple manner.

3. Evaluation of the Isgur–Wise function

Here we have presented the evaluation of the Isgur-Wise function in the same manner as suggested in ref. [14]. In the large N_c limit the light baryons n, p, Δ etc. can be viewed as solitons in the chiral Lagrangian for pion self interaction [18]. The baryons containing a single heavy charm (or bottom) quark are bound states of these solitons with D and D^* (or B and B^*) mesons [19–22]. In this paper we use the bound state soliton picture to estimate the value of the baryonic Isgur–Wise function. In the ground state of Λ_Q baryons, the light

quarks are in the spin 0 state [23]. Hence in the bound state soliton picture, Λ_Q-type bound state arise when the spin of the light degrees of freedom of the heavy meson and the spin of the nucleon are combined into a spin zero configuration whereas the isospin of the heavy meson and that of the nucleon are combined into an isospin zero state. Other baryons (e.g, the Δ) only contribute to the bound states with higher isospin.

Let the light degrees of freedom of the heavy baryon be denoted by $|I, I_3; s_l, m_l\rangle$, where I and s_l denote their isospin and spin quantum numbers while I_3 and m_l are their third components respectively. Hence the light degrees of freedom of Λ_Q baryon is denoted by $|0, 0; 0, 0\rangle$. The chiral soliton is denoted by $|R, b; R, n\rangle$, where $R = 1/2$ for the nucleon. On the other hand the light degrees of freedom of the heavy meson is given as $|1/2, c; 1/2, p\rangle$.

In the large N_c limit, the binding potential between the chiral soliton and heavy meson is independent of both the isospin and spin of the particles. Hence for the light degrees of freedom of Λ_Q baryon, we have the decomposition as

$$
|0, 0; 0, 0(v)\rangle = \int d^3q \Phi_Q(\mathbf{q})(1/2, b; 1/2, c|0, 0)(1/2, n; 1/2, p|0, 0)
$$
$$
\times |1/2, b; 1/2, n(-\mathbf{q} + M_B\mathbf{v}))|1/2, c; 1/2, p(\mathbf{q} + M_H\mathbf{v})\},
$$
(12)

where $(j_1, m_1; j_2, m_2|J, M)$'s are the Clebsch–Gordan coefficients. $\Phi_Q(\mathbf{q})$ is the ground state wave function, M_B and M_H are the masses of the chiral soliton and heavy meson respectively.

The spin-1/2 Λ_Q baryon is composed of a spin-1/2 heavy quark and spin-0 light degrees of freedom. Hence the matrix element of the current $\bar{c}\gamma_\mu(1 - \gamma_5)b$ between Λ_b and Λ_c baryons is given as

$$
\langle \Lambda_c^+(v', s')|\bar{c}\gamma_\mu(1 - \gamma_5)b|\Lambda_b(v, s)\rangle = \langle 0, 0; 0, 0(v')|0, 0; 0, 0(v)\rangle \bar{u}_c\gamma_\mu(1 - \gamma_5)u_b.
$$
(13)

Comparing (9) and (13) we obtain the expression for the baryonic IW function as

$$
\eta(v \cdot v') = \langle 0, 0; 0, 0(v')|0, 0; 0, 0(v)\rangle
$$
$$
= \int d^3q' d^3q \Phi_c^*(\mathbf{q'})\Phi_b(\mathbf{q})(1/2, b'; 1/2, c'|0, 0)^*
$$
$$
\times (1/2, n'; 1/2, p'|0, 0)^*(1/2, b; 1/2, c|0, 0)(1/2, n; 1/2, p|0, 0)
$$
$$
\times (1/2, b'; 1/2, n'(-\mathbf{q'} + M_B\mathbf{v'})|1/2, b; 1/2, n(-\mathbf{q} + M_B\mathbf{v}))
$$
$$
\times \{1/2, c'; 1/2, p'(\mathbf{q'} + M_H\mathbf{v'})|1/2, c; 1/2, p(\mathbf{q} + M_H\mathbf{v})\}.
$$
(14)

Using the normalization conditions for the chiral soliton and heavy meson states, it is found that all the Clebsch–Gordan coefficients in (14) turn out to be unity and the Isgur–Wise function is given as

$$
\eta(v \cdot v') = \int d^3q \Phi_c^*(\mathbf{q})\Phi_b(\mathbf{q} + M_B(\mathbf{v} - \mathbf{v'})).
$$
(15)

It is noted from (15) that the IW function depends on the spatial wave function $\Phi_Q(\mathbf{q})$ of the Λ_Q baryon. In the large N_c limit, the binding potential between the heavy meson and

Table 1. Prediction for the branching ratios BR (in %) for the two body nonleptonic Λ_b decays in the large N_c limit.

Decay process	|p| in MeV	$\eta(v \cdot v')$	BR Present calculation	BR Ref. [13]	BR_{max} Ref. [13]
$\Lambda_b \to \Lambda_c^+ \pi^-$	2355.343	0.456	0.342	$0.46^{+0.20}_{-0.31}$	2.0
$\Lambda_b \to \Lambda_c^+ D_s^-$	1849.748	0.596	1.156	$2.3^{+0.30}_{-0.40}$	6.5
$\Lambda_b \to \Lambda_c^+ \rho^-$	2284.288	0.474	0.954	$0.66^{+0.24}_{-0.40}$	2.5
$\Lambda_b \to \Lambda_c^+ D_s^{*-}$	1765.854	0.621	1.769	$1.73^{+0.20}_{-0.30}$	4.7

the chiral soliton is simple harmonic [15], and hence the wave function is taken as

$$\Phi_Q(\mathbf{q}) = \frac{1}{(\pi^2 \mu_Q \kappa)^{3/8}} \exp\left(-\frac{\mathbf{q}^2}{2\sqrt{\mu_Q \kappa}}\right), \tag{16}$$

where $\mu_Q = M_B M_H / (M_B + M_H)$, is the reduced mass of the bound state, M_H denotes the masses of D/B mesons for Φ_c/Φ_b wave functions. κ is the spring constant and its value is taken to be $(440\,\text{MeV})^3$ [24]. In the rest frame of the initial state, $v = (1, \vec{0})$ and $\mathbf{v}'$ directed along z-axis we obtain the Isgur–Wise function (15) using (16) for non-relativistic recoils i.e., $|\mathbf{v}'|^2 \approx 2(v \cdot v' - 1)$, as

$$\eta(v \cdot v') = \left[\frac{4\sqrt{\mu_b \mu_c}}{(\sqrt{\mu_b} + \sqrt{\mu_c})^2}\right]^{3/4} \exp\left(-\frac{(v \cdot v' - 1)M_B^2}{\sqrt{\kappa}(\sqrt{\mu_b} + \sqrt{\mu_c})}\right). \tag{17}$$

It should be noted from (17) that the Isgur–Wise function slightly deviates from unity at the point of zero recoil. This violation of normalization condition can be explained as follows. The heavy quark symmetry arises in the limit of QCD, where the heavy quark mass m_Q is taken formally to be infinite and in this limit all the hadronic form factors can be expressed in terms of the IW function. However here we have used finite masses for the heavy mesons (i.e., B and D mesons), and hence heavy quark symmetry breaks down. Thus breaking of the heavy flavor symmetry causes a violation of the normalization condition $\eta(1) = 1$.

The product $(v \cdot v')$ is determined by considering the kinematics of the system. Since we are dealing with the two body decays $\Lambda_b(v) \to \Lambda_c^+(v')P^-(p)/V^-(p)$, from momentum conservation we obtain

$$v \cdot v' = \frac{M_{\Lambda_b}^2 + M_{\Lambda_c}^2 - M_{P/V}^2}{2M_{\Lambda_b}M_{\Lambda_c}}. \tag{18}$$

Taking the masses of the particles from [25] the values of the Isgur–Wise functions are calculated with (17) and (18) as presented in table 1.

4. Results and discussion

Having obtained the values of the Isgur–Wise function, we use the following data to estimate the decay widths for the processes $\Lambda_b \to \Lambda_c^+ P^-$ and $\Lambda_b \to \Lambda_c^+ V^-$ using (10) and

(11). The CKM matrix elements $V_{cb} = 0.041$, $V_{ud} = 0.0976$ and $V_{cs} = 0.9743$ are taken from ref. [25] and the values of the decay constants used are $f_\pi = 130.7\,\text{MeV}$, $f_{D_s} = f_{D_s^*} = 232\,\text{MeV}$ [25] and $f_\rho = 210\,\text{MeV}$ [13]. With these values we have evaluated the branching ratios for several two body non-leptonic Λ_b decays as presented in table 1.

In this work we have estimated the branching ratios for $\Lambda_b \to \Lambda_c^+ P^-$ and $\Lambda_b \to \Lambda_c^+ V^-$ decays in the heavy quark effective theory with factorization approximation. In fact factorization method works well for the description of non-leptonic decays of heavy baryons in the large N_c limit [13]. The use of HQET allows us to write the weak decay form factors in terms of the Isgur–Wise function. However it does not predict the shape of the IW function except at the point of zero recoil, where it is normalized to unity. Therefore to know the value of Isgur–Wise function at the particular kinematic point of interest, we have evaluated it in the large N_c limit [14, 15] considering the bound state soliton picture. These decays have been previously studied in ref. [13] in which they have parametrized the Isgur–Wise function in three different forms (eq. (1)) and used the values of the unknown parameters from the work on B meson decays [16]. However in our case we have evaluated the Isgur–Wise function in the large N_c limit where factorization approximation is valid and the results came out in a straightforward manner. Therefore our predicted results differ from theirs as noted from table 1. They have also predicted the upper limit of the branching fractions for these decays, by considering the normalized value of the Isgur–Wise function. The results of the present investigation lie well below their corresponding upper limit. As the experimental data are expected in the future from the colliders, these results can be verified, which will definitely enrich our understanding in this sector to a greater extent.

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