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# Motion of test particles around domain walls

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**Abstract.** We present a detailed analysis of the motion of test particles around domain walls. The study of the trajectories of the test particles has been done using the Hamilton–Jacobi formalism. In most of the cases we show that the particles can not be trapped by the walls.

**Keywords.** Domain walls; test particles; trajectories; Hamilton–Jacobi formalism.

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## 1. Introduction

The explanation of the formation of large scale structure of the Universe is one of the basic problem of cosmology even today. In literature, the widely used mechanisms for this structure formation are the gravitational perturbations generated by topological defects namely, domain walls, cosmic strings, monopoles and their hybrids [1]. It is generally assumed that these topological defects are formed at the very early stages of the evolution of the Universe when the Universe exhibits a series of phase transitions [2]. In particular, the appearance of domain walls is associated with the breaking of a discrete symmetry [3] i.e. the vacuum manifold consists of several disconnected components.

From the cosmological standpoint, domain walls have become important after a proposal for a new scenario of galaxy formation by Hill, Schramn and Fry [4]. According to them, the formation of galaxies are due to domain walls produced during a phase transition after the time of recombination of matter and radiation [5]. In fact, the phase transition occurs by the breaking of a discrete symmetry of a weakly coupled scalar field of pseudo-gold stone bosons ([3, 5]).

In this paper, we study the motion of test particles in the gravitational field of static and non-static planar domain walls and of plane symmetric domain walls using Hamilton–Jacobi (H–J) formalism and examine whether bound orbits are possible or not. This study will be interesting in the relevance of structure formation because it gives us some idea about the behaviour of the particles (created at the early Universe) in the gravitational field of the domain walls.

## 2. Plane symmetric domain walls

### 2.1 *Thick walls*

The line element describing plane symmetric thick domain walls is given by [5]

$$dS^2 = \frac{\mathrm{sech}^{2n}(z)}{L^2}\,[dt^2 - n^2 dz^2 - e^{2t}(dx^2 + dy^2)], \tag{1}$$

where $L$ and $n$ are constants. The horizon in all directions are of the order of $1/L$ and $n$ is involved in the relation between pressure perpendicular to the wall and the energy density.

We consider a relativistic particle with mass $m$ moving in the field of the plane symmetric thick domain wall (1). The H–J equation has the expression

$$L^2 \cosh^{2n}(z)[(\partial S/\partial t)^2 - 1/n^2(\partial S/\partial z)^2 - e^{-2t}\{(\partial S/\partial x)^2 + (\partial S/\partial y)^2\}] + m^2 = 0. \tag{2}$$

As the metric (1) is independent of $x$ and $y$ co-ordinates, a natural choice for H–J function in separable form will be

$$S(x,y,z,t) = p_x \cdot x + p_y \cdot y + S_1(z) + S_2(t). \tag{3}$$

Here, the constants $p_x$ and $p_y$ can be termed as the momentum of the particle in the plane of the wall. If we substitute the ansatz (3) for $S$ in the H–J equation (2) then the expression (in integral form) for the unknown functions $S_1$ and $S_2$ are

$$S_1(z) = \epsilon n \cdot \int \left[-E^2 + \frac{m^2}{L^2}\,\mathrm{sech}^{2n}(z)\right]^{1/2} dz, \tag{4a}$$

$$S_2(t) = \epsilon \cdot \int [-E^2 + p^2 e^{-2t}]^{1/2} dt, \tag{4b}$$

where $P^2 = p_x^2 + p_y^2$, $E$ is a constant and $\epsilon = \pm 1$ stands for the sign changing whenever $z$ (or $t$) passes through a zero of the integrand in (4a) or (4b) [6, 7].

In the H–J formalism, the path of the particle is characterized by [7, 8]

$$\frac{\partial S}{\partial p_x} = \text{constant}, \quad \frac{\partial S}{\partial p_y} = \text{constant and } \frac{\partial S}{\partial E} = \text{constant}.$$

Thus we get

$$x = \epsilon \int \frac{p_x \cdot e^{-2t}}{(p^2 e^{-2t} - E^2)^{1/2}}\, dt, \tag{5}$$

$$y = \epsilon \int \frac{p_y \cdot e^{-2t}}{(p^2 e^{-2t} - E^2)^{1/2}}\, dt, \tag{6}$$

and

$$\int \frac{dz}{[(m^2/L^2)\mathrm{sech}^{2n}(z) - E^2]^{1/2}} = \epsilon \int \frac{dt}{n} \cdot \frac{1}{\sqrt{P^2 \cdot e^{-2t} - E^2}}, \tag{7}$$

where we have taken the constants to be zero without any loss of generality. From (7) the velocity of the particle transverse to the wall is

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{1}{n}\frac{[(m^2/L^2)\mathrm{sech}^{2n}(z) - E^2]^{1/2}}{[P^2\,\mathrm{e}^{-2t} - E^2]^{1/2}}. \tag{8}$$

The turning points of the trajectory are given by $\mathrm{d}z/\mathrm{d}r = 0$ and as a consequence the potential curves are characterized by

$$\frac{E}{m} = \frac{1}{L}\,\mathrm{sech}^n(z). \tag{9}$$

As there are no real extrema (except for $z = 0$) for the potential curve, the particles can not be trapped by plane symmetric thick domain walls. This conclusion is also supported by Goetz [5] that wall described by metric (1) exerts repulsive gravitational field on the matter around it.

## 2.2 *Thin wall*

We now consider a plane thin domain wall located at $z = 0$. The metric ansatz describing the gravitational field of the Wall is [9]

$$\mathrm{d}S^2 = \mathrm{e}^{-4\pi\sigma|z|}(\mathrm{d}t^2 - \mathrm{d}z^2) - \mathrm{e}^{4\pi\sigma(t-|z|)}(\mathrm{d}r^2 + r^2\mathrm{d}\phi^2), \tag{10}$$

where $\sigma$ is the surface tension of the wall.

As the metric coefficients are independent of $\phi$ co-ordinate, the solution of the H–J equation

$$\begin{aligned}&\mathrm{e}^{4\pi\sigma|z|}[(\partial S/\partial t)^2 - (\partial S/\partial z)^2]\\ &\quad = \mathrm{e}^{-4\pi\sigma(t-|z|)}[(\partial S/\partial r)^2 + (1/r)^2(\partial S/\partial\phi)^2] + m^2 = 0,\end{aligned} \tag{11}$$

can be expressed in the following separable form

$$S(t,z,r,\phi) = S_1(t) + S_2(z) + S_3(r) + J\cdot\phi. \tag{12}$$

The constant $J$ can be identified as the angular momentum of the test particle and the unknown functions have the integral expressions

$$\begin{aligned}S_1 &= \epsilon\int(p_1^2\cdot\mathrm{e}^{-4\pi\sigma t} - E_1^2)^{1/2}\mathrm{d}t,\\ S_2 &= \epsilon\int(m^2\mathrm{e}^{-4\pi\sigma|z|} - E_1^2)^{1/2}\mathrm{d}z,\\ S_3 &= \epsilon\int(p_1^2\cdot\tau J^2/r^2)^{1/2}\mathrm{d}r,\end{aligned} \tag{13}$$

where $E_1$ and $p_1$ are separable constants. Now, the equation of the trajectories can be written as

$$\int(p_1^2\cdot\mathrm{e}^{-4\pi\sigma t} - E_1^2)^{-1/2}\mathrm{d}t = \epsilon\int(m^2\cdot\mathrm{e}^{-4\pi\sigma|z|} - E_1^2)^{-1/2}\mathrm{d}z, \tag{14}$$

$$\int (p_1^2 - J^2/r^2)^{-1/2} \mathrm{d}r = \epsilon \int \frac{\mathrm{e}^{-4\pi\sigma t \cdot \mathrm{d}t}}{(p_1^2\, \mathrm{e}^{-4\pi\sigma t} - E_1^2)^{1/2}}, \tag{15}$$

$$\phi = \epsilon \int \frac{J}{r^2} \cdot \frac{\mathrm{d}r}{\sqrt{p_1^2 - J^2/r^2}}. \tag{16}$$

Hence from (14), the expression for transverse velocity is

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \left\{ \frac{(m^2 \cdot \mathrm{e}^{-4\pi\sigma|z|} - E_1^2)}{(p_1^2\, \mathrm{e}^{-4\pi\sigma t} - E_1^2)} \right\}^{1/2}. \tag{17}$$

The turning points are given by

$$\frac{E_1}{m} = \mathrm{e}^{-2\pi\sigma|z|}. \tag{18}$$

As the expression on the right hand side is monotonic decreasing for $z > 0$ and $z < 0$ so bound states are not possible and particle experiences a repulsive force near the wall.

Therefore, particles cannot be trapped by plane domain wall whether it is thin or thick.

## 3. Planar domain wall

We now consider static planar wall with metric ansatz of the form [10]

$$\mathrm{d}S^2 = f^2(|x|)\mathrm{d}t^2 - \frac{1}{f^2(|x|)}\, \mathrm{d}x^2 - g^2(|x|)(\mathrm{d}y^2 + \mathrm{d}z^2), \tag{19}$$

where

$$f^2(x) = \frac{C}{x - x_0} - \frac{\Lambda}{3}\,(x - x_0)^2, \quad g^2(x) = K^2(x - x_0)^2,$$

and $C$, $K$ and $x_0$ are constants satisfying the inequality

$$C + \frac{\Lambda}{3}\, x_0^3 < 0.$$

In this case the H–J equation has the expression

$$\frac{1}{f^2(|x|)}\left(\frac{\partial S}{\partial t}\right)^2 - f^2(|x|)\left(\frac{\partial S}{\partial x}\right)^2 - \frac{1}{g^2(|x|)}\left\{\left(\frac{\partial S}{\partial y}\right)^2 + \left(\frac{\partial S}{\partial z}\right)^2\right\} + m^2 = 0. \tag{20}$$

The solution can be written as

$$S = -E_s t + S_1(x) + p_y \cdot Y + p_z \cdot Z, \tag{21}$$

where $E_s$, $p_y$ and $p_z$ are constants and $S_1(x)$ has the expression (in integral form)

$$S_1(x) = \epsilon \int \frac{1}{f(x)} \left[ \frac{E_s^2}{f^2} = \frac{p^2}{g^2} + m^2 \right]^{1/2} \mathrm{d}x. \tag{22}$$

The trajectory of the particle is characterized by

$$t = \epsilon \int \left(\frac{E}{f^3}\right) \cdot \frac{\mathrm{d}x}{[E_s^2/f^2 - p^2/g^2 + m^2]^{1/2}}, \tag{23}$$

$$y = \epsilon \int \left(\frac{p_y}{fg^2}\right) \cdot \frac{\mathrm{d}x}{[E_s^2/f^2 - p^2/g^2 + m^2]^{1/2}}, \tag{24}$$

$$z = \epsilon \int \left(\frac{p_z}{fg^2}\right) \cdot \frac{\mathrm{d}x}{[E_s^2/f^2 - p^2/g^2 + m^2]^{1/2}}. \tag{25}$$

Hence the expression for transverse velocity is

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \left(\frac{f^3}{E_s}\right) \cdot \left(\frac{E_s^2}{f^2} - \frac{p^2}{g^2} + m^2\right)^{1/2}. \tag{26}$$

The turning points are given by

$$\frac{E_s}{m} = \left\{f^2 \cdot \left(\frac{p^2}{m^2 g^2} - 1\right)\right\}^{1/2} \tag{27}$$

which determines the potential curves.

As the radicand in the above expression may have real extremals, depending on the nature of the arbitrary constants in the metric co-efficients, the trajectory of the test particle is bounded i.e. particle can be trapped by Walls.

Finally, we note that $x = x_0 + (3C/\Lambda)^{1/3} = x$, corresponds to an event horizon where spatial and temporal co-ordinates interchange their roles and the regions $x < \bar{x}$ and $x > \bar{x}$ cannot influence each other.

Therefore, we conclude that domain walls are not always exert repulsive forces. There are situations where there is a force of attraction and particles are trapped by the walls.

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