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Schrödinger picture formalism of Φ^6 theory

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Abstract. The static effective potential for a scalar field with Φ^6 interaction is calculated using the effective action in Schrödinger picture formalism. It is found that the effective potential obtained is same as the Gaussian effective potential as far as static case is concerned. Equivalence with the CJT formalism can also be established. As in CJT formalism after renormalization an unrenormalized mass term persists. Nonzero turning points are obtained both for positive and negative λ. Results are analysed numerically. Graphical analysis indicates a behaviour similar to that obtained for CJT formalism at zero temperature.

Keywords. Gaussian trial function; variational principle; Schrödinger picture; static effective potential; effective action; renormalization; CJT formalism; Gaussian effective potential.

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1. Introduction

Variational methods in functional Schrödinger picture have been found to be very useful in the study of detailed structures of the quantum fields, both for the bosonic and fermionic field theories. Recently various interesting applications of this formalism have been presented [1–3]. In $(2+1)$ dimensional Thirring model, Gaussian approximation provides better information than the large-N approximation [4]. The formalism has been successfully used to derive effective potential in $(2+1)$ dimensional Liouville model [5]. Quantum field theoretic analysis of inflation dynamics in a $(2+1)$ dimensional universe has been worked out using this method. Taking into consideration the recent applications of the formalism in $(2+1)$ dimensional theories we propose to apply the method to the most general renormalizable scalar theory in $(2+1)$ dimensions that is a Φ^6 model. We find that the effective potential expression that emerges using functional Schrödinger picture is same as that derived using CJT formalism [6, 7]. The functional Schrödinger picture formalism for quantum field theory is a generalization from ordinary quantum mechanics to infinite number of degrees of freedom that comprise a field [8]. The method is suitable for both static and time-dependent problems at zero and finite temperatures and for quantum fields far from equilibrium. It has also been shown that renormalization in this model does not pose any special difficulties for static or time-dependent problems [9–15]. Among the applications of this approach are scalar QED, quantum mechanics of inflation, quantum roll process and quantum processes in non-euclidean space-time [16].

The functional Schrödinger picture formalism for quantum field theory has been found to be superior to conventional Fock space methods for analysing detailed structural properties of a quantum field. In this method one need not choose a vacuum and normal ordering is not required. Schrödinger picture method has been extensively used in studying solitons and other collective phenomena, topological defects in gauge theories and confinement. Time dependent problems relevant to early universe studies and non-equilibrium thermal physics cannot be studied using conventional Greens's function methods which require an initial condition for the solution. In these areas time-dependent Schrödinger picture has been profitably used. Also for analysing representations of transformation groups the method is very useful [9]. One can achieve intrinsic regularization and renormalization without any reference to vacuum state and an unique representation is obtained. In this method a quantum mechanical state $|\Psi(t)\rangle$ is replaced by a functional of the c-number field $\phi(x)$

$$|\Psi(t)\rangle \rightarrow \Psi[\phi, t]. \tag{1}$$

The action of an operator can be realized as a product and that of a canonical momentum as a functional differentiation

$$\Phi(x)|\Psi(t)\rangle \rightarrow \Psi(x)\Psi(\phi, t), \tag{2}$$

$$\Pi(x)|\Psi(t)\rangle \rightarrow -i\frac{\delta}{\delta\phi(x)}\Psi(\phi, t). \tag{3}$$

The dynamical evolution of a given initial state is described by the functional Schrödinger equation. This equation can be derived using variational principles if we define a time-dependent effective action [12]

$$\Gamma = \int \mathrm{d}t\langle\Psi(t)|i\partial_t - H|\psi(t)\rangle \tag{4}$$

and impose the condition that $|\Psi(t)\rangle$ is stationary against arbitrary variations we get

$$i\frac{\partial\Psi(\phi,t)}{\partial t} = H\Psi(\phi,t) = \int_x \left[-\frac{1}{2}\frac{\delta^2}{\delta\phi^2(x)} + \frac{1}{2}(\nabla\phi)^2 + V(\phi)\right]\Psi(\phi,t). \tag{5}$$

For applying variational method we have to prescribe a trial state. In the Gaussian approximation we assume a Gaussian trial state

$$\Psi(\phi,t) = \exp\left[-\int_{x,y}(\phi(x) - \hat{\phi}(x,t))\left[\frac{G^{-1}(x,y,t)}{4} - i\Sigma(x,y,t)\right]\right]$$
$$\times (\phi(y) - \hat{\phi}(y,t)) + i\int_x \hat{\Pi}(x,t)[\phi(x) - \hat{\phi}(x,t)]. \tag{6}$$

It can be seen that the Gaussian is centered at $\hat{\phi}$ and the width is given by G. Σ plays the role of the conjugate momentum of G and $\hat{\Pi}$ that of $\hat{\phi}$. $\hat{\phi}, \hat{\Pi}, G$ and Σ are the variational parameters as well as the expectation values:

$$\langle\phi(x)\rangle = \hat{\phi}(x,t), \tag{7}$$

$$\left\langle -i\frac{\delta}{\delta\phi(x)}\right\rangle = \Pi(\hat{x},t), \tag{8}$$

$$\langle\phi(x)\phi(y)\rangle = \phi(\hat{x},t)\phi(\hat{y},t) + G(x,y,t), \tag{9}$$

$$\left\langle i\frac{\partial}{\partial\phi(x)}\right\rangle = \int_x \hat{\Pi}(x,t)\dot{\hat{\phi}}(x,t) + \int_{x,y}\Sigma(x,y,t)\dot{G}(y,x,t), \tag{10}$$

$$V^{(n)}(\hat{\phi}) \equiv \frac{\mathrm{d}^n V(\hat{\phi})}{\mathrm{d}\hat{\phi}^n}. \tag{11}$$

For applying the formalism to a Φ^4 model, the following expression for effective action can be written up to two loop level [10, 11]

$$\Gamma = \int \mathrm{d}t\left[\int_x \hat{\Pi}\dot{\hat{\phi}} - \frac{1}{2}(\nabla\hat{\phi})^2 - V(\hat{\phi}) + \int_{x,y}\Sigma\dot{G} - 2\int_{x,y,z}\Sigma G\Sigma - \int_x \frac{1}{8}G^{-1}(x,x,t)\right.$$
$$\left. - \frac{1}{2}\nabla_x^2 G(x,y,t)|_{x=y} + \frac{1}{2}V^{(2)}(\hat{\phi})G(x,x,t)\right] - \frac{1}{8}V^{(4)}(\hat{\phi})\int_x G(x,x,t)^2. \tag{12}$$

Identifying the first term as the classical action and performing variations we get

$$\frac{\delta\Gamma}{\delta\dot{\hat{\phi}}(x,t)} = 0 \to \dot{\hat{\Pi}}(x,t) = \nabla_x^2\hat{\phi}(x,t) - V^{(1)}(\hat{\phi}) - \frac{1}{2}V^{(3)}(\hat{\phi})G(x,x,t), \tag{13}$$

$$\frac{\delta\Gamma}{\delta\dot{\hat{\Pi}}(x,t)} = 0 \to \dot{\Sigma}(x,y,t) + 2\int_x \Sigma(x,z,t)\Sigma(x,y,t)$$
$$= \frac{1}{8}G^{-2}(x,y,t) + \left[\frac{1}{2}\nabla_x^2 - \frac{1}{2}V^{(2)}(\hat{\phi}) - \frac{1}{4}V^{(4)}(\hat{\phi})G(x,x,t)\right]\delta^{\nu}(x-y), \tag{14}$$

$$\frac{\delta\Gamma}{\delta\Sigma(x,y,t)} = 0 \to \dot{G}(x,y,t) = 2\left[\int_x G(x,z,t)\Sigma(x,y,t) + \Sigma(x,z,t)G(z,y,t)\right]. \tag{15}$$

The static effective potential can be obtained by taking $\hat{\phi}$ to be x-independent and by putting $\Sigma = 0$. By performing variation for G, a gap equation for G could be written. A slightly different but equivalent method has been used in $(2+1)$ dimensional Liouville model [5].

2. Effective action

We propose to apply the formalism to a Φ^6 model and since Φ^6 coupling effects show up only at the three loop level we write the expression up to three loops

$$\Gamma = \int \mathrm{d}t\left[\int_x \hat{\Pi}\dot{\hat{\phi}} - \frac{1}{2}(\nabla\hat{\phi})^2 - V(\phi) + \int_{x,y}\Sigma\dot{G} - 2\int_{x,y,z}\Sigma G\Sigma\right.$$
$$\left. - \int_x \frac{1}{8}G^{-1}(x,x,t) - \frac{1}{2}\nabla_x^2 G(x,y,t)|_{x=y} + \frac{1}{2}V^{(2)}(\hat{\phi})G(x,x,t)\right]$$
$$- \frac{1}{8}V^{(4)}(\hat{\phi})\int_x G(x,x,t)^2 - \frac{1}{16}V^{(6)}(\hat{\phi})\int_x G^3(x,x,t). \tag{16}$$

We represent expectation values as $\hat{\phi}$ equivalent to the shift ϕ. Identifying the first term as the classical action and performing variations we obtain

$$\frac{\delta\Gamma}{\delta\dot{\hat{\phi}}(x,t)} = 0 \rightarrow \dot{\hat{\Pi}}(x,t) = \nabla_x^2\hat{\phi}(x,t) - V^{(1)}(\hat{\phi}) - \frac{1}{2}V^{(3)}(\hat{\phi})G(x,x,t) - \frac{1}{8}V^{(5)}(\hat{\phi})G^2(x,x,t), \tag{17}$$

$$\frac{\delta\Gamma}{\delta\dot{\hat{\Pi}}(x,t)} = 0 \rightarrow \dot{\Sigma}(x,y,t) + 2\int_x \Sigma(x,z,t)\Sigma(x,y,t) = \frac{1}{8}G^{-2}(x,y,t) + \left[\frac{1}{2}\nabla_x^2 - \frac{1}{2}V^{(2)}(\hat{\phi}) - \frac{1}{4}V^{(4)}(\hat{\phi})G(x,x,t) - \frac{1}{4}V^{(6)}(\hat{\phi})G^2(x,x,t)\right]\delta^\nu(x-y), \tag{18}$$

$$\frac{\delta\Gamma}{\delta\Sigma(x,y,t)} = 0 \rightarrow \dot{G}(x,y,t) = 2\left[\int_x G(x,z,t)\Sigma(x,y,t) + \Sigma(x,z,t)G(z,y,t)\right]. \tag{19}$$

3. Static effective potential

It has been shown that renormalization of time-dependent equations can be done along the same lines of the renormalization of the static effective potential. We therefore evaluate the static effective potential for Φ^6 model at zero temperature. The static effective potential takes the form

$$V_{\mathrm{eff}}(\hat{\phi},G) = \frac{1}{2}m^2\hat{\phi}^2 + \frac{\lambda}{4!}\hat{\phi}^4 + \frac{\xi}{6!}\hat{\phi}^6 + \left(\frac{1}{2}m^2 + \frac{\lambda}{4}\hat{\phi}^2 + \frac{\xi}{48}\hat{\phi}^4\right)G(x,x) + \left(\frac{\lambda}{8} + \frac{\xi}{16}\hat{\phi}^2\right) \times G^2(x,x) + \frac{\xi}{48}G^3(x,x) + \frac{1}{8}\mathrm{tr}\,G^{-1}(x,x) - \frac{1}{2}\nabla_x^2 G(x,x). \tag{20}$$

By performing variation with respect to G the gap equation is obtained.

$$\frac{1}{4}G^{-2}(x,y) = \left[-\delta_x^2 + m^2 + \frac{\lambda}{2}\hat{\phi}^2 + \frac{\xi}{24}\hat{\phi}^4 + \frac{\lambda}{2}G(x,y) + \frac{\xi}{4}\hat{\phi}^2 G^2(x,y) + \frac{\xi}{48}G^3(x,y)\right]\delta(x-y). \tag{21}$$

Assuming translation invariance the Fourier transform of a function is defined as

$$f(x) \equiv \int \frac{\mathrm{d}^\nu k}{2\pi^\nu}\mathrm{e}^{ik,x}f(k), \tag{22}$$

$$G(x,x) = \int \frac{\mathrm{d}^\nu k}{2\pi^\nu}\frac{1}{2}\left[\left[k^2 + m^2 + \frac{\lambda}{2}\hat{\phi}^2 + \frac{\xi}{24}\hat{\phi}^4\right] \times \frac{\lambda}{2}G(x,x) + \frac{\xi}{4}\hat{\phi}^2 G(x,x) + \frac{\xi}{48}G^2(x,x)\right]^{-1/2}, \tag{23}$$

$$G(x,x)=\int\frac{\mathrm{d}^\nu k}{2\pi^\nu}\frac{1}{2(k^2+M^2)^{1/2}}, \tag{24}$$

where an ansatz is fixed for G in terms of an effective mass. The effective mass M is treated here as a variational parameter which is $\hat{\phi}$ dependent. The static effective potential can be written as

$$\begin{aligned}V_{\mathrm{eff}}(\phi,M)=&\frac{1}{2}\int\frac{\mathrm{d}^\nu k}{2\pi^\nu}(k^2+M^2)^{1/2}+\left(\frac{1}{2}m_{\mathrm{B}}^2\hat{\phi}^2+\frac{\lambda}{4!}\hat{\phi}^4+\frac{\xi_{\mathrm{B}}}{6!}\hat{\phi}^6\right)\\&+\frac{1}{2}\left[M^2-m_{\mathrm{B}}^2-\frac{\lambda_{\mathrm{B}}}{2}\hat{\phi}^2-\frac{\xi_{\mathrm{B}}}{24}\hat{\phi}^4\right]G(x,x)\\&+\left[\frac{\lambda}{8}+\frac{\xi}{16}\hat{\phi}^2\right]G^2(x,x)+\frac{\xi}{48}G^3(x,x).\end{aligned} \tag{25}$$

Equation (25) shows that the effective potential expression obtained here is the same as the one obtained using Gaussian effective potential approach [17]. This is only natural since when time dependence is not taken into account definitions of effective action in both the approaches coincide. It also becomes clear that the formalism is equivalent to CJT approach at zero temperature. Equations obtained here differ from those obtained in CJT formalism by a $\xi\phi^2$ term. This term does not contribute when daisy and super daisy diagrams alone are considered through Hartee–Fock approximation which is the popular approximation employed in CJT formalism. At ϕ^4 level both the approaches are exactly identical.

Considering the first and second terms alone of (25) it can be seen that one loop effective potential result is contained in the expression with the mass term replaced by the effective mass. Identity with the Gaussian effective potential results become more transparent if we make the following correspondence in notation.

$$G(x,x)\to I_0=\int\frac{\mathrm{d}^\nu k}{2\pi^\nu}\frac{1}{2(k^2+m^2)^{1/2}}, \tag{26}$$

$$\frac{1}{2}\int\frac{\mathrm{d}^\nu k}{2\pi^\nu}(k^2+m^2)^{1/2}\to I_1, \tag{27}$$

$$M\to\Omega. \tag{28}$$

Since the effective potential is an ordinary function (not a functional) stationary requirements with respect to ϕ and M^2 is obtained by ordinary differentiation

$$\frac{\partial V}{\partial\phi}=\phi\left[m_{\mathrm{B}}^2+\frac{\lambda_{\mathrm{B}}}{6}\phi^2\frac{\xi_{\mathrm{B}}}{120}\phi^4+\frac{\lambda_{\mathrm{B}}}{2}G(x,x)+\frac{\xi_{\mathrm{B}}}{12}\Phi^2G(x,x)+\frac{\xi}{8}G(x,x)G(x,x)\right]=0 \tag{29}$$

$$\begin{aligned}\frac{\partial V}{\partial M^2}=-\frac{1}{2}&\left[M^2-m_{\mathrm{B}}^2-\frac{\lambda_{\mathrm{B}}}{2}\phi^2-\frac{\xi_{\mathrm{B}}}{24}\phi^4-\frac{\lambda_{\mathrm{B}}}{2}G(x,x)-\frac{\xi_{\mathrm{B}}}{4}\hat{\phi}^2G(x,x)\right.\\&\left.-\frac{\xi_{\mathrm{B}}}{8}G(x,x)G(x,x)\right]\frac{\partial G(x,x)}{\partial M^2}=0.\end{aligned} \tag{30}$$

Conventional effective potential is defined at the solution of (30). The effective mass is given by

$$M^2(\phi) = \left[m_B^2 + \frac{\lambda_B}{2}\phi^2 + \frac{\xi_B}{24}\phi^4 + \frac{\lambda_B}{2}G(x,x) + \frac{\xi_B}{8}G(x,x)G(x,x)\right]. \tag{31}$$

Required expression for the effective potential is obtained by replacing the effective mass M by $M(\phi)$ in eq. (25). Equation (25) shows certain very important peculiarities of Φ^6 and Φ^4 theories relevant at zero temperature.

$$V'(\phi) = \phi\left[M^2(\phi) - \left(\frac{\lambda_B}{3}\phi^2\right)\right]. \tag{32}$$

For Φ^4 theory $M^2(\phi)$ is intrinsically positive. Hence if $\lambda_B < 0$ only solution to the above equation is $\phi = 0$ (or potential is unbounded from below). That is for negative λ_B non-zero turning points do not exist. In the case of Φ^6 theory

$$V'(\phi) = \phi\left[M^2(\phi) - \left(\frac{\lambda_B}{3}\phi^2 + \frac{\xi_B}{30}\phi^4\right)\right] - \frac{\xi}{4}\left[1 - \frac{\phi^2}{3}\right]G^2 \tag{33}$$

non-zero turning points are possible also for $\lambda_B < 0$. Φ^6 theory in Hartee–Fock approximation requires up to three loops for obtaining the effects of ξ_B coupling. We have four parts for the effective potential

$$V_\beta(\phi, M(\phi)) = V^0 + V^1 + V^2 + V^3, \tag{34}$$

$$V^0 = \left[\frac{1}{2}m_B^2\phi^2 + \frac{\lambda_B}{4!}\phi^4 + \frac{\xi_B}{6!}\phi^6\right], \tag{35}$$

$$V^1 = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\ln[k^2 + M^2(\phi)], \tag{36}$$

$$V^2 = -\frac{\lambda_B}{8}G(x,x)G(x,x) - \frac{xi}{16}(\phi)^2 G(x,x)G(x,x), \tag{37}$$

$$V^3 = -\frac{\xi_B}{24}G(x,x)G(x,x)G(x,x), \tag{38}$$

which are obtained by substituting $M(\phi)$. Effective potential for both Φ^4 and Φ^6 theories can be obtained from this equation. In the following we do not consider the $\hat{\phi}^2$ dependent term to establish the equivalence with the Hartee–Fock approximation commonly used in the CJT formalism [6].

4. Renormalization

The effective mass term $M(\phi)$ defining the effective potential is divergent mainly due to the presence of $G(x,x)$. Following re-normalization prescription is employed in $(2+1)$ dimensions to regularize $M(\phi)$. Define

$$G(M(\phi)) \equiv -\frac{M(\phi)}{4\pi} \tag{39}$$

with

$$E \equiv [k^2 + M^2(\phi)]^{1/2}. \tag{40}$$

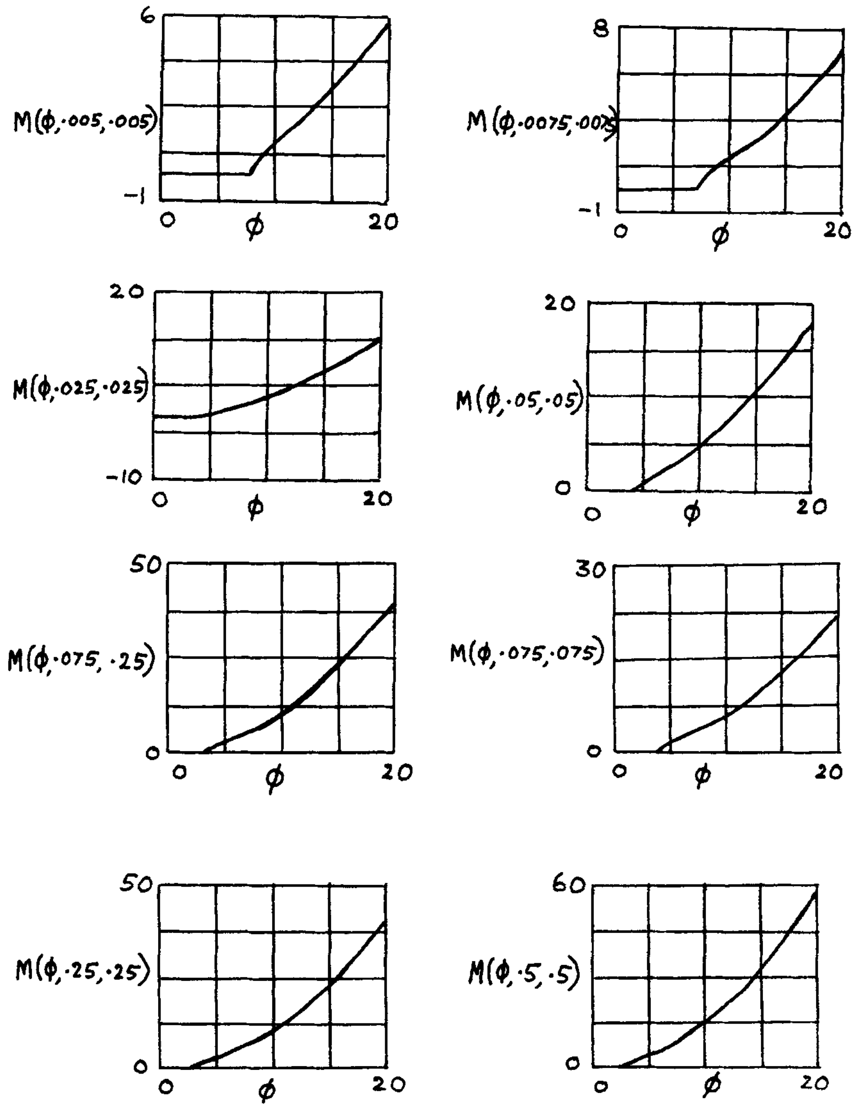

Figure 1. Effective mass for various couplings-Φ^6 theory.

In $(2+1)$ dimensions coupling constant renormalization is not required. Define

$$m_r^2 \equiv m_{\mathrm{B}}^2 + \frac{1}{2}\lambda I_1 + \frac{\xi'}{4} I_1 + \frac{\xi}{8} I_1^2, \tag{41}$$

$$I_1 \equiv \int \frac{\mathrm{d}^2 k}{(2\pi)^2} \frac{1}{2k} = \lim_{\Lambda\to\infty} \left(\frac{\Lambda}{4\pi}\right). \tag{42}$$

$\xi' \equiv \xi G(M(\phi))$,

$$G(x,x) = \int \frac{\mathrm{d}^2 k}{(2\pi)^2} \frac{1}{2E}. \tag{43}$$

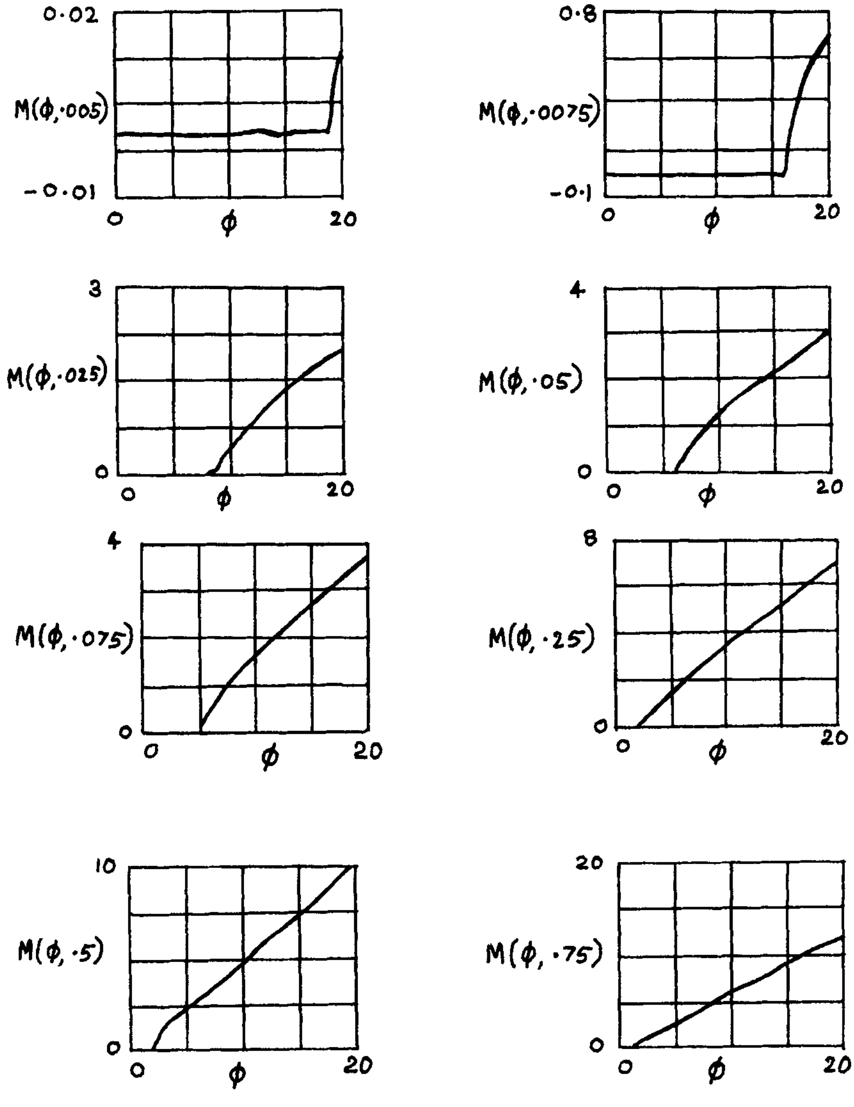

Figure 2. Effective mass for various couplings-Φ^4 theory.

By actual evaluation, introducing a cut-off parameter Λ

$$G(x,x) = G(M(\phi)) + I_1. \tag{44}$$

Equation (34) shows that $G(M(\phi))$ is the finite part of the vacuum propagator. A finite expression for $M(\Phi)$ is obtained by expressing it in terms of the renormalized parameters:

$$M^2(\phi) = -m_r^2 + \frac{\lambda}{2}\phi^2 + \frac{\xi}{24}\phi^4 + \frac{\lambda}{2}G(M(\phi)) + \frac{\xi}{8}G(M(\phi))G(M(\phi)). \tag{45}$$

Second derivative of the tree level potential is defined as m_{tree} (tree level mass)

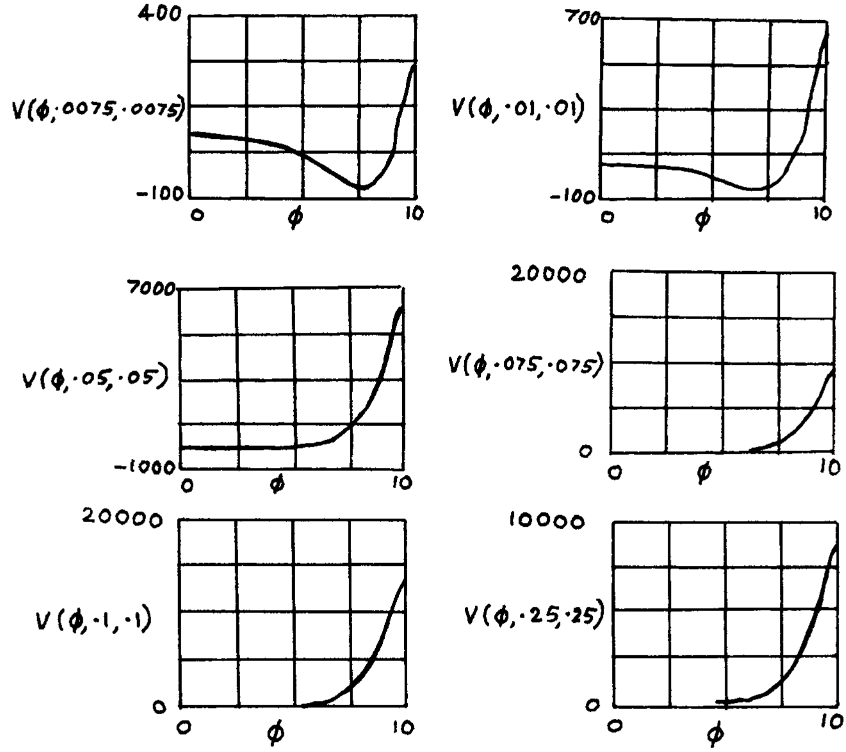

Figure 3. Effective potential for various couplings-Φ^6 theory.

$$M^2(\phi) = m^2_{\text{tree}}(\phi) + \frac{\lambda}{2}G(M(\phi)) + \frac{\xi}{8}G(M(\phi))G(M(\phi)), \tag{46}$$

$$V^1(M(\phi)) = -\frac{M^3}{6\pi} + \frac{\Lambda^3}{6\pi}. \tag{47}$$

The last term is cut-off dependent. Cancellation of this divergence is obtained by combining V^0, V^2 and V^3:

$$V(M) = -\frac{M^3}{6\pi} + \frac{M^4}{2\lambda} - \frac{1}{2}M^2G(M) - \frac{M^2}{24\eta}G^2(M) - F(\phi) \tag{48}$$

with

$$\eta^{-1} = \frac{\xi}{\lambda} \text{ and } F(\phi) = \left[\frac{m^2}{24\eta} - \frac{\lambda}{12} - \frac{29}{6!}\xi\phi^2\right]\phi^4. \tag{49}$$

$\xi = 0$ reproduces the result of Camellia and Pi at zero temperature. Using unrenormalized gap equation we combine V^0, V^2 and V^3 and writing them in terms of renormalized parameters

$$V^0 + V^2 + V^3 = \frac{\lambda_r}{8}\left[\phi^2 - \frac{2m_r^2}{\lambda}\right]^2 - \frac{\eta}{24}m_r^2\phi^4 - \frac{\lambda_r}{8}G^2(M) - \frac{\xi_r}{48}G^3(M) - F(\phi). \tag{50}$$

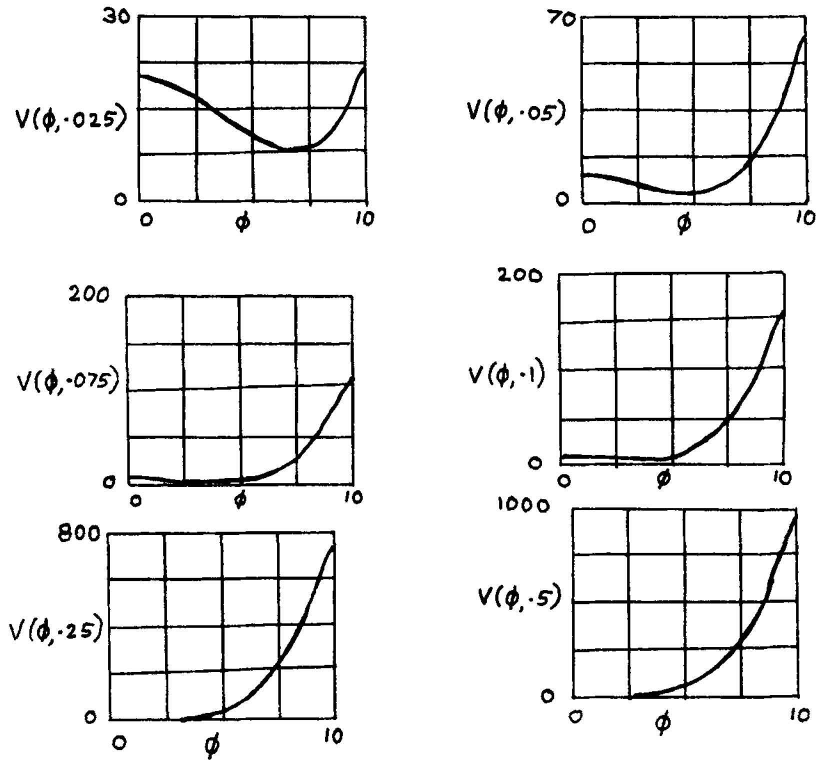

Figure 4. Effective potential for various couplings-Φ^4 theory.

In the case of $(2+1)$ dimensional Φ^4 theory $F(\phi) = \lambda/12$ which is finite. Thus unlike $(3+1)$ dimensional Φ^4 theory the effective potential do not contain any unrenormalized parameters. But in the case of Φ^6 theory $F(\phi)$ contains m which is an unrenormalized mass parameter. But here we can make $F(\phi) = 0$ by adjusting the parameters suitably and make the unrenormalized parameters vanish.

5. Conclusions

Static effective potential for Φ^6 theory is obtained using the functional Schrödinger picture formalism for calculating the effective action. Regarding the presence of turning points it is shown that for $(2+1)$ dimensional Φ^6 theory turning points can exist both for positive and negative λ. This indicates the possibility of bound states in the case of Φ^6 theories. The equations obtained for the effective potential and effective mass are found to be identical with the more popular Gaussian effective potential. Apart from a difference arising due to the use of Hartee–Fock approximation the formalism is identical to the CJT formalism.

Graphical analysis of the effective mass and effective potential is performed by numerically solving the equations for effective mass and effective potential. Graphs plotted compared with earlier works show close similarity with CJT formalism.

Application of the formalism to finite temperature situation is presently under investigation.

Comparing the graphs obtained for Φ^4 and Φ^6 theories. (figures 1 and 2) we can see the similarity in behaviour. Value of the effective mass for Φ^4 theory is smaller as expected. Straight portion of the graph indicating imaginary value of effective mass is more pronounced in the case of Φ^4 theory. The straight portion of the graph is more pronounced also for weak couplings. Comparison between effective potential values can be seen from figures 3 and 4.

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