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Bulk viscosity of neutron stars

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Abstract. Viscosity of neutron stars has been a continuing area of research for many years now. Recently interest in this field has revived because of the possibility of URCA processes in neutron stars. In this paper we report calculation of the bulk viscosity of neutron stars from these processes. For this purpose we have used the β-decay rates which were calculated without making the usual approximations of neglecting the neutrino momentum and using the nuclear mean field theory for the description of interacting nuclear matter. Also we have not restricted our calculation to the linear regime which corresponds to the assumption that fluctuations in the chemical potential away from β-equilibrium remain small: $\Delta\mu/kT \ll 1$. We find that for large amplitude fluctuations, where the linear approximation is not valid, bulk viscosity increases by many orders of magnitude. Also, as against strange matter stars, where the viscosity first increases with increasing temperature and then starts decreasing beyond 0.1 MeV, we find that the viscosity increases uniformly with temperature at least up to 2 MeV. We discuss the implications of these results for the stability of neutron stars.

Keywords. Bulk viscosity; radial damping; stability of neutron stars.

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1. Introduction

The viscosity of neutron star matter has been studied for more than a decade now and it still remains an active area of research [1]. Viscosity determines the damping time scales of radial vibrations of neutron stars [2] which may have been excited at the time of their formation from supernovae explosions. Viscosity also determines the criteria for the gravitational wave instabilities in rapidly rotating neutron stars and thereby the maximum rates of their rotation [3].

Another area of ongoing research is that of strange quark matter. Matter composed of comparable number of u, d and s quarks is conjectured to be the stable (or meta-stable) state of matter [4, 5]; if so, some of the neutron stars may turn out to be strange stars (composed of strange matter) or hybrid stars i.e., those having strange matter cores enveloped by ordinary nuclear matter.

Observationally, it is not easy to distinguish strange matter, hybrid and ordinary neutron stars since in the observed mass region of around 1.4 times the solar mass, all the three have similar radii. The neutrino cooling rates, considered to be one of the best candidates for this purpose, have lost much of their discriminating power since the revival of ordinary URCA processes in nuclear matter has narrowed down the difference in the expected neutrino cooling rates from the two types of matter [6, 7].

Many studies of the viscosities of the two types of matter have been reported in literature [8–13]. Recently there has been a revival of interest in this problem mainly because of the possibility of the direct URCA processes in nuclear matter. Consequently, the values of quantities determining nuclear transport properties are bound to change considerably.

The bulk viscosity of neutron star matter from direct URCA processes viz,

$$\begin{aligned} n &\rightarrow p + e + \bar{\nu}_e \\ p + e &\rightarrow n + \nu_e \end{aligned} \tag{1}$$

have been calculated by Haensel and Schaeffer [10]. The source of bulk viscosity of neutron star matter is the deviation from β-equilibrium and the ensuing non-equilibrium reactions implied by the compressions and rarefactions of the matter in the pulsating neutron star; thus they are driven by the non-zero value of $\Delta = \mu_n - \mu_p - \mu_e$, away from β-equilibrium. Haensel and Schaeffer [10] assumed that $\Delta/kT \ll 1$. As has been pointed out by Madesen [12], this assumption is certainly not correct at low temperatures. It is also not expected to be valid far away from equilibrium. In the context of the viscosity of strange quark matter Madesen [12] and Goyal *et al* [13] have shown that relaxation of this condition could lead to very significant modifications in the results. Moreover, Haensel and Schaeffer have made the usual simplifying assumptions [14] regarding the rates of various β-decay processes involved. In particular, the angular integrals involved in the calculation of the matrix element are evaluated only approximately and the neutrino momentum in the momentum-conserving δ-function is ignored. These assumptions though necessary to obtain the results in a simple analytic form can, however, alter the rates significantly [7, 15]. This has been demonstrated both for strange [15] as well as nuclear matter [7]. In view of the importance of bulk viscosity in the damping of radial vibrations, in determining the maximum rotation rates of the neutron stars and as means of observationally distinguishing between strange and neutron stars, we feel that a more accurate calculation of this quantity without these simplifying assumptions is in order.

In this paper we report the calculation of the bulk viscosity of neutron stars from the URCA processes (1). For this purpose we have not made use of the linear approximation viz., $\Delta\mu \ll kT$ which is valid for small perturbations only. The direct URCA processes in dense nuclear matter are possible only if the proton fraction $x = n_p/n_B$ is greater than a critical value of 1/9. Lattimer *et al* [6] have shown that there are simple models of nuclear matter where x is indeed greater than this critical value. Following the revival of interest in the URCA processes [6], we calculated the improved rates of energy loss by neutrino emission from neutron stars [7], without making the usual approximation of neglecting the neutrino momentum. We have followed the same approach for calculation of the rates of various β-decay processes involved. For the description of dense nuclear matter, we have used Walecka's mean field theory [16–18] (including the ρ-meson contribution), as extended to include strong interactions among the particle species involved. As is well-known this theory gives a fairly good description of strong interactions among the various particles and can be easily extended to include higher resonances as well. In § 2 we derive the expressions for bulk viscosity; in § 3 we present our results followed by a brief discussion.

2. Derivation of the bulk viscosity

In this section we derive an expression for the bulk viscosity of a neutron star using an approach similar to that of Wang and Lu [8], Sawyer [11] and Madesen [12]. Let there be a periodic fluctuation in the specific volume v of the star according to the following relation

$$v(t) = v_0 + \Delta v \sin(2\pi t/\tau) = v_0 + \delta v(t), \tag{2}$$

where v_0 is the equilibrium volume and Δv the amplitude of the perturbation. The mean dissipation rate of energy per unit mass can be expressed as

$$(\mathrm{d}w/\mathrm{d}t)_{\mathrm{av}} = (-1/\tau)\int_0^{\tau} P(t)(\mathrm{d}v/\mathrm{d}t)\mathrm{d}t \tag{3}$$

where the pressure $P(t)$ can be expressed near its equilibrium value P_0 as

$$P(t) = P_0 + (\partial P/\partial v)_0\delta v + (\partial P/\partial n_p)_0\delta n_p + (\partial P/\partial n_e)_0\delta n_e + (\partial P/\partial n_n)_0\delta n_n. \tag{4}$$

The change in the number of neutrons, protons and electrons per unit mass is given by the rate of reactions (1)

$$\delta n_p = \delta n_e = -\delta n_n = \int_0^t (\mathrm{d}n_p/\mathrm{d}t)\mathrm{d}t. \tag{5}$$

The reaction rates of the two reactions have already been derived in ref. [7] wherein the angular integrals involved in the phase space integrals have been evaluated exactly. The results for the case of degenerate matter ($\mu_n, \mu_p, \mu_e \gg T$) are

$$\Gamma_1 = \frac{G^2\cos^2\theta_c}{4\pi^5}T^3\int_0^{\infty}\mathrm{d}x\,\frac{(x-\alpha)^2+\pi^2}{1+e^{x-\alpha}}I(\mu_n, Tx, \mu_p, \mu_e) \tag{6}$$

and

$$\Gamma_2 = \frac{G^2\cos^2\theta_c}{4\pi^5}T^3\int_0^{\infty}\mathrm{d}x\,\frac{(x+\alpha)^2+\pi^2}{1+e^{x+\alpha}}J(\mu_n, Tx, \mu_p, \mu_e), \tag{7}$$

where G is the Fermi constant, θ_c the Cabibbo angle, and $\alpha = \delta\mu/T$. In these equations I and J are complicated expressions given by (for detials of steps leading to these expressions see ref. [7])

$$\begin{aligned}I(E_1,E_2,E_3,E_4) = {} & k_-^{12}k_+^{34}I_1(E_1,E_2,E_3,E_4) + \tfrac{1}{6}(k_+^{34} - k_-^{12}) \\ & \times I_2(E_1,E_2,E_3,E_4) - \tfrac{1}{20}I_3(E_1,E_2,E_3,E_4),\end{aligned} \tag{8}$$

where

$$k_{\pm}^{ij} = E_iE_j \pm \tfrac{1}{2}(p_i^2 + p_j^2), \tag{9}$$

$$\begin{aligned}I_n(E_i,E_j,E_k,E_l) = {} & (P_{ij}^{2n-1} - p_{ij}^{2n-1})\theta(P_{kl} - P_{ij})\theta(p_{ij} - p_{kl}) \\ & + (P_{ij}^{2n-1} - p_{kl}^{2n-1})\theta(P_{kl} - P_{ij})\theta(p_{kl} - p_{ij})\theta(P_{ij} - p_{kl}) \\ & + (P_{kl}^{2n-1} - p_{kl}^{2n-1})\theta(P_{ij} - P_{kl})\theta(p_{kl} - p_{ij}) \\ & + (P_{kl}^{2n-1} - p_{ij}^{2n-1})\theta(P_{ij} - P_{kl})\theta(p_{ij} - p_{kl})\theta(P_{kl} - p_{ij}),\end{aligned} \tag{10}$$

$$P_{ij} = p_i + p_j, \quad p_{ij} = | p_i - p_j |, \tag{11}$$

$$P_i = (E_i, \mathbf{p}_i), \quad | \mathbf{p}_i | = p_i. \tag{12}$$

The net rate for the production of protons is therefore

$$\frac{dx_p}{dt} = \Gamma_1 - \Gamma_2 = Af(\delta\mu), \tag{13}$$

where

$$A = \frac{G^2 \cos^2 \theta_c}{4\pi^5} T^3 \tag{14}$$

and

$$f(\delta\mu) = \int_0^\infty dx \left[\frac{(x-\alpha)^2 + \pi^2}{1+e^{x-\alpha}} I - \frac{(x+\alpha)^2 + \pi^2}{1+e^{x+\alpha}} J \right]. \tag{15}$$

For calculating energy dissipation according to eq. (3), only last three terms in eq. (4) contribute. Now

$$\frac{\partial P}{\partial n_i} = -\frac{\partial \mu_i}{\partial v} \tag{16}$$

and

$$\delta\mu(t) = \delta\mu(0) + \left(\frac{\partial(\delta\mu)}{\partial v}\right)_0 \delta v + \left(\frac{\partial\delta\mu}{\partial n_n}\right)_0 \delta n_n + \left(\frac{\partial\delta\mu}{\partial n_p}\right) \delta n_p + \left(\frac{\partial\delta\mu}{\partial n_e}\right) \delta n_e. \tag{17}$$

This leads to

$$\frac{d\delta\mu(t)}{dt} = \omega B\left(\frac{\Delta v}{v_0}\right) \cos\omega t - CAf(\delta\mu), \tag{18}$$

where

$$B = v_0 \left(\frac{\partial\delta\mu}{\partial v}\right)_0; \quad C = -v_0 \left(\frac{\partial\delta\mu}{\partial n_p} + \frac{\partial\delta\mu}{\partial n_e} - \frac{\partial\delta\mu}{\partial n_n}\right)_0 \tag{19}$$

and

$$w = \frac{2\pi}{\tau}. \tag{20}$$

Defining the bulk viscosity ζ by

$$\zeta = 2\frac{(dw/dt)_{av}}{v_0} \left(\frac{v_0}{\Delta v}\right)^2 \frac{1}{\omega^2} \tag{21}$$

and using eqs (3), (4) and (16)–(18), we obtain

$$\zeta = \frac{1}{\pi\omega} \left(\frac{v_0}{\Delta v}\right) \int_0^\tau Af(\delta\mu) \sin(\omega t) dt. \tag{22}$$

For the description of dense nuclear matter we have used Walecka's mean field theory [16–18]. In the mean field approximation we allow the meson fields σ, ω and ρ to acquire density dependent average values. From the effective Lagrangian one can then read off the effective masses and chemical potentials $\bar{m}_i$ and $\bar{\mu}_i$. The fermi momenta are, in turn, related to these effective quantities by

$$\bar{k}_i = (\bar{\mu}_i^2 - \bar{m}_i^2)^{1/2}. \tag{23}$$

The various chemical potentials and masses required in these expressions are obtained from the self-consistent solution of the mean field equations:

$$m_\sigma^2\bar{\sigma} + bm_n g_{\sigma N}^3\bar{\sigma}^2 + cg_{\sigma N}^4\bar{\sigma}^3 - g_{\sigma N}(n_p^s + n_n^s) = 0, \tag{24}$$

$$m_\omega^2\bar{\omega}_0 = g_{\omega N}(n_p + n_n), \tag{25}$$

$$m_\rho^2\bar{\rho}_0 = \tfrac{1}{2}g_{\rho N}(n_p - n_n), \tag{26}$$

$$m_n - \bar{m}_n = m_p - \bar{m}_p = g_{\sigma N}\bar{\sigma}_0, \tag{27}$$

$$\bar{\mu}_n = \mu_n - g_{\omega N}\bar{\omega}_0 + \tfrac{1}{2}g_{\rho N}\bar{\rho}_0, \tag{28}$$

$$\bar{\mu}_p = \mu_p - g_{\omega N}\bar{\omega}_0 - \tfrac{1}{2}g_{\rho N}\bar{\rho}_0. \tag{29}$$

The charge densities n_i and the scalar densities, n_i^s (see ref. [17]) are given in terms of these quantities by the expressions

$$n_i = 2\int\frac{\mathrm{d}^3p}{(2\pi)^3}\,[1 + \mathrm{e}^{(E_i-\bar{\mu}_i)/T}]^{-1}, \tag{30}$$

$$n_i^s = 2\bar{m}_i\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{E_i}[1 + \mathrm{e}^{(E_i-\bar{\mu}_i)/T}]^{-1}, \tag{31}$$

where

$$E_i = (p^2 + \bar{m}_i^2)^{1/2}. \tag{32}$$

Since the chemical potentials μ_n and μ_p are of the order of a few hundred MeV, even for temperatures of up to a few MeV the matter is completely degenerate leading to the following expansions (up to T^2 terms):

$$n_i = \frac{1}{3\pi^2}\left\{(\bar{\mu}_i^2 - \bar{m}_i^2)^{3/2} + \frac{\pi^2T^2}{2}\frac{(2\bar{\mu}_i^2 - \bar{m}_i^2)}{(\bar{\mu}_i^2 - \bar{m}_i^2)^{1/2}}\right\} \tag{33}$$

and

$$n_i^s = \frac{\bar{m}_i}{2\pi^2}\left\{(\bar{\mu}_i^2 - \bar{m}_i^2)^{1/2}\bar{\mu}_i - \bar{m}_i^2\cdot\ln\left(\frac{\bar{\mu}_i + \sqrt{\bar{\mu}_i^2 - \bar{m}_i^2}}{\bar{m}_i}\right) + \frac{\pi^2}{3}\frac{T^2\bar{\mu}_i}{(\bar{\mu}_i^2 - \bar{m}_i^2)^{1/2}}\right\}, \tag{34}$$

where $i = n, p$. To evaluate n_e, set $\bar{\mu}_e = \mu_e$ and $\bar{m}_i = m_e$ in (33). Using these expansions in (19) and going through some straightforward but tedious algebra one obtains

$$B = \frac{1}{3}\left(\frac{\bar{m}_n^2}{\bar{\mu}_n} - \frac{\bar{m}_p^2}{\bar{\mu}_p} - \frac{m_e^2}{\mu_e}\right) + \frac{2\pi^2}{9}T^2\left(\frac{\bar{\mu}_p}{\bar{\mu}_p^2 - \bar{m}_p^2} + \frac{\mu_e}{\mu_e^2 - m_e^2} - \frac{\bar{\mu}_n}{\bar{\mu}_n^2 - \bar{m}_n^2}\right)$$
$$-\frac{2}{3}\left(\frac{g_{\rho N}}{m_\rho}\right)^2(n_p - n_n) \tag{35}$$

and

$$C = \frac{1}{3}\left(\frac{\bar{k}_p^2}{\bar{\mu}_p n_p} + \frac{k_e^2}{\mu_e n_e} + \frac{\bar{k}_n^2}{\bar{\mu}_n n_n}\right) + \frac{2\pi^2 T^2}{9}\left(\frac{\bar{\mu}_p}{n_p \bar{k}_p^2} + \frac{\mu_e}{n_e k_e^2} + \frac{\bar{\mu}_n}{n_n \bar{k}_n^2}\right) + \left(\frac{g_{\rho N}}{m_\rho}\right)^2. \tag{36}$$

We have used the values of the various couplings $g_{\sigma n}, g_{\omega n}, g_{\rho n}, b$ and c fixed by Ellis *et al* [18], namely

$$g_{\sigma N}/m_\sigma = 0.0152502\,\mathrm{MeV}^{-1},$$
$$g_{\omega N}/m_\omega = 0.011\,\mathrm{MeV}^{-1},$$
$$g_{\rho N}/m_\rho = 0.011\,\mathrm{MeV}^{-1},$$
$$b = 3.418 \times 10^{-3},$$
$$c = 0.0146.$$

With this set of parameters, one obtains [18]

n_B(saturation) $= 0.153\,\mathrm{fm}^{-3}$,
binding energy$(E/n_B - m_N) = -16.3\,\mathrm{MeV}$,
charge symmetry energy $= 32.5\,\mathrm{MeV}$,
bulk modulus $= 300\,\mathrm{MeV}$,
Landau mass $= 0.83 m_N$.

To obtain bulk viscosity, we solve the differential equation (18) for $\delta\mu(t)$, obtain $f(\delta\mu)$ by evaluating the integral in (15) and finally evaluate the integral in (21). These steps of course have to be done numerically. In the linear approximation which is valid for small perturbations, (21) can be solved analytically and we obtain

$$\zeta = \frac{A' B^2}{(A'C)^2 + \omega^2}\left[1 - \frac{\omega A' C}{\pi} \cdot \frac{1 - e^{-A' c \tau}}{(A'C)^2 + \omega^2}\right], \tag{37}$$

where

$$A' = \frac{A}{T}\int_0^\infty \mathrm{d}x (I + J)\left[\frac{(x^2 + \pi^2)e^x}{(1 + e^x)^2} - \frac{2x}{1 + e^x}\right]. \tag{38}$$

This integral of course still has to be evaluated numerically.

3. Results and discussion

The results for the nuclear matter viscosity as derived from eq. (21) are given in figures 1–3 for various values of temperatures, nuclear densities n_B, and periods of harmonic perturbations, τ. In each figure we have plotted the viscosity as a function of relative volume perturbation $(\Delta v/v_0)$ for different temperatures. These figures correspond to $n_B = 0.3\,\mathrm{fm}^{-3}$ and $0.6\,\mathrm{fm}^{-3}$ (figures 1a and 1b respectively) and $\tau = 0.0005$ s. In

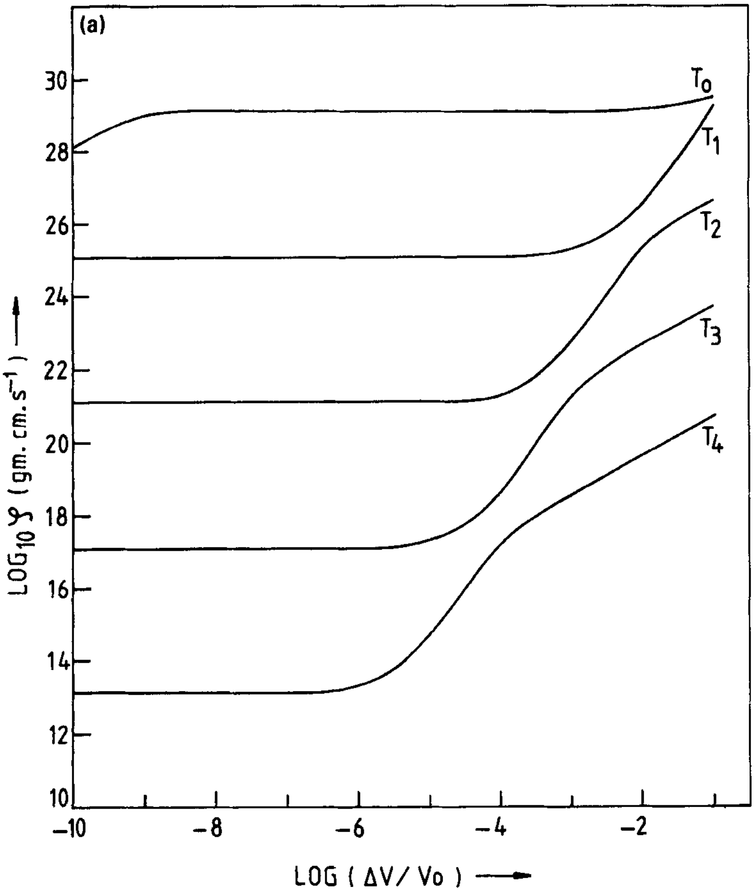

Figure 1a. Log of viscosity ζ vs log of amplitude of perturbation $(\Delta v/v_0)$ for $n_B = 0.3\,\mathrm{fm}^{-3}, \tau = 0.0005$ s. The five curves T_0, T_1, T_2, T_3, T_4 correspond to temperatures $T = 10^0, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}$ MeV respectively.

figure 2 we show our results for three different time periods, viz., $\tau_1 = 0.001$ s, $\tau_2 = 0.0005$ s and $\tau_3 = 0.0001$ s and two different temperatures, viz., 1 MeV and 0.001 MeV. We find that by and large the dependence of viscosity on the perturbation amplitude is similar to that obtained by Madesen [12] for strange quark matter. For small perturbations the viscosity is nearly independent of the amplitude of perturbation whereas for large perturbations there is a sharp increase in the viscosity by many orders of magnitude. As far as dependence on temperature is concerned, Madesen [12] found that the viscosity increases with increasing temperature initially but for $T > 0.1$ MeV it starts decreasing. A similar behaviour was reported by Goyal *et al* [13] in a more detailed calculation of the strange matter viscosity. In the present calculation we find that the nuclear matter viscosity continues to increase with temperature at least up to 2 MeV.

We now compare our results with those of Haensel and Schaeffer [10]. In the perturbation independent regime the viscosity is nearly proportional to the fourth power of temperature in agreement with the results of URCA processes as given by Haensel and Schaeffer [10]. However, at higher amplitudes of perturbation, the behaviour of viscosity

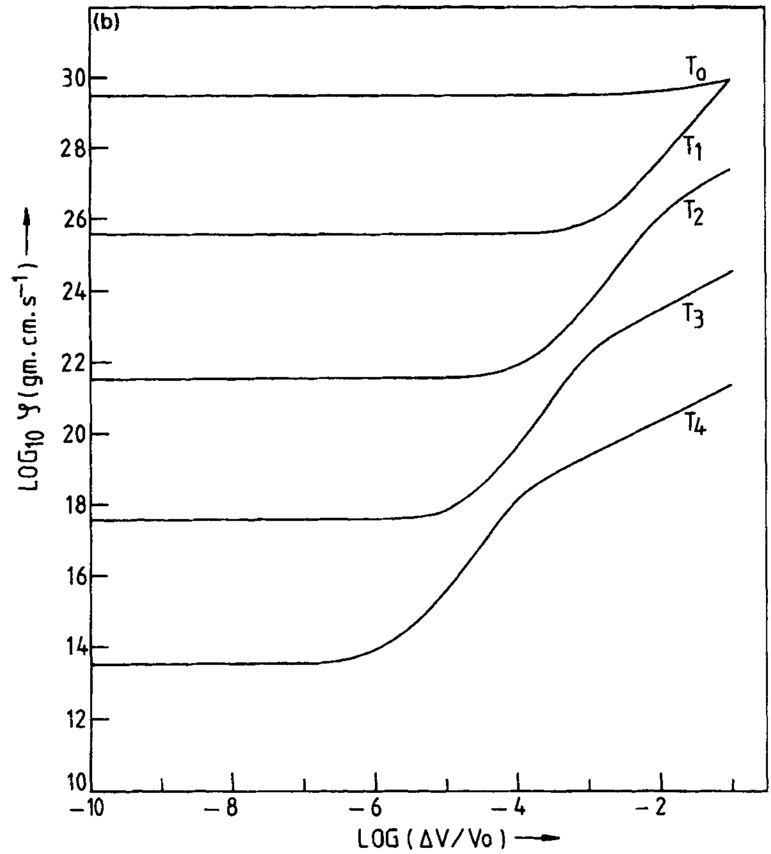

Figure 1b. Same as figure 1a for $n_B = 0.6\,\mathrm{fm}^{-3}$, $\tau = 0.0005\,\mathrm{s}$. The five curves correspond to the same five temperatures.

becomes markedly different. It increases very rapidly with $\Delta v/v_o$ except at the highest temperatures considered ($T \gtrsim 1\,\mathrm{MeV}$). This clearly indicates that the linear approximation is no longer valid at temperatures $\lesssim 0.1\,\mathrm{MeV}$. The lower the temperature, the smaller is the region of validity of linear approximation. Thus for $T = 10^{-4}\,\mathrm{MeV}$ the linear approximation is valid only up to $\Delta v/v_0 \lesssim 10^{-6}$, whereas for $T = 0.1\,\mathrm{MeV}$, it is valid up to $\Delta v/v_0 \lesssim 10^{-3}$.

At higher amplitudes, even the temperature dependence becomes quite different and more involved than the simple T^4 behaviour expected in the linear range. For $\Delta v/v_0 \gtrsim 10^{-3}$ the viscosity increases much less rapidly than what is dictated by the T^4 law. As T increases by four orders (from $10^{-4}\,\mathrm{MeV}$ to $1\,\mathrm{MeV}$), viscosity increases by only 10–11 orders of magnitude instead of sixteen.

From figure 2 we find that even the τ^2 behaviour of viscosity is valid only at high temperatures. At lower temperatures ($T = 10^{-4}\,\mathrm{MeV}$), increase of τ from 0.0001 s to 0.001 s increases viscosity by six orders instead of just two. This is so even for small perturbations. As can be seen from (37) which is valid for small perturbations, the frequency ($\omega = 2\pi/\tau$) dependence of viscosity is far more complicated than the simple relation, $\rho \propto \omega^{-2}$ obtained by Haensel and Schaeffer [10].

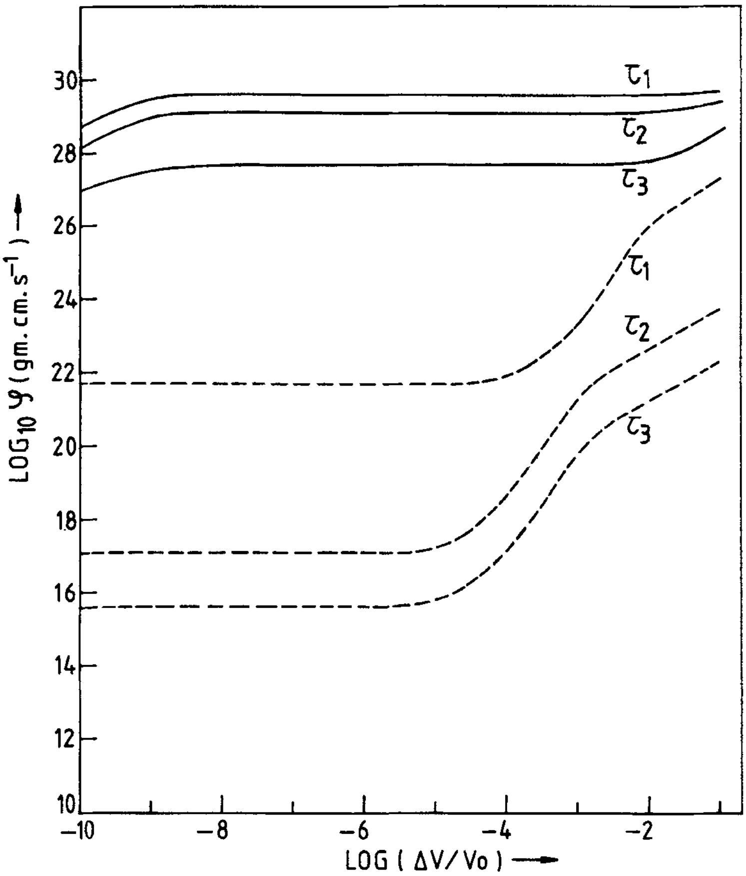

Figure 2. Log of viscosity ζ vs log of amplitude of perturbation $(\Delta v/v_0)$ for three time periods $\tau_1 = 0.001\,\mathrm{s}$, $\tau_2 = 0.0005\,\mathrm{s}$ and $\tau_3 = 0.0001\,\mathrm{s}$, and two different temperatures, $T = 1\,\mathrm{MeV}$ (solid curves) and $T = 0.001\,\mathrm{MeV}$ (dashed curves). $n_B = 0.3\,\mathrm{fm}^{-3}$ for all the sets.

The dependence of viscosity on nuclear density is rather weak (figure 3), and is adequately described by the $\rho \propto n^{1/3}$ relation given by Haensel and Schaeffer. As for the dependence on proton fraction $x(= n_p/n_B)$, in our calculation it cannot be judged as x appears as a parameter in our expressions only through its dependence on n_B. As n_B changes from n_0 to $10n_0$, x increases by a factor of five (figure 3). Thus the variation in the viscosity with the nuclear density n_B includes the implied dependence on x as well.

One of the main applications of the bulk viscosity is in determining the damping time of star vibrations. A rough estimate of the damping time can be obtained in the manner of Sawyer [11] who has obtained the following result:

$$\tau_d = \frac{1}{60}\left(\frac{\delta\rho}{\rho_0}\right)^2 \rho_m \omega^2 R^2 \frac{2}{\zeta(\delta\rho/\rho_0)\omega^2} = R^2\zeta^{-1}$$
$$= \frac{1}{30}\rho_m R^2 \zeta^{-1}. \tag{39}$$

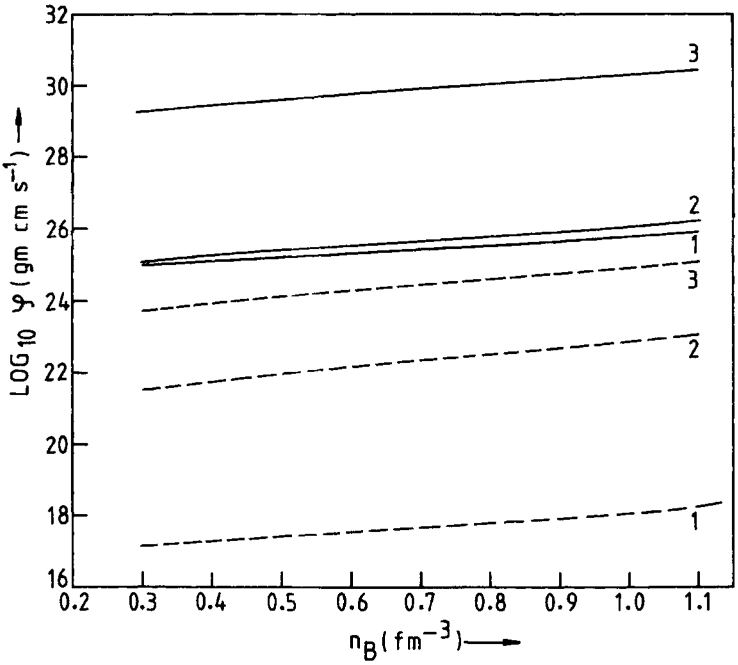

Figure 3. Log of viscosity ζ vs the nuclear density n_B. Solid curves correspond to $T = 0.1$ MeV and dashed curves to $T = 0.001$ MeV. Curves labeled 1, 2, 3 correspond respectively to $(\Delta v/v_0) = 10^{-5}, 10^{-3}, 10^{-1}$ for both temperatures. $\tau = 0.0005$ s for all the sets.

For a typical neutron star with mass $1.4 M_\theta, \rho_m = 4 \times 10^{14}$g/cc and radius $R \sim 10$ km, $\tau_d \approx 1.5 \times 10^{25}\zeta^{-1}$ s. It is clear that since the viscosity increases by many orders of magnitude as the amplitude of perturbation increases, particularly for low temperatures, the high amplitude perturbations will die out very rapidly. For low amplitude perturbations our results for viscosity are somewhat larger (by about an order of magnitude) than those obtained in [10], thus yielding considerably shorter damping times.

A comparison with the results for the strange quark matter is not so straightforward. In our case the viscosity increases with temperature whereas in the case of strange stars viscosity exhibits a more complicated behaviour: it first increases with temperature but for $T > 0.1$ MeV it decreases with further increase of temperature [12, 13]. Also as a function of n_B, the viscosity exhibits the same peculiar behaviour i.e. increases with n_B for low temperatures (< 0.1 MeV), but decreases with increasing n_B for higher temperatures (> 0.1 MeV) [13]. However, generally for n_B around $3n_0$ and for a temperature around 0.1 MeV, the strange star viscosity is about an order of magnitude lower than our result [13].

As compared to the modified URCA process which has a T^6 dependence on temperature, for the URCA process it is close to T^4. At a temperature close to 0.1 MeV, our calculation yields a value of bulk viscosity of 9 to 10 orders of magnitude more than for the modified URCA process as obtained by Sawyer [9].

The dramatic difference between the time scales of cooling of neutron star cores with $x > x_{\text{crit}}$ and $x < x_{\text{crit}}$ cases have already been highlighted in ref. [7]. Since our emissivity rates [7] are the same as those obtained by Lattimer *et al* [6] within a factor of two or so, the time scales for cooling are also nearly the same. However our larger values of viscosity imply a higher stability for the rapidly rotating star immediately after it is born.

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