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Electromagnetic properties of a chiral-plasma medium

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Abstract. The theoretical properties of a composite chiral-plasma medium are developed. By using the reaction theorem for a magnetized chiroplasma, we obtain the proof of nonreciprocity based upon the constitutive relationships between electromagnetic vectors **E**, **B**, **H**, **D**. Using the Maxwell's equations and the proposed constitutive relations for a chiral-plasma medium, we derive the vectors **E** and **H** and from these equations, dispersion relations and **E**-field polarizations are based. The obtained results for waves propagating parallel to the external magnetic field in a cold magnetized chiro-plasma are compared with typical results obtained for a cold plasma. For circulary polarized waves, a new mode conversion is founded with the chiral effect. The chiral rotation is obtained and compared with the Faraday rotation. For waves propagating across the magnetic field, we found a shift of the cut-offs of ordinary and extraordinary waves. On the lower branch of the extraordinary wave mode there is no bands of forbidden frequencies and the reflection point vanishes when the chiral parameter increases.

Keywords. Chiral waves; modes; polarization; chiral-plasma medium; Faraday rotation.

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1. Introduction

Chirality is a geometrical notion which refers to the lack of symmetry of an object. The electromagnetic chirality is also known as optical activity. The handedness of the uniformly distributed and randomly oriented chiral objects, which compose the chiral medium, is responsible for the observed optical activity. Optically active objects are three-dimensional and chiral. A three-dimensional object is chiral if it is not superimposable in its mirror image by translation and rotation.

A chiral medium when interacting with an electromagnetic wave can rotate the plane of polarization of the wave to the right or to the left depending on the handedness of the media. This fact is expected to play an important role in the potential application of chiral media in the microwave and optical regimes.

Chiral media [1, 2] and ferrite media [3] have been studied over the last decade for many applications. Chiral media has been examined as a coating for reducing radar cross section, for antennas and radomes, in waveguides and for microstrip substrates, guided-wave structures and potential application as reflection or antireflection, thin coating and shielding. The principal problem in working with a chiral medium is on the control of the degree of the chirality.

A chiral-plasma medium is examined. The plasma part of the composite medium is non-reciprocal due to an external magnetic field. To find the general dispersion relation, vector phasor Helmholtz based equations are derived, giving the ω against $\mathbf{k}$ behavior. The modal eigenvalue properties in the chiral-plasma medium, which is doubly anisotropic, are determined. We compare our results for the case of waves that propagate parallel to the magnetic field in a cold magnetized chiro-plasma with the typical ones obtained for a cold plasma [4]. For some values of the chirality parameter a new mode conversion appears. Also we obtain the chiral-Faraday rotation which can be compared with the typical Faraday rotation for a pair of right and left-hand circular polarized waves. For waves which propagate perpendicular to the magnetic field, there is no mode conversion but a lower band of forbidden frequencies disappears. Besides we found a shift of the cut-offs of ordinary and extraordinary waves across the external magnetic field.

2. Theoretical foundations

We propose the followings constitutive relations for chiral-plasma media:

$$\mathbf{D} = \overleftrightarrow{\varepsilon}\cdot\mathbf{E} + t_1\mathbf{H},$$
$$\mathbf{B} = \mu\mathbf{H} + t_2\mathbf{E}. \tag{1}$$

The plasma media constitutive relations are [4]

$$\mathbf{D}_{\mathrm{p}} = \overleftrightarrow{\varepsilon}_{\mathrm{p}}\cdot\mathbf{E}_{\mathrm{p}},$$
$$\mathbf{B}_{\mathrm{p}} = \mu_0\mathbf{H}_{\mathrm{p}}, \tag{2}$$

where

$$\overleftrightarrow{\varepsilon} = \begin{vmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{vmatrix}. \tag{3}$$

Here $\overleftrightarrow{\varepsilon}$, $t_{1,2}$ represents the permittivity tensor and the chirality parameters of the composite medium respectively.

The lossless character of the magnetized cold plasma medium is implied by the Hermitian nature of the tensor $(\overleftrightarrow{\varepsilon}_{\mathrm{p}}^{*})^{\mathrm{T}} = \overleftrightarrow{\varepsilon}_{\mathrm{p}}$. The superscripts * and T denote complex conjugate and transpose respectively.

In the search for new media which display nonreciprocal properties, it is essential to establish the nature of the chirality parameter t_1 and t_2.

The anisotropic reaction theorem [5] is

$$\int \mathbf{E}_b\cdot\mathbf{J}_a\,\mathrm{d}v = \int \tilde{\mathbf{E}}_a\cdot\mathbf{J}_b\,\mathrm{d}v. \tag{4}$$

Here, the sources current $\mathbf{J}_a$, $\mathbf{J}_b$ produce fields $\mathbf{E}_a$, $\mathbf{E}_b$ respectively. The tilde over the fields indicates a modified medium. Thus, we can obtain 6×6 constitutive tensors

$$\tilde{\mathbf{A}} = \begin{vmatrix} \hat{\varepsilon}^{\mathrm{T}} & -\hat{t}'^{\mathrm{T}} \\ -\hat{t}^{\mathrm{T}} & \hat{\mu}^{\mathrm{T}} \end{vmatrix} \tag{5}$$

and

$$\hat{\mathbf{A}} = \begin{vmatrix} \hat{\varepsilon} & -\hat{t} \\ -\hat{t} & \hat{\mu} \end{vmatrix} \tag{6}$$

with $\hat{t}$ and $\hat{t}'$ being the optical activity 3×3 tensors.

Reciprocity occurs only if

$$\int \mathbf{E}_b \cdot \mathbf{J}_a \, \mathrm{d}v = \int \mathbf{E}_a \cdot \mathbf{J}_b \, \mathrm{d}v. \tag{7}$$

By (4) it requires

$$\tilde{\mathbf{A}} = \hat{\mathbf{A}}. \tag{8}$$

For chiral media we must obtain

$$\hat{\varepsilon} = \overleftrightarrow{\varepsilon}, \ \hat{t} = t_1 I, \ \hat{t}' = t_2 I, \ \hat{\mu} = \mu, \tag{9}$$

where I is the unity tensor. To obtain reciprocity (8) imposes

$$-t_2 I^{\mathrm{T}} = t_1 I, \ -t_1 I^{\mathrm{T}} = t_2 I. \tag{10}$$

That is

$$t_1 = -t_2. \tag{11}$$

In the case of plasma media equations (2) hold, leading to

$$\hat{\varepsilon} = \overleftrightarrow{\varepsilon}_{\mathrm{p}}, \ \hat{t} = \hat{t}' = 0, \ \hat{\mu} = \mu_0, \tag{12}$$

Then, for the proposed constitutive relations, eq. (1), we have

$$\mathbf{D} = \overleftrightarrow{\varepsilon}_{\mathrm{p}} \cdot \mathbf{E} + t_1 \mathbf{H}, \tag{13}$$

$$\mathbf{B} = \mu_0 \mathbf{H} + t_2 \mathbf{E}. \tag{14}$$

A complete study that characterize electromagnetic waves in general bianisotropic media was developed by Kong [6] and our approach is a particular case.

3. Vector Helmholtz equations

The E-field vector Helmholtz equations is derived by inserting the constitutive relation, (13) and (14), into Maxwell's equations

$$\nabla \times \mathbf{E} = -i\omega \mathbf{B}, \tag{15}$$

$$\nabla \times \mathbf{H} = i\omega \mathbf{D} + \mathbf{J}, \tag{16}$$

so

$$\nabla \times \mathbf{E} = -i\omega\mu_0 \mathbf{H} - i\omega t_2 \mathbf{E}, \tag{17}$$

$$\nabla \times \mathbf{H} = i\omega \overleftrightarrow{\varepsilon} \cdot \mathbf{E} + i\omega t_1 \mathbf{H}. \tag{18}$$

Solving for $\mathbf{H}$, (17) gives

$$\mathbf{H} = \frac{1}{\mu_0}\left(\frac{i}{\omega}\nabla \times \mathbf{E} - t_2 \mathbf{E}\right) \tag{19}$$

and putting this into (18) we obtain

$$\nabla \times \mathbf{H} = \frac{1}{\mu_0}\frac{i}{\omega}(\nabla \times \nabla \times \mathbf{E}) - \frac{t_2}{\mu_0}\nabla \times \mathbf{E}. \tag{20}$$

Then the **E** field vector equation becomes

$$\nabla\times\nabla\times\mathbf{E}+i\omega(t_2-t_1)\nabla\times\mathbf{E}-\omega^2\mu_0\varepsilon_0\left(\frac{\overleftrightarrow{\varepsilon}}{\varepsilon_0}-\frac{t_1t_2}{\mu_0\varepsilon_0}\right)\cdot\mathbf{E}=0. \tag{21}$$

Here the plasma current is included in the permittivity tensor $\overleftrightarrow{\varepsilon}$. Similarly the **H** field vector equation is

$$\nabla\times\overleftrightarrow{\varepsilon}^{-1}\nabla\times\mathbf{H}+i\omega(t_2\overleftrightarrow{\varepsilon}^{-1}\nabla\times\mathbf{H}$$
$$-t_1\nabla\times\overleftrightarrow{\varepsilon}^{-1}\times\mathbf{H})-\omega^2\mu_0\left(I-\frac{t_1t_2}{\mu_0}\overleftrightarrow{\varepsilon}^{-1}\right)\mathbf{H}=0. \tag{22}$$

The inverse permittivity tensor is

$$\overleftrightarrow{\varepsilon}^{-1}=\left|\begin{matrix}\varepsilon_1 & -i\varepsilon_2 & 0\\ i\varepsilon_2 & \varepsilon_1 & 0\\ 0 & 0 & \dfrac{\varepsilon_1^2-\varepsilon_2^2}{\varepsilon_3}\end{matrix}\right| \Bigg/ (\varepsilon_1^2-\varepsilon_2^2).$$

4. Dispersion relation

The dispersion relation for the propagation vector **k** against ω can be obtained from **E** or **H** vector equations. We start with the **E** field relation which is simpler than **H** vector equation.

Setting **E** as follows,

$$\mathbf{E}=\mathbf{E}_0e^{-i\mathbf{k}\cdot\mathbf{r}}, \tag{23}$$

we obtain

$$-\mathbf{k}\times\mathbf{k}\times\mathbf{E}_0+\omega(t_2-t_1)\mathbf{k}\times\mathbf{E}_0-\omega^2\mu_0\varepsilon_0\left(\frac{\overleftrightarrow{\varepsilon}}{\varepsilon_0}-\frac{t_1t_2}{\mu_0\varepsilon_0}\right)\mathbf{E}_0=0. \tag{24}$$

Putting $\mathbf{E}_0$ into rectangular coordinates

$$\mathbf{E}_0=E_x\hat{x}+E_y\hat{y}+E_z\hat{z}, \tag{25}$$

we obtain three component systems of equations which determine the eigenvector. The determinant of the coefficient component matrix M_k will determine the eigenvalues, thereby yielding the ω against **k** dispersion diagram in phase-space.

Writing det $M_k=0$, with $k_x=0$ and with symmetry about the z-axis we obtain

$$\left|\begin{matrix}1-\dfrac{\varepsilon_1}{n^2\varepsilon_0}\left(1-\dfrac{t_1t_2}{\mu_0\varepsilon_1}\right) & -\dfrac{i\varepsilon_2}{n^2\varepsilon_0}-\dfrac{\cos\theta(t_2-t_1)}{n\sqrt{\mu_0\varepsilon_0}} & \dfrac{\sin\theta(t_2-t_1)}{n\sqrt{\mu_0\varepsilon_0}}\\ \dfrac{i\varepsilon_2}{n^2\varepsilon_0}+\dfrac{\cos\theta(t_2-t_1)}{n\sqrt{\mu_0\varepsilon_0}} & \cos^2\theta-\dfrac{\varepsilon_1}{n^2\varepsilon_0}\left(1-\dfrac{t_1t_2}{\mu_0\varepsilon_1}\right) & -\sin\theta\cos\theta\\ -\dfrac{\sin\theta(t_2-t_1)}{n\sqrt{\mu_0\varepsilon_0}} & -\sin\theta\cos\theta & \sin^2\theta-\dfrac{\varepsilon_3}{n^2\varepsilon_0}\left(1-\dfrac{t_1t_2}{\varepsilon_3\mu_0}\right)\end{matrix}\right|=0. \tag{26}$$

Here the refractive index n is defined as $n=ck/\omega$, $c=1/\sqrt{\mu_0\varepsilon_0}$.

If $\mu_0 = 1, \varepsilon_0 = 1, t_1 = t_2 = 0$, we obtain the same result given by Krall and Trivelpiece for a magnetized plasma [4].

For a collisionless chiroplasma, using (11) we have $t_1 = -it\sqrt{\mu_0\varepsilon_0}$, $t_2 = it\sqrt{\mu_0\varepsilon_0}$, the nontrivial solution of this system comes from setting the determinant of the coefficients equal to zero giving

$$\omega = \omega(n^2, \omega_p, \omega_c, \varepsilon_1, \varepsilon_2, t\ \varepsilon_3, \theta, k). \tag{27}$$

Equation (27) is then the general dispersion relation for waves propagating in a cold collisionless homogeneous chiroplasma in a uniform magnetic field. For a given plasma frequency ω_p, cyclotron frequency ω_c, wave frequency ω and the direction of propagation θ, equation (24) can be solved for the index of refraction n having as parameter the chirality t. From here, for the permittivities we will use relative values. In terms of k, the dispersion relation is

$$a_1k^4 + a_2k^3 + a_3k^2 + a_4k + a_5 = 0, \tag{28}$$

where

$$a_1 = -\left[\omega^2\varepsilon_1\left(1 - \frac{t^2}{\varepsilon_1}\right) - \omega^2\varepsilon_3\left(1 - \frac{t^2}{\varepsilon_3}\right)(\sin^2\theta - 1)\right],$$

$$a_2 = 0,$$

$$a_3 = \omega^2\varepsilon_1\left(1 - \frac{t^2}{\varepsilon_1}\right)\omega^2\varepsilon_3\left(1 - \frac{t^2}{\varepsilon_3}\right)(2 - \sin^2\theta) + \left[\omega^2\varepsilon_1\left(1 - \frac{t^2}{\varepsilon_1}\right)\right]^2$$
$$- (\omega^2\varepsilon_2)^2\sin^2\theta - 4\omega^2t^2\left[\omega^2\varepsilon_1\left(1 - \frac{t^2}{\varepsilon_1^2}\right)\right]\sin^2\theta + \omega^2\varepsilon_3\left(1 - \frac{t^2}{\varepsilon_3}\right)\cos^2\theta,$$

$$a_4 = -2i\omega^2\varepsilon_2\omega^2\varepsilon_3\left(1 - \frac{t^2}{\varepsilon_3}\right)(-2it\omega)\cos\theta,$$

$$a_5 = \omega^2\varepsilon_1\left(1 - \frac{t^2}{\varepsilon_1}\right)\left[\omega^4\varepsilon_2^2 - \omega^4\varepsilon_1^2\left(1 - \frac{t^2}{\varepsilon_1}\right)^2\right].$$

Here, there are four different eigenmodes for **k** implied by (28)

$$\varepsilon_1 = 1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2}, \tag{29}$$

$$\varepsilon_2 = -\frac{\omega_c}{\omega}\frac{\omega_p^2}{\omega^2 - \omega_c^2}, \tag{30}$$

$$\varepsilon_3 = 1 - \frac{\omega_p^2}{\omega^2}, \tag{31}$$

where ω_p is the plasma frequency and ω_c is the gyrofrequency given by

$$\omega_p^2 = \frac{4\pi n_e e^2}{m_e}, \tag{32}$$

$$\omega_c = \frac{eB_0}{m_e c}. \tag{33}$$

5. High-frequency wave propagation along the magnetic field

Setting $\theta = 0$, it is posible from eq. (21) different wave modes, writing the E-field vector equation in the form

$$(n^2 - \varepsilon_R) E_R = 0,$$

$$(n^2 - \varepsilon_L) E_L = 0,$$

$$\varepsilon_3 \left(1 - \frac{t^2}{\varepsilon_3}\right) E_z = 0. \tag{34}$$

From these equations, the last one gives a longitudinal mode. When $\varepsilon_3 = t^2$, we obtain the longitudinal electron plasma oscillations modified by the chiral parameter t. Since there is no wave propagation along the magnetic field, these chiroplasma oscillations do not constitute a propagation mode. For the other modes we have

$$\varepsilon_{R,L} = \varepsilon_1 \left(1 - \frac{t^2}{\varepsilon_1}\right) \pm \varepsilon_2 \left(1 + \frac{2tn}{\varepsilon_2}\right) \tag{35}$$

and

$$E_{R,L} = E_x \pm iE_y. \tag{36}$$

It is useful to explore these solutions in terms of the wave numbers k

$$k_R = \frac{t\omega}{c} \pm \frac{\omega}{c}\sqrt{\varepsilon_1 + \varepsilon_2} \tag{37}$$

$$k_L = -\frac{t\omega}{c} \pm \frac{\omega}{c}\sqrt{\varepsilon_1 - \varepsilon_2}, \tag{38}$$

where k_R is the wave number for a circularly polarized wave which drive electrons in the direction of their cyclotron motion, i.e., right-hand circularly polarized waves (RCP) and k_L is the wave number for a circularly polarized wave which drive electrons in the direction opposite of their cyclotron motion, i.e., left-hand circularly polarized waves (LCP). The t parameter modifies the typical plot of $\omega(k)$ shown by Krall and Trivelpiece, where the cutoff frequencies are shifted. Also the reflection points of the RCP and LCP are shifted. However the resonance which occurs when the wave phase velocity goes to zero is not modified by the chiral parameter. In figure 1 we present the modifications introduced by the parameter t in the dispersion relations of the right and left polarized waves. In this figure, the dispersion relations of the right and left circularly polarized waves are indicated by circles and stars, respectively. When $t \neq 0$, ε_1 and ε_3 depend on t and k_R and k_L have a linear term, $t\omega/c$, as can be seen in (37) and (38). In this way instead of modifying the curves that exist for $t = 0$, the parameter t permits the wave to propagate in a region of frequencies that is forbidden in the case $t = 0$. The RCP wave mode, in the lower branch, also known as the electron cyclotron wave is weakly modified by the chiral parameter, but the upper branch is strongly shifted when t increases. We can observe in figure 1 that for $t = 0$, there is no intersection of the dispersion relations of the right and left circularly polarized waves. Here the frequency bands for which there is no wave propagation can be identified in the plots. When $t \neq 0$ we can observe that there is an intersection of these curves, indicating that the presence

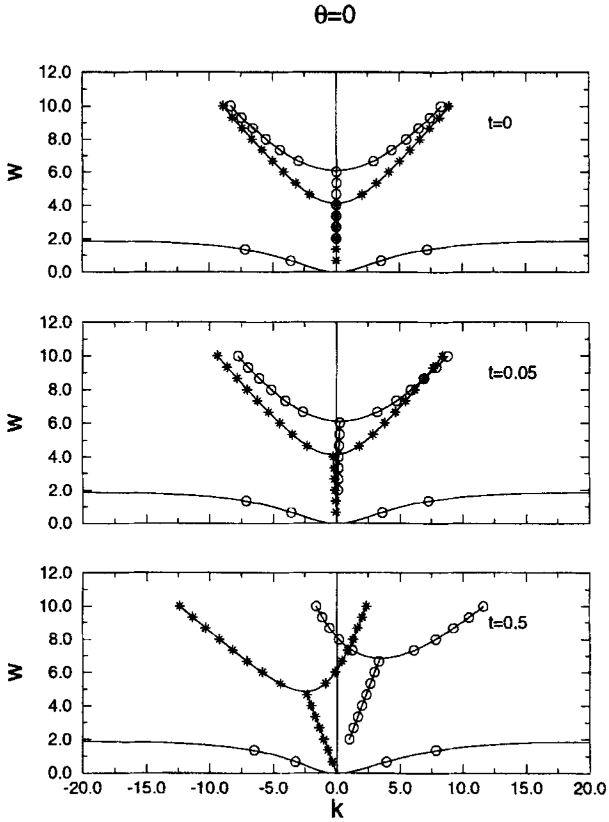

Figure 1. Dispersion relations for various values of the parameter t when the direction of propagation is parallel to the magnetic field ($\theta = 0$). The curves indicated by circles and stars correspond to the right and left circularly polarized waves, respectively.

of the t parameter permits that a wave changes its polarization. In figure 1, for $t = 0$ we can also observe that there is a region where only right circularly polarized waves propogate, a region where only left circularly polarized waves propagate and a region where both propagate. If their amplitudes are equal, the effect of the superposition of a left and right circularly polarized wave is to produce a plane wave with a particular plane of polarization. Because the two polarizations propagate at different velocities, the plane of polarization rotates as the wave propagates along the magnetic field. This effect is called Faraday-chiral rotation [7].

The global rotation of a plane of polarization as a function of distance in the direction of propagation is given by

$$\frac{E_x}{E_y} = \cot\left(\frac{k_L - k_R}{2}\right) z, \tag{39}$$

which means, the presence of the t parameter also affects the Faraday rotation. This chiral-Faraday rotation can be used as a plasma probe. In laboratory experiment this would be done by launching a planewave along the magnetic field in a chiroplasma.

Considering that the plane of polarization of this wave can be determined by an antenna and that we know the magnetic field, the density of the plasma and the frequency of the launched wave, the measurement of the plane of polarization away from the source can determine the value of the parameter t. For instance, considering the plasma frequency, $\omega_p = 5\cdot 10^7\,\mathrm{s}^{-1}$, for the electron gyrofrequency, $\omega_c = 2\cdot 10^7\,\mathrm{s}^{-1}$, and for the launched wave, $\omega = 6\cdot 5\cdot 10^7\,\mathrm{s}^{-1}$, the value of E_x/E_y, 1 cm away from the source is $E_x/E_y = 15\cdot 38$, $E_x/E_y = 11\cdot 48$ and $E_x/E_y = 9\cdot 15$ for $t = 0$, $t = 0\cdot 05$ and $t = 0\cdot 1$, respectively. The chiral rotations are 3·72, 4·97 and 6·24 degrees respectively. Another important effect caused by the presence of the parameter t is the conversion of modes [8]. In figure 1 for $t = 0\cdot 5$, we can observe that there is a region where both RCP and LCP propagate, for $k = 1\cdot 0$ we have a mode conversion from RCP to LCP wave. The explicit expression for the frequency around which intersection takes place can be obtained from $(\varepsilon_2^*)^2 = 4t^2(\varepsilon_1)^*$, where the quantitites with $*$ are modified forms of those of a collisionless biased plasma [9], that is, $\varepsilon_1^* = \varepsilon_1 - t^2$, $\varepsilon_3^* = \varepsilon_3 - t^2$ and $\varepsilon_2^* = \varepsilon_2$. This means that the energy of the electrons, obtained from the RCP electromagnetic wave at the electron cyclotron frequency can be transferred to the ions and this mode conversion can be used as a means of heating of the plasma. Note that the RCP wave rotates in the same direction as the electrons about the magnetic field and near the resonance, the energy is transferred from the wave field to the electrons but the mode conversion allows the absorption of energy by the ions.

In connection with the mode conversion and crossover frequencies in plasma, we note that a plasma with only two species may also show lack of symmetry, change of polarization and intermode coupling in the presence of the coriolis force. A detailed study of wave propagation in a rotating cold plasma including the coriolis force has been made by Uberoi and Das [10, 11]. Our results correspond to the very high frequency limit where the rotation and dynamic of ions are not taken into account. As the chiral effect is important at microwave and optical frequencies, if we extend the study by considering the Uberoi and Das [10, 11] approach, we can find that the result of these authors on the polarization reversal, intermode coupling of magnetic and inertial mode in a rotating magnetoplasma are not modified essentially.

In terms of the index of refraction, all the waves supported by the medium can be obtained from the following matrix if n_x and n_y are set equal to zero:

$$\mathbf{M} = \begin{bmatrix} n^2 - \varepsilon_1^* & -2itn - i\varepsilon_2^* & 0 \\ 2itn + i\varepsilon_2^* & n^2 - \varepsilon_1^* & 0 \\ 0 & 0 & \varepsilon_3^* \end{bmatrix} \tag{40}$$

where $t_1 = -it\sqrt{\mu_0\varepsilon_0}$, $t_2 = it\sqrt{\mu_0\varepsilon_0}$, $\varepsilon_3^* = \varepsilon_3 - t^2$, $\varepsilon_1^* = \varepsilon_1 - t^2$, $\varepsilon_2^* = \varepsilon_2$, $\mu_0 = 1$, $\varepsilon_0 = 1$. The quantities with $*$ are modified forms of those of a collisionless biased plasma [9].

Solving det $M_n = 0$, under these conditions, it is possible to obtain

$$\begin{aligned} n_{\|p+} &= t + \sqrt{\varepsilon_1 + \varepsilon_2} = t + \sqrt{\mathrm{R}}, \\ n_{\|p-} &= t - \sqrt{\varepsilon_1 + \varepsilon_2} = t - \sqrt{\mathrm{R}}, \\ n_{\|a+} &= -t + \sqrt{\varepsilon_1 - \varepsilon_2} = -t + \sqrt{\mathrm{L}}, \\ n_{\|a-} &= -t - \sqrt{\varepsilon_1 - \varepsilon_2} = -t - \sqrt{\mathrm{L}}. \end{aligned} \tag{41}$$

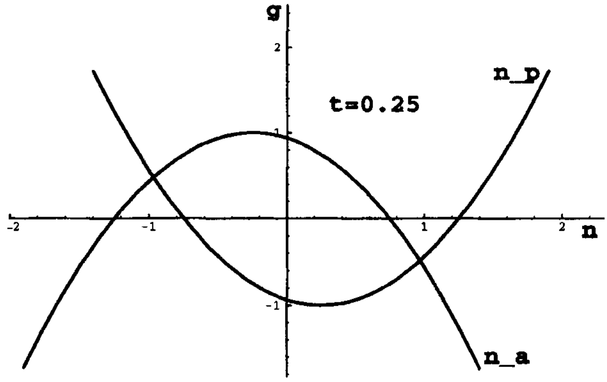

Figure 2. A graph of the $g = \varepsilon_2$ versus n representing the four wavenumbers along the magnetostatic field of a chiroplasma, the chiral parameter $t = 0{\cdot}25$.

Therefore, four wavenumbers are found which are dependent on the permittivity and chirality parameters of the composite medium.

The subscripts 'p' and 'a' refer to the parallel and antiparallel directions of energy propagation, that is the direction of the real part of the Poynting's vector with respect to the static magnetic field, while the plus (R) and minus (L) subscripts denote right-circular polarized (RCP) and left-circular polarized (LCP) propagating waves. For $t = 0{\cdot}25$, figure 2 show the $g = \varepsilon_2$ dependence of the four solutions of the index of refraction present along the magnetostatic field of a chiroplasma. It is possible to compare these results with those obtained by Engheta *et al* [7], in which the description of the Faraday chiral media is obtained using the chirality admittance. Here similar results are obtained in a simpler manner [12].

The helicity and polarization state corresponding to each of the wave numbers can be found by substituting (39) into the condition that the electric field must satisfy:

$$\overleftrightarrow{M} \cdot \mathbf{E} = 0. \tag{42}$$

We note that $n_{\|p+}$ and $n_{\|a+}$ are of positive helicity while $n_{\|p-}$ and $n_{\|a-}$ are of negative helicity.

Also the wave impedances of the positive and negative helicities, η_1 and η_2, respectively, are found to be

$$\eta_1 = \frac{1}{\sqrt{(\varepsilon_1^* + \varepsilon_2)}}, \tag{43}$$

$$\eta_2 = \frac{1}{\sqrt{(\varepsilon_1^* - \varepsilon_2)}}. \tag{44}$$

6. Wave propagation across the magnetic field

By making $\theta = 90°$ the solution to $\det M = 0$ yields a wave with elliptical polarization in the plane transverse to the direction of propagation. The angle θ is defined as the clockwise angle between the z-axis and n. Under this condition we have the following dispersion relation:

$$k_{\perp \text{extr.}} = \pm \frac{\sqrt{A - \sqrt{B}}}{\sqrt{2(\varepsilon_1 - t^2)}} \tag{45}$$

and

$$k_{\perp \text{ord.}} = \pm \frac{\sqrt{A + \sqrt{B}}}{\sqrt{2(\varepsilon_1 - t^2)}}, \tag{46}$$

where

$$A = \frac{\omega^2}{c^2}[\varepsilon_1^2 - \varepsilon_2^2 + \varepsilon_1 \varepsilon_3 + t^2(\varepsilon_1 - \varepsilon_3) - 2t^4] \tag{47}$$

and

$$\begin{aligned} B = \frac{\omega^4}{c^4}\Big[&\big((\varepsilon_1^2 - \varepsilon_2^2) - \varepsilon_1 \varepsilon_3\big)^2 \\ &+ t^2\big(6\varepsilon_1^3 - 6\varepsilon_1\varepsilon_2^2 - 2\varepsilon_1\varepsilon_3^2 + 12\varepsilon_1^2\varepsilon_3 - 2\varepsilon_2^2\varepsilon_3\big) \\ &+ t^4(-15\varepsilon_1^2 + 8\varepsilon_2^2 - 18\varepsilon_1\varepsilon_3 + \varepsilon_3^2) \\ &+ 8t^6(\varepsilon_1 + \varepsilon_3)\Big]. \end{aligned} \tag{48}$$

It should be pointed out that the electric field of the extraordinary wave, $k_{\perp \text{extr.}}$ is perpendicular to the magnetic field and the electric field of the ordinary wave, $k_{\perp \text{ord.}}$ is parallel to the magnetic field. In figure 3 we present the effect of the parameter t on the dispersion relations for the case $\theta = \pi/2$. In this figure the ordinary and extraordinary waves are indicated by circles and stars, respectively. When $t = 0{\cdot}05$, for $\theta = \pi/2$, the effect of the parameter is very small. We can observe that the dispersion relations are a little modified, but the parameter is not able to break up the forbidden regions that exist when $t = 0$. When $t = 0{\cdot}5$, the dispersion relations present very different curves with respect to the curves for $t = 0$. The difference in the way the t parameter acts in the parallel and perpendicular directions is due to the kind of equations we have. In (37) and (38) the t parameter appears as a linear term and in (45) and (46) the t parameter appears just inside a square root [6]. Also, we see that for $\theta = \pi/2$ the parameter t does not lead to the conversion of modes, as happens when $\theta = 0$. However, at $t = 0{\cdot}5$, the dispersion relation for the ordinary wave mode is flat and the reflection point is shifted. Now the ordinary and extraordinary wave mode do not have the same value of ω when k is very large. The more important effect is on the lower branch of the extraordinary wave mode because there is no more bands of forbidden frequencies, and the reflection point vanishes.

To see the effects of the chirality parameter on the cut-off frequencies, we can reduce the factors A and B in terms of the Stix's parameters P, R, and L [13], as

$$n_{\perp \text{ord.}} = \pm \sqrt{\mathrm{P}\left(1 + \frac{2t^2}{R + L}\right)}, \tag{49}$$

$$n_{\perp \text{extr.}} = \pm \sqrt{\frac{2RL}{R + L}\left(1 - \frac{2t^2}{R + L}\right)}. \tag{50}$$

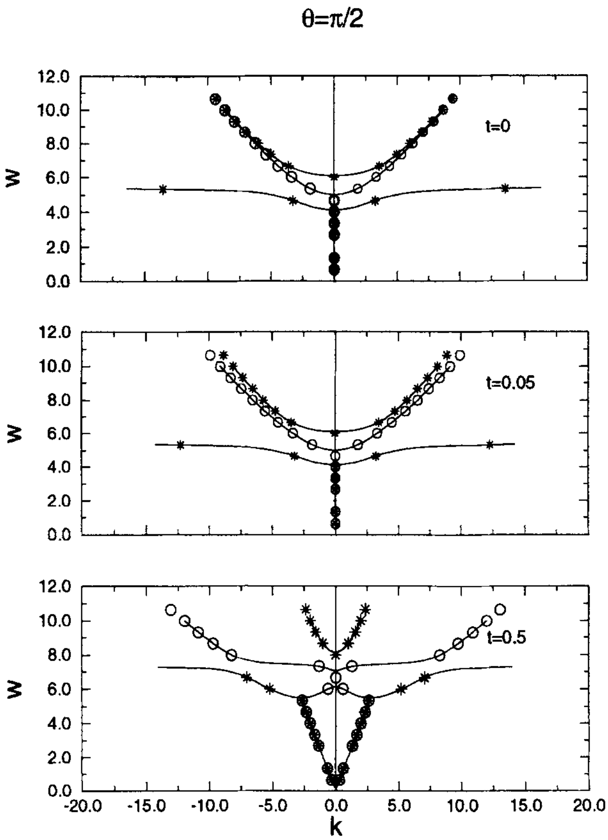

Figure 3. Dispersion relations for various values of the parameter t when the direction of propagation is perpendicular to the magnetic field ($\theta = \pi/2$). The curves indicated by circles and stars correspond to the ordinary and extraordinary waves, respectively.

As t goes to zero we obtain the typical results for a collisionless magnetized plasma. It follows from the equations that the cold plasma cut-offs $P = 0$, $R = 0$ and $L = 0$ are shifted for a chiroplasma in the domain $(\omega_c^2/\omega^2, \omega_p/\omega)$. The respective localization of the lines are schematically presented in figure 4, the vertical dashed lines correspond to $P = 0$.

7. Conclusion

We have examined the problem of high frequency waves in a magnetized chiral plasma. In the case of waves which propagate parallel to the magnetic field, we found a new mode conversion as the chiral parameter increases. This phenomenon is similar to polarization reversal and intermode coupling of slow and fast mode of propagation at low-frequency wave in a rotating magnetoplasma studied by Uberoi and Das. Here we obtain a combined effect Faraday-chiral rotation. Also, we found a shift in the cut-offs of ordinary and extraordinary waves across the external magnetic field.

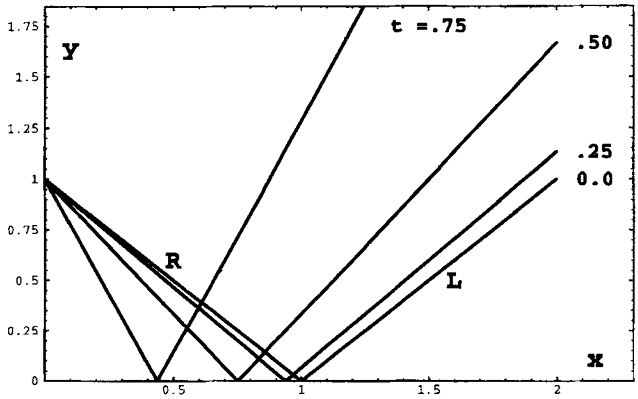

Figure 4. The lines $P = 0$, $R = 0$ and $L = 0$ in the plane $(\omega_c^2/\omega^2, \omega_p/\omega)$ corresponding to the cut-offs in a cold plasma ($t = 0$) and chiroplasma for some values of t.

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