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Investigation of the boundary value problem corresponding to a generalized Skyrme lagrangian

RAJESH KUMAR and P K CHATTOPADHYAY
Department of Physics, Maharshi Dayanand University, Rohtak, Haryana 124 001, India

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Abstract. A detailed numerical analysis of the boundary value problem resulting from the most general Skyrme type lagrangian containing up to quartic terms in field gradients is presented. The additional parameters in the lagrangian can be related to pion-pion scattering lengths. It is found that solutions to the boundary value problem does not exist for all values of the parameters and in particular, for the values predicted from pion-pion scattering data. Physical quantities of the nucleon are calculated for the highest possible values of the parameters admitting a solution and are compared with the corresponding values for the Skyrme model and experimental values.

Keywords. Skyrmion; Skyrme lagrangian; chiral symmetry; heavier mesons.

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1. Introduction

The Skyrme model [1,2] has emerged as an attractive model of baryons. In this model, baryons are solitonic solutions of non-linear pion fields. Since the original work of Skyrme and the work of Adkins *et al* [3] who calculated the physical quantities of the nucleon from the model, various generalizations have been attempted. On the one hand there are theories involving mesons heavier than the pion [4–8], on the other, terms higher order in field gradients have been included [9,10]. The intimate connection between the higher order terms and heavier mesons has also been discussed [11,12]. Weinberg [13] has written down the most general Skyrme type lagrangian involving quartic terms in the field gradients. In this paper we report results of a systematic investigation of the boundary value problem arising out of the Weinberg lagrangian. We also throw light on the inter-relationship of various Skyrme type lagrangians. Comparison of various calculated physical quantities with experimental results show that though there is not much change in the fitted value of the parameter F_π (the pion decay constant), there is some improvement over the Skyrme value in the case of other physical quantities.

In § 2 the inter-relationship of various Skyrme lagrangians is discussed. The boundary value problem is set up in § 3 and the solution of the boundary value problem is discussed in § 4. This is followed by § 5 which contains the calculated values of the physical quantities of the nucleon and their comparisons with the Skyrme and experimental values. § 6 contains the concluding remarks.

2. Generalized Skyrme lagrangians

The Skyrme lagrangian density is given by

$$\mathcal{L}_{\text{sk}} = \frac{1}{16} F_\pi^2 \,\text{Tr}(L_\mu L^{\mu+}) + \frac{1}{32e^2}\,\text{Tr}\,[L_\mu, L_\nu]_-^2, \tag{1}$$

where $L_\mu = u^+ \partial_\mu u, u$ being a $SU(2)$ matrix whose elements are meson fields and F_π is the pion decay constant. Donoghue, Golowich and Holstein [14] (DGH) modified this by adding an additional term quartic in fields gradients;

$$\mathcal{L}_{\text{DGH}} = \mathcal{L}_{\text{sk}} + \frac{\gamma}{8e^2}\,[\text{Tr}\,(\partial_\mu L^\mu)]^2. \tag{2}$$

The parameter γ can be related to pion-pion scattering data.

Weinberg [13] has proposed that the most general Skyrme type lagrangian density involving up to quartic terms in the field gradients, under very general restrictions of locality, Lorentz invariance, chiral symmetry and CPT theorem is given by

$$\mathcal{L} = \mathcal{L}_{\text{DGH}} + \frac{\beta}{32e^2}\,\text{Tr}[L_\mu, L_\nu]_+^2. \tag{3}$$

Other combinations of L_μ, L_ν up to quartic terms in field gradients can be reduced to the form (3) by using the Maurer–Cartan identity

$$\partial_\mu L_\nu - \partial_\nu L_\mu + [L_\mu, L_\nu]_- = 0. \tag{4}$$

With $\beta = 0$ and $\gamma = 0$, $\mathcal{L}$ reduces to the Skyrme lagrangian density and one obtains $\mathcal{L}_{\text{DGH}}$ from $\mathcal{L}$ by putting $\beta = 0$ and $\gamma \neq 0$.

The first term in (1) constitute the non-linear sigma model lagrangian. The other terms in (3) are quartic-derivative corrections to the model. An important observation by Gasser and Leutwyler [15] is that in the chiral limit, there exist only two independent quartic-derivative correction terms to the non-linear sigma model lagrangian viz. $[\text{Tr}(\partial_\mu u \partial^\mu u^+)]^2$ and $[\text{Tr}(\partial_\mu u \partial^\nu u^+)]^2$.

It is therefore necessary to analyse lagrangians (1), (2) and (3) in the light of this observation. $\mathcal{L}$ in (3) can be expressed in terms of u and u^+ using the identity $u^+ \partial_\mu u = -\partial_\mu u^+ u$. Thus

$$\begin{aligned}\text{Tr}(L_\mu L_\nu L^\nu L^\mu) &= \text{Tr}(\partial^\mu u \partial_\mu u^+ \partial_\nu u \partial^\nu u^+)\\ &= \tfrac{1}{2}[\text{Tr}(\partial^\mu u \partial_\mu u^+)]^2,\end{aligned} \tag{5}$$

$$\begin{aligned}\text{Tr}(L_\mu L_\nu L^\mu L^\nu) &= \text{Tr}(\partial_\mu u^+ \partial_\nu u \partial^\mu u^+ \partial^\nu u)\\ &= -\tfrac{1}{2}[\text{Tr}(\partial_\mu u^+ \partial^\mu u)]^2 + [\text{Tr}(\partial_\mu u^+ \partial^\nu u)]^2\end{aligned} \tag{6}$$

and

$$[\text{Tr}(\partial_\mu L^\mu)]^2 = [\text{Tr}(\partial_\mu u^+ \partial^\mu u)]^2, \tag{7}$$

since in the chiral limit $\partial_\mu \partial^\mu u = 0$. The last steps in (5) and (6) are the consequences of $SU(2)$ trace identities [16]. Using (5), (6) and (7), $\mathcal{L}$ in eq. (3) can be written as

$$\mathcal{L} = \frac{1}{16} F_\pi^2 \mathrm{Tr}(\partial_\mu u \partial^\mu u^+) - \frac{1}{16e^2}(1-2\gamma)[\mathrm{Tr}(\partial_\mu u^+ \partial^\mu u)]^2 + \frac{1}{16e^2}(1+\beta)[\mathrm{Tr}(\partial_\mu u^+ \partial^\nu u)]^2. \tag{8}$$

This shows that indeed there are only two independent quartic terms in $\mathcal{L}$.

3. The boundary value problem

By using the hedgehog ansatz

$$u = \exp(i\boldsymbol{\tau} \cdot \hat{\mathbf{r}} F(r)), \tag{9}$$

where $\boldsymbol{\tau}$ are the Pauli spin matrices and $F(r)$ is the chiral angle. One can obtain from the lagrangian (3), the following differential equation satisfied by $F(r)$:

$$\left[1 + \frac{8\sin^2 F}{\tilde{r}^2}(1-2\gamma) - 12(\beta+2\gamma)F'^2\right]F'' + \frac{2}{\tilde{r}}F' + \frac{4\sin 2F}{\tilde{r}^2} \times \left[F'^2(1-2\gamma) - \frac{1}{4} + \frac{\sin^2 F}{\tilde{r}^2}(\beta+4\gamma-1)\right] - \frac{8}{\tilde{r}}(\beta+2\gamma)F'^3 = 0, \tag{10}$$

where $\tilde{r}$ is the dimensions variable $\tilde{r} = eF_\pi r$. Equation (10) has to be solved subject to the boundary conditions

$$F(r) = \pi \quad \text{at } r=0 \quad \text{and } F(r) = 0 \quad \text{as } r \to \infty. \tag{11}$$

The static mass M of the soliton can be obtained in terms of $F(r)$ and its derivatives:

$$M = 4\pi\left(\frac{F\pi}{e}\right)\int_0^\infty \tilde{r}^2\left\{\frac{1}{8}\left[F'^2 + \frac{2\sin^2 F}{\tilde{r}^2}\right] + (1+\beta)\frac{\sin^2 F}{2\tilde{r}^2}\left[\frac{\sin^2 F}{\tilde{r}^2} + 2F'^2\right] - \frac{1}{4}(\beta+2\gamma)\left[F'^2 + \frac{2\sin^2 F}{\tilde{r}^2}\right]^2\right\} \mathrm{d}\tilde{r}. \tag{12}$$

The solitonic solutions obtained from (9), (10) and (11) are not eigenstates of spin and iso-spin. To obtain solutions with definite spin and iso-spin one makes the transformation $u(t) = A(t)u_0 A^{-1}(t)$ where u_0 is the static solution and $A(t)$ is an arbitrary, time dependent $SU(2)$ matrix. With this transformation, the lagrangian (3) gets transformed to

$$L = \int \mathcal{L}\mathrm{d}^3x = -M + \lambda \mathrm{Tr}[\partial_0 A(t) \partial_0 A^{-1}(t)], \tag{13}$$

where λ is given by

$$\lambda = \frac{4\pi}{6}\frac{1}{e^3 F_\pi}\int_0^\infty \tilde{r}^2 \sin^2 F\left[1 + 4(1+\beta)\left(F'^2 + \frac{\sin^2 F}{\tilde{r}^2}\right) - 4(\beta+2\gamma)\left(F'^2 + \frac{2\sin^2 F}{\tilde{r}^2}\right)\right]\mathrm{d}\tilde{r}. \tag{14}$$

The corresponding energy eigenvalues are obtained by projection to the proper spin and iso-spin eigenstates and are given by

$$m_{\mathrm{N}} = M + \frac{3}{8\lambda}, \tag{15a}$$

$$m_{\Delta} = M + \frac{15}{8\lambda}, \tag{15b}$$

where m_{N} and m_{Δ} are nucleon and delta isobar masses respectively.

4. Solutions of the boundary value problem

Before embarking on the numerical solution of the boundary value problem (BVP) defined by equations (10) and (11) it is useful to analyse the nature of the solutions analytically in the limits $r = 0$ and $r \to \infty$. In the vicinity of $r = 0, F(r)$ can be expanded in a power series of the form

$$F(r) = \pi + Cr^n + \cdots. \tag{16}$$

Substituting in eq. (10) one obtains

$$\begin{aligned} &Cr^{n-2}[(n-1)(n+2)] + C^3r^{3n-4}[-12(\beta+2\gamma)n^4 + 4(\beta+2\gamma)n^3 \\ &\quad + 16(1-2\gamma)n^2 - 8(1-2\gamma)n + 8(\beta+4\gamma-1)] + \cdots = 0. \end{aligned} \tag{17}$$

For $n = 1$ both the first and second terms in the l.h.s vanish identically. This implies that a power series solution for $F(r)$ in the form $F(r) = \pi + Cr + \cdots$ is possible near $r = 0$ for all values of β and γ.

To study the solutions at $r \to \infty$, eq. (10) is first transformed by the substitution $\xi = 1/r$ and the resulting equation studied at $\xi = 0$. One obtains the following equation:

$$\begin{aligned} &[1 + 8\sin^2 K \cdot \xi^2(1-2\gamma) - 12(\beta+2\gamma)\xi^4 K'^2]\xi^2 K'' \\ &\quad + 2[8\sin^2 K \cdot \xi^2(1-2\gamma) - 12(\beta+2\gamma)\xi^4 . K'^2]\xi K' \\ &\quad + 4\sin 2K \cdot [\xi^4 K'^2(1-2\gamma) - \tfrac{1}{4} + \sin^2 K \cdot \xi^2(\beta+4\gamma-1)] \\ &\quad - 8\xi^5(\beta+2\gamma)K'^3 = 0, \end{aligned} \tag{18}$$

where $K(\xi) = F(1/\xi)$.

As before, we try a power series solution of (18) of the form

$$K(\xi) = C'\xi^n + \cdots. \tag{19}$$

Substituting (19) in (18) and equating the coefficient of ξ^n, the lowest power of ξ to zero one obtains

$$C'[n(n-1) - 2] = 0. \tag{20}$$

Equation (20) cannot be satisfied for $n = 0$ and $n = 1$ ($n = 0$ also does not satisfy the boundary condition), but can be satisfied for $n = 2$. This shows that eq. (10) admits of a power series solution of the form $F(r) = C'/r^2 + \cdots$ at $r \to \infty$ irrespective of the values of β and γ. Thus the asymptotic behaviour of $F(r)$ is identical with those for the Skyrme problem.

The boundary value problem was solved numerically by the shooting procedure for the following cases: (a) $\beta = 0$, $\gamma \neq 0$; (b) $\beta \neq 0$, $\gamma = 0$. It was found that solutions to the BVP exist only for values of the parameters β and γ lying in a range. The stepsize for numerical integration was taken to be 0.1. Since the interval of integration extends from 0 to ∞, a cut off $\tilde{r}_{\max}$ for the upper limit has to be introduced. We have chosen $\tilde{r}_{\max} = 150.1$. This choice of $\tilde{r}_{\max}$ is reasonable as it is about 38 times the pion Compton wavelength and $r^2F(r)$ becomes almost constant in this range of r as is expected from the asymptotic behaviour of $F(r)$ as $r \to \infty$.

(a) *Case* $\beta = 0$, $\gamma \neq 0$. The lagrangian (3) with $\beta = 0$ was investigated by DGH [14] by treating the term proportional to γ as a perturbation. The unperturbed solution of (10) corresponding to $\beta = 0$, $\gamma = 0$, when substituted in (12) and (14) yields [14]

$$M = \left(\frac{73F_\pi}{2e}\right)(1 - 0.77\gamma + \cdots), \tag{21}$$

$$\lambda = \left(\frac{106.6}{e^3F_\pi}\right)(1 - 1.1\gamma + \cdots). \tag{22}$$

By partial wave analysis of the T-matrix for $\pi\,\pi$ scattering, the parameters F_π, e and γ can be expressed in terms of scattering lengths and slopes [14]. In particular γ is given by

$$\gamma = \frac{1}{4} \cdot \frac{(a_2^0 + 2a_2^2)}{(a_2^0 - a_2^2)}, \tag{23}$$

where a_l^{I} is the scattering length corresponding to partial wave l and iso-spin channel I. Using the experimental values of the $\pi\pi$ scattering parameters, the value of γ is found [14] to be 0.16. M and λ can be obtained from (21) and (22) which, when substituted in (15a) and (15b) yields the values of nucleon and Δ-isobar masses.

Instead of the perturbative approach, we have solved the full boundary value problem (10) and (11) for various values of γ. The chiral angle $F(r)$ obtained this way, when substituted in (12) and (14) gives M and λ for each value of γ. The nucleon and Δ-isobar masses can then be obtained in an analogous manner. It was found that the solution to the boundary value problem (10) and (11) does not exist for $\gamma > 0.11$. In a non-linear boundary value problem, the addition of even a small non-linear term may drastically affect the nature of the solution and perturbative methods may not work.

(b) *Case* $\beta \neq 0, \gamma = 0$. In this case solutions to the BVP exist only for β lying in the range 0 to 0.29. As in the previous case, the parameter β was expressed in terms of $\pi\pi$ scattering parameters by partial wave analysis of the T-matrix. We obtain

$$\beta = \frac{(2a_2^2 + a_2^0)}{(a_2^0 - 4a_2^2)}. \tag{24}$$

Using the experimental values of the scattering lengths quoted in ref. [14], one obtains $\beta = 0.44$. However, the solution to the BVP for this value of β does not exist.

One might suspect that the instabilities encountered in the solution of the BVP for $\gamma > 0.11 (\beta = 0)$ and $\beta > 0.29 (\gamma = 0)$ are due to the behaviour of the static mass under

Derrick's scaling. From eq. (12) the static mass is given by

$$M = M_2 + (1+\beta)M_4 - (\beta + 2\gamma)M_4', \tag{25}$$

where

$$M_2 = 4\pi \cdot \left(\frac{F_\pi}{e}\right) \int_0^\infty \frac{1}{8} \tilde{r}^2 \left[F'^2 + \frac{2\sin^2 F}{\tilde{r}^2}\right] \mathrm{d}\tilde{r}, \tag{26}$$

$$M_4 = 4\pi \cdot \left(\frac{F_\pi}{e}\right) \int_0^\infty \frac{1}{2} \sin^2 F \left[\frac{\sin^2 F}{\tilde{r}^2} + 2F'^2\right] \mathrm{d}\tilde{r}, \tag{27}$$

and

$$M_4' = 4\pi \cdot \left(\frac{F_\pi}{e}\right) \int_0^\infty \frac{1}{4} \tilde{r}^2 \left[F'^2 + \frac{2\sin^2 F}{\tilde{r}^2}\right] \mathrm{d}\tilde{r}. \tag{28}$$

Under scaling $\tilde{r} \to \lambda\tilde{r}$, eq. (25) transforms to

$$M_\lambda = \lambda M_2 + [(1+\beta)M_4 - (\beta + 2\gamma)M_4'] \cdot \frac{1}{\lambda}. \tag{29}$$

When there is no quartic term in the lagrangian (non-linear sigma model), the term in square brackets in (29) is zero and $M = \lambda M_2$. This does not have a minimum with respect to λ, indicating that there is no stable soliton. With Skyrme lagrangian $\beta = \gamma = 0$, $M_\lambda = \lambda M_2 + M_4 \cdot (1/\lambda)$ which has a minimum for $\lambda = \sqrt{M_4/M_2}$. Since M_4 and M_2 are positive definitive, there is always a stable soliton solution for the Skyrme model. With $\beta, \gamma \neq 0$ the term in square brackets in (29) may again vanish for some values of the parameters. When this happens $M_\lambda = \lambda M_2$ and again there is no stable soliton. To check whether the instabilities encountered for $\gamma > 0.11(\beta = 0)$ and $\beta > 0.29(\gamma = 0)$ are due to this effect, we have studied the variation of the term in square brackets in (29) as γ approaches 0.11 for $\beta = 0$ and as β approaches 0.29 for $\gamma = 0$. The term accounts for half

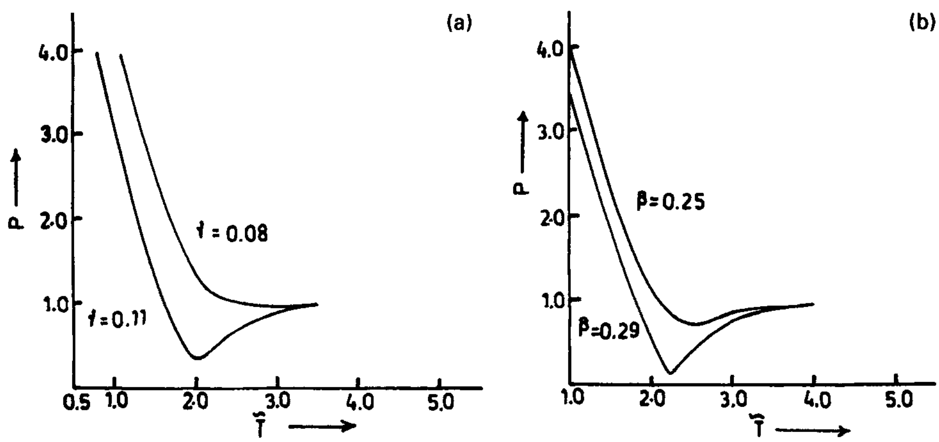

Figure 1. (**a**) and (**b**) show plots of $P = 1 + (8\sin^2 F/\tilde{r}^2)\,(1-2\gamma) - 12(\beta + 2\gamma)F'^2$ as a function of $\tilde{r}$ for various values of β and γ, P is the coefficient of F'' in eq. (10).

the static mass and does not approach zero as the upper limits of the parameters are reached indicating that the instabilities are not due to Derrick's scaling behaviour.

The origin of the instabilities can be understood from the differential equation (10). For some values of the parameters β and γ, the coefficient of F'' in (10) may become zero at some point $\tilde{r}$. Then the differential equation, at this particular point becomes first order and the numerical method breaks down. A study of the coefficient of F'' in (10) as a function of β and γ supports this conjecture. Figures 1(a) and 1(b) show plots of the coefficient $P = 1 + (8\sin^2 F/\tilde{r}^2(1-2\gamma) - 12(\beta+2\gamma)F'^2$ of F'' as a function of $\tilde{r}$ for (a) $\gamma = 0.08$ and $\gamma = 0.11$ with $\beta = 0$ and (b) $\beta = 0.25$ and $\beta = 0.29$ with $\gamma = 0$. In both the cases P is positive throughout the interval and has pronounced minima for the limiting values of the parameters β and γ, the minimum values of P being close to zero. Away from the limiting values of β and γ [e.g. for (a) $\gamma = 0.08$, $\beta = 0$ and (b) $\gamma = 0, \beta = 0.25$] the minima are rather flat and the minimum values are higher. It is interesting to note that the coefficient of F'' is never zero for the Skyrme problem.

5. Physical quantities of the nucleon

Various physical quantities of the nucleon can be expressed in terms of the chiral angle $F(r)$ and the parameters F_π, e, β and γ. The analytical expressions are given in ref. [3] for the Skyrme model ($\beta = 0, \gamma = 0$). For $\beta, \gamma \neq 0$, the expressions for iso-scalar mean radius $\langle r^2\rangle^{1/2}_{I=0}$, the iso-scalar magnetic mean radius $\langle r^2\rangle^{1/2}_{M,I=0}$, the proton and neutron magnetic moments are unchanged except that the expressions for M and λ occurring in them are changed as given in (12) and (14). The axial coupling constant g_{A} is given by

$$g_{\mathrm{A}} = -\frac{\pi}{3e^2}D, \tag{30}$$

where

$$D = \int_0^\infty \mathrm{d}\tilde{r}\,\tilde{r}^2\Bigg[\left(F' + \frac{\sin 2F}{\tilde{r}}\right) + 4\left(\frac{\sin 2F}{\tilde{r}}F'^2 + \frac{2\sin^2 F}{\tilde{r}^2}F' + \frac{\sin^2 F\sin 2F}{\tilde{r}^3}\right) - 4\beta\left(F'^3 + \frac{\sin^2 F\sin 2F}{\tilde{r}^3}\right) - 8\gamma\left(F'^3 + \frac{\sin 2F}{\tilde{r}}F'^2 + \frac{2\sin^2 F}{\tilde{r}^2}F' + \frac{2\sin^2 F}{\tilde{r}^3}\sin 2F\right)\Bigg] \tag{31}$$

and the pion nucleon coupling constant $g_{\pi\mathrm{NN}}$ is related to g_{A} by the relation

$$g_{\pi\mathrm{NN}} = \frac{2m_{\mathrm{N}}}{F_\pi}\,g_{\mathrm{A}}. \tag{32}$$

Table 1 summarizes the calculated values of the physical quantities of the nucleon for $\beta = 0$, $\gamma = 0.11$ and $\beta = 0.29$, $\gamma = 0$ (the highest values for which solution exists). The corresponding values for the Skyrme model and the experimental values are quoted for comparison. It is found that the pion decay constant decreases a little from the Skyrme value making the agreement with experiment slightly worse. But in case of other physical quantities like iso-scalar mean square radius, iso-scalar magnetic mean square radius,

Table 1. Calculated physical quantities of the nucleon for (a) $\beta = 0.29, \gamma = 0$ and (b) $\beta = 0, \gamma = 0.11$. The values of the parameter e in eq. (1) are found to be 4.87 and 4.32 respectively in cases (a) and (b).

	Our calculation			
Quantity	$\beta = 0.29,\ \gamma = 0$	$\beta = 0,\ \gamma = 0.11$	Skyrme model	Experiment
Nucleon mass (m_N)	Input	Input	Input	939 Mev
Δ-isobar mass (m_Δ)	Input	Input	Input	1232 Mev
Pion decay constant (F_π)	126.2 Mev	126.6 Mev	129 Mev	186 Mev
Iso-scalar, mean radius ($\langle r^2 \rangle_{I=0}^{1/2}$)	0.60 fm	0.60 fm	0.59 fm	0.72 fm
Iso-scalar magnetic mean radius ($\langle r^2 \rangle_{M,I=0}^{1/2}$)	0.86 fm	0.87 fm	0.92 fm	0.81 fm
Proton magnetic moment (μ_p)	1.89	1.89	1.87	2.79
Neutron magnetic moment (μ_n)	−1.32	−1.32	−1.31	−1.91
$\lvert \mu_p / \mu_n \rvert$	1.43	1.43	1.43	1.46
Axial coupling constant (g_A)	0.69	0.67	0.61	1.23
Pion nucleon coupling constant ($g_{\pi NN}$)	10.20	9.96	8.9	13.5

proton magnetic moment, axial coupling constant and pion nucleon coupling constant $g_{\pi NN}$, the agreement with experiment improves as compared to the Skyrme value.

6. Conclusion

We have given a detailed, systematic numerical analysis of the boundary value problem resulting from the most general Skyrme type lagrangian containing quartic terms in field gradients. From our discussion in § 2 it is clear that in the chiral limit, either the lagrangian with $\beta = 0, \gamma \neq 0$ or the one with $\beta \neq 0, \gamma = 0$ can be taken as a suitable quartic generalization of the Skyrme model. In both the cases, solutions to the BVP do not exist for values of the parameters β and γ predicted from pion-pion scattering data. In the case $\beta = 0$, Lacombe *et al* [17] have pointed out that the term proportional to γ in the lagrangian is necessary to produce the right nucleon-nucleon potential in the middle range. With $\beta \neq 0, \gamma = 0$, the lagrangian can be written entirely in terms of commutators and anti-commutators of the current L_μ, a feature which can be useful in the sixth order generalization of the lagrangian.

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