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The effect of environment induced pure decoherence on the generalized Jaynes–Cummings model

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Abstract. We have studied the effect of environment induced pure decoherence on the generalized Jaynes–Cummings model (JCM). This generalized JCM is introduced to take into account both atom-field interaction and a class of spin-orbit interactions in the same framework. For generalized JCM with atom-field interaction, it is shown that along with the suppression of the oscillatory behaviour of the atomic and field variables, in the steady state, atomic energy is transferred to the field or vice versa through the dressed state coherence depending on the initial condition of the atom-field system and the model under consideration. It is also shown that initial Poissonian field acquires a sub-Poissonian character in the steady state and thus all the nonclassical properties are not erased by the decoherence in JCM. An interesting effect of this decoherence mechanism is that it affects the population and coherence properties of the individual subsystem in a different way. As an example of generalized JCM with spin-orbit interaction, the dynamics of spin of the hydrogen atom in a magnetic field is studied to show the effect of decoherence.

Keywords. Decoherence; Jaynes–Cummings model; generalized JCM.

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1. Introduction

The Jaynes–Cummings model (JCM) is a two-state system interacting with a single mode field in the rotating wave approximation (RWA) [1]. This model was first used to examine the classical aspects of spontaneous emission but it was subsequently discovered that in the JCM atomic population histories presented direct evidence for the discreteness of photons. Also many nonclassical features of photon statistics were discovered through this model. More recently, JCM has been used to elucidate quantum correlation and the formation of macroscopic superposed quantum states. Apart from exact solutions of different variants of JCM, the remarkable advancement in cavity quantum electrodynamics experiments [2] involving single atoms within single mode cavities have made the model a very good testing ground of the quantum phenomena of the atom-field entangled system.

It is well-known that the surroundings have profound effects on the evolution of a system of interest. In particular, quantum noise due to the surroundings may result in nonunitary evolution of the system resulting in decoherence and relaxation. One can naturally ask the question what will be the effect of decoherence on the atom and field variables which are interacting through the JCM. Here unlike the normal case of spontaneous emission we are looking at a situation where there is no energy dissipation but only a decoherence from the composite system, i.e, a case of pure decoherence. Although in many cases the effect of decoherence is studied for single systems of interest [3–5], it may be worthwhile to understand the effect of decoherence on the joint subsystems [6]. We examine the fate of the individual atom and field variables when the decoherence acts on the composite atom-field system.

In this work we have given a solution for generalized JCM undergoing environment induced pure decoherence in terms of the atom-field dressed states. By this approach one can observe that the decoherence does not affect the dressed state population but affect only the coherence of the dressed state levels. But when one looks at the individual atomic or field population it decays to a steady state and in the steady state depending on the initial condition and specific model under study, atomic energy is transferred to the field or vice-versa. This is due to the fact that the dressed state coherence has an effect on the atomic and field population and as the decoherence model is energy conserving, the total energy is distributed in this way.

We have also shown that the nonclassical properties of the field get affected by the decoherence. Although squeezing and other phase related nonclassical properties are suppressed by the decoherence, the sub-Poissonian [7] nature of the field is not completely suppressed by the decoherence. This is exemplified with some variants of atom-field interaction models.

We have pointed out that in JCM with atom-field interaction, the decoherence mechanism as proposed affects the population and coherence properties of the individual subsystem in a different way.

We have also introduced a more generalized JCM which not only accounts for the atom-field interaction but also for a class of spin-orbit interactions. As an explicit example, it is shown that a hydrogen atom in a magnetic field can also be solved. This system undergoing decoherence can also be studied using dressed states. The characteristic spin dynamics is studied for a particular value of orbital angular momentum.

The paper is organized as follows: In § 2(a) we present the master equation for pure decoherence and its effect on generalized Jaynes–Cummings (JC) model with atom-field interaction. The effect of decoherence mechanism on the transient atom-field dynamics is discussed in § 2(b). Taking a particular nonlinear JC model in § 3 we show how the nonclassical properties of the field and and atom evolves in time. In § 4 we have given a more general Hamiltonian which describes not only the atom-field interaction but also a class of spin-orbit interactions, in general. Section 5 is devoted to show how the dynamics of spin is affected by the decoherence in the case of hydrogen atom in presence of a magnetic field. In § 6 we conclude the paper.

2a. Master equation for pure decoherence and its effect on the generalized JCM with atom-field interaction

To start with, we allow an interaction between the system and its environment reservoir which gives only a decoherence in the system but no energy dissipation from the system.

We take the total Hamiltonian H_T as

$$H_T = H + H_R + V, \tag{2.1}$$

where H and H_R are the Hamiltonians of the system of interest and of the reservoir respectively; V is the interaction between them. The reservoir Hamiltonian is given by

$$H_R = \sum_{j=0}^{\infty} \hbar\omega_j b_j^\dagger b_j, \tag{2.2}$$

where ω_j is the frequency and $b_j^\dagger(b_j)$ is the creation (annihilation) operator for the jth reservoir mode. We consider the interaction part V as a quantum non-demolition coupling satisfying

$$[H, V] = 0. \tag{2.3}$$

Such couplings have been previously examined by others and the examples include, optical four-wave mixing [8], many particle bosonic systems [9] etc.

Equation (2.3) implies that V is a constant of motion generated by H and V can be taken as some function of H. We take for example,

$$V = \hbar H \sum_j [\kappa(\omega_j) b_j^\dagger + \kappa^*(\omega_j) b_j], \tag{2.4}$$

where $\kappa(\omega_j)$ is a c-function.

Thus one can write the Liouville equation for the system and the bath in the interaction picture (IP) as

$$\frac{\partial\eta}{\partial t} = -\frac{i}{\hbar}[V_I(t), \eta(t)], \tag{2.5}$$

where $\eta(t)$ and $V_I(t)$ are the joint system-bath density and coupling term in IP respectively and $V_I(t)$ is given by

$$V_I(t) = \exp[i(H + H_R)t/\hbar] V \exp[-i(H + H_R)t/\hbar]. \tag{2.6}$$

After integrating eq. (2.5) and iterating twice one can write

$$\begin{aligned}\eta(t) - \eta(0) = {} & -(i/\hbar)\int_0^t [V_I(t'), \eta(0)]\mathrm{d}t' \\ & + (-i/\hbar)^2 \int_0^t \mathrm{d}t' \int_0^{t'} \mathrm{d}t''[V_I(t'), [V_I(t''), \eta(t'')]].\end{aligned} \tag{2.7}$$

The first term in the RHS of eq. (2.7) will not contribute due to the random initial condition of the bath which is often used. Now we make a number of approximations as follows [10].

(i) We integrate eq. (2.7) with respect to t thereby making a coarse graining approximation.
(ii) We take $\eta(t) = \eta_s(t)\eta_b(t)$ where s and b correspond to system and bath. Further assume the bath is in thermal equilibrium and thus we can write $\eta_b(t) = \eta_b$ (at equilibrium).

(iii) We use the Born–Markov approximation whereby we take $\eta_s(t') = \eta_s(t)$ i.e, truncate the series after second order and make the limits of integration up to infinity.

Now taking trace over the bath varibles one arrives at the equation of motion of the reduced system density operator $\eta_s(t)$ in IP as

$$\frac{d\eta_s}{dt} = -\left\{[HH\eta_s - 2H\eta_s H + \eta_s HH]\sum_j |\kappa(\omega_j)|^2 \int_0^\infty dt' \langle b_j b_j^\dagger\rangle \exp[i\omega_j(t-t')] + [HH\eta_s - 2H\eta_s H + \eta_s HH]\sum_j |\kappa(\omega_j)|^2 \int_0^\infty dt' \langle b_j^\dagger b_j\rangle \exp[-i\omega_j(t-t')]\right\}. \tag{2.8}$$

Thus returning back to the Schrödinger picture (SP) we can write

$$\frac{d\rho}{dt} = -(i/\hbar)[H,\rho] - \left\{[HH\rho - 2H\rho H + \rho HH]\sum_j |\kappa(\omega_j)|^2 \times \int_0^\infty dt' \langle b_j b_j^\dagger\rangle \exp[i\omega_j(t-t')] + [HH\rho - 2H\rho H + \rho HH] \times \sum_j |\kappa(\omega_j)|^2 \int_0^\infty dt' \langle b_j^\dagger b_j\rangle \exp[-i\omega_j(t-t')]\right\}, \tag{2.9}$$

where ρ is the reduced system density operator in SP

Now the survival of the integrals in eq. (2.9) within the Markovian approximation means that approximately zero frequency bath modes are important and very often the characteristic frequency of the system is much larger than that. Thus to attain a finite relaxation constant one needs to have the thermal average number density of bath at approximately zero frequency modes high enough which is possible only at high temperatures. A typical integral can be shown as

$$\sum_j |\kappa(\omega_j)|^2 \int_0^\infty d\tau \langle b_j b_j^\dagger\rangle \exp[i\omega_j\tau] = \sum_j \int_0^\infty d\tau \exp[i\omega_j\tau]|\kappa(\omega_j)|^2[1+\bar{n}(\omega_j)]. \tag{2.10}$$

Now converting the sum into an integral and assuming a continuous spectral density of reservoir modes, $s(\omega)$ and taking the thermal average excitation number in the bath as

$$\bar{n}(\omega_j) = [\exp(\hbar\omega_j/kT) - 1]^{-1}.$$

The RHS of eq. (2.10) can be written as

$$\text{RHS of eq.(2.10)} = (1+\bar{n}(\omega)) \lim_{\omega\to 0} \frac{2\pi|\kappa(\omega)|^2 s(\omega)}{\omega} = \gamma kT/\hbar,$$

where k is the Boltzmann constant and $\gamma = \lim_{\omega\to 0}(2\pi|\kappa(\omega)|^2 s(\omega))/\omega$ and is assumed to be finite, i.e., $\gamma > 0$. We have taken the high temperature limit i.e, $\bar{n}(\omega) \approx kT/\hbar\omega$ and thus neglecting the zero temperature term in eq. (2.9), we arrive at the desired master equation for the reduced system as

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\frac{i}{\hbar}[H, \rho] - c[HH\rho + \rho HH - 2H\rho H]. \tag{2.11}$$

Here c is given by $\gamma kT/\hbar$. The frequency shift term is neglected here.

We first note that eq. (2.11) is a typical model for studying decoherence. The decoherence arising due to the assumption that there exists a classical measurement apparatus during which the measurement process destroys quantum coherences [4] has also been considered. Another approach relies on the assumption in addition to conventional quantum mechanics that in a sufficiently short time scale, the quantum system does not evolve continuously under a unitary transformation but in a stochastic sequence of identical unitary transformation [5, 11]. In that case the rate of decoherence does not depend on temperature but on the inverse of the time scale of stochastic perturbation.

In what follows we consider a two state system with levels $|b\rangle$(ground) and $|a\rangle$(excited) coupled to a single mode field. A basis for the whole Hilbert space is given as $\{|n\,b\rangle, |n\,a\rangle, n = 0, 1, 2, \ldots\}$, where $|n\,b\rangle(|n\,a\rangle)$ denotes a state with n photons with the atom in its ground (excited) state. We assume that the atomic transition is mediated by k photons; i.e, the Hamiltonian in terms of this basis is given, in the RWA, by

$$H = \sum_{n=0}^{\infty} b_n |n\,b\rangle\langle n\,b| + a_n |n\,a\rangle\langle n\,a| + \sum_{n=0}^{\infty} R_n |n\,a\rangle\langle n+k\,b| + R_n^* |n+k\,b\rangle\langle n\,a|, \tag{2.12}$$

where b_n and a_n are the energies of the state $|n\,b\rangle$ and $|n\,a\rangle$ respectively and R_n describes the coupling. This Hamiltonian is diagonalized as follows:

$$H|n\,b\rangle = b_n|n\,b\rangle, \quad n < k, \quad \text{and} \quad H|\Psi_n^{\pm}\rangle = H_n^{\pm}|\Psi_n^{\pm}\rangle, \quad n \geq k, \tag{2.13}$$

where

$$H_n^{\pm} = \frac{b_{n+k} + a_n}{2} \pm \Omega_n, \tag{2.14}$$

and

$$\Omega_n^2 = (\Delta_n/2)^2 + |R_n|^2; \tag{2.15}$$

with detuning $\Delta_n = a_n - b_{n+k}$. The eigenstates $|\Psi_n^{\pm}\rangle$ are the well-known dressed states [12]

$$\begin{aligned} |\Psi_n^+\rangle &\equiv |+n\rangle = \cos\theta_n |b\,n+k\rangle + \sin\theta_n |a\,n\rangle, \\ |\Psi_n^-\rangle &\equiv |-n\rangle = -\sin\theta_n |b\,n+k\rangle + \cos\theta_n |a\,n\rangle, \end{aligned} \tag{2.16}$$

with θ_n given as

$$\tan\theta_n = \frac{R_n}{\Delta_n/2 + \Omega_n}. \tag{2.17}$$

This structure of the Hamiltonian and its solution are valid for many variants of the atom-field interaction models. This model Hamiltonian can be further extended to describe a hydrogen atom in an arbitrarily strong magnetic field and also for systems where the field or the atom can be replaced by a finite dimensional multilevel system which is discussed in § 4.

Here we show that the equation of motion (2.11) is exactly solvable for the Hamiltonian of generalized JC model. To solve the equation of motion we used the dressed state density matrix elements as

$$\rho_{1nm} = \langle +n|\rho| + m\rangle, \quad \rho_{4nm} = \langle -n|\rho| - m\rangle,$$
$$\rho_{2nm} = \langle +n|\rho| - m\rangle, \quad \rho_{3nm} = \langle -n|\rho| + m\rangle. \tag{2.18}$$

The population and coherence of the dressed state elements evolve as

$$\dot{\rho}_{1nn} = 0 = \dot{\rho}_{4nn}$$

and

$$\dot{\rho}_{2nm} = [-i(H_n^+ - H_m^-) - c(H_n^+ - H_m^-)^2]\rho_{2nm},$$
$$\dot{\rho}_{3nm} = [-i(H_n^- - H_m^+) - c(H_n^- - H_m^+)^2]\rho_{3nm},$$
$$\dot{\rho}_{1nm} = [-i(H_n^+ - H_m^+) - c(H_n^+ - H_m^+)^2]\rho_{1nm},$$
$$\dot{\rho}_{4nm} = [-i(H_n^- - H_m^-) - c(H_n^- - H_m^-)^2]\rho_{4nm}. \tag{2.19}$$

It is immediately apparent that the decoherence does not affect the dressed state populations but only induces a decay in the dressed state coherence. Nevertheless the decay of dressed state coherence terms depends on the free evolution of the composite system.

A variety of initial conditions of the atom-field system can be studied but here we restrict ourselves to simple initial conditions that the atom is initially in the excited state and the field is in a coherent state

$$|\alpha\rangle = \sum_{n=0}^{\infty} C_n|n\rangle, \tag{2.20}$$

where

$$C_n = \exp(-|x|^2/2)x^n/\sqrt{(n!)}, \tag{2.21}$$

with $|x|^2$ as the average number of photons. Initial conditions in terms of the dressed state elements can be written as follows:

$$\rho_{1nm}(0) = \sin\theta_n \sin\theta_m C_n C_m^*,$$
$$\rho_{4nm}(0) = \cos\theta_n \cos\theta_m C_n C_m^*,$$
$$\rho_{2nm}(0) = \sin\theta_n \cos\theta_m C_n C_m^*,$$
$$\rho_{3nm}(0) = \cos\theta_n \sin\theta_m C_n C_m^*. \tag{2.22}$$

We define the atomic density matrix as

$$\kappa = \mathrm{Tr}_f[\rho] \tag{2.23}$$

and for the field

$$P = \mathrm{Tr}_a[\rho]. \tag{2.24}$$

Using the above initial conditions, atomic inversion $W(t)$ is given by

$$W(t) = \kappa_{aa} - \kappa_{bb} = \sum_{n=0}^{\infty} [\cos^2 2\theta_n |C_n|^2 + \sin^2 2\theta_n |C_n|^2 \cos(H_n t) \exp[-cH_n^2 t], \tag{2.25}$$

and population distribution in the field is given by

$$\begin{aligned} P_{nn}(t) = \tfrac{1}{2} [&|C_n|^2 + |C_{n-k}|^2 + \cos^2 2\theta_n |C_n|^2 - \cos^2 2\theta_{n-k} |C_{n-k}|^2 \\ &+ \sin^2 2\theta_n |C_n|^2 \cos(H_n t) \exp[-cH_n^2 t] - \sin^2 2\theta_{n-k} |C_{n-k}|^2 \\ &\times \cos(H_{n-k} t) \exp[-cH_{n-k}^2 t]], \end{aligned} \tag{2.26}$$

where H_n is given by

$$H_n = H_n^+ - H_n^-. \tag{2.27}$$

Off-diagonal elements of the atomic and field density matrix can also be obtained similarly. For example atomic coherence properties can be studied if we know the off-diagonal density matrix κ_{ab},

$$\begin{aligned} \kappa_{ab} =(1/2) \sum_{n=0}^{\infty} \{&[\sin(\theta_{n+k} - \theta_n) + \sin(\theta_{nk} + \theta_n)] \rho_{1n\ n+k}(0) \exp[A_{n\ m+k}^+ t] \\ &+ [\sin(\theta_{n+k} - \theta_n) - \sin(\theta_{nk} + \theta_n)] \rho_{4n\ n+k}(0) \exp[A_{n\ m+k}^- t] \\ &+ [\cos(\theta_{n+k} - \theta_n) + \cos(\theta_{nk} + \theta_n)] \rho_{2n\ n+k}(0) \exp[B_{n\ m+k}^+ t] \\ &+ [-\cos(\theta_{n+k} - \theta_n) + \cos(\theta_{nk} + \theta_n)] \rho_{3n\ n+k}(0) \exp[B_{n\ m+k}^- t]\}, \end{aligned} \tag{2.28}$$

where

$$\begin{aligned} A_{nm}^{\pm} &= i(H_n^{\pm} - H_m^{\pm}) - c(H_n^{\pm} - H_m^{\pm})^2, \\ B_{nm}^{\pm} &= \mp i(H_n^+ - H_m^-) - c(H_n^+ - H_m^-)^2. \end{aligned} \tag{2.29}$$

Field coherences can also be evaluated similarly as follows:

$$\begin{aligned} P_{nm}(t) =(1/2)[\{&[\cos(\theta_{m-k} - \theta_{n-k}) + \cos(\theta_{m-k} + \theta_{n-k})] \rho_{1n-k\ m-k}(t), \\ &+ [\cos(\theta_{m-k} - \theta_{n-k}) - \cos(\theta_{m-k} + \theta_{n-k})] \rho_{4n-k\ m-k}(t), \\ &+ [-\sin(\theta_{m-k} - \theta_{n-k}) - \sin(\theta_{m-k} + \theta_{n-k})] \rho_{2n-k\ m-k}(t), \\ &+ [-\sin(\theta_{m-k} - \theta_{n-k}) + \sin(\theta_{m-k} + \theta_{n-k})] \rho_{3n-k\ m-k}(t)\}, \\ &+ \text{similar terms with } m-k \to m \text{ and } n-k \to n]. \end{aligned} \tag{2.30}$$

From eq. (2.25) it is evident that the population inversion in the generalized JC model decays to a steady state value

$$W(\infty) = \sum_{n=0}^{\infty} \cos^2 2\theta_n |C_n|^2. \tag{2.31}$$

Depending on the model, $\cos^2 2\theta_n$ assumes some finite value. For the ordinary JCM at exact resonance, $W(\infty) = 0$.

As the decoherence model is energy conserving, the loss of atomic energy is gained by the field and thus the average energy of the field is saturated to a higher value. Average photon number evolves as

$$\begin{aligned}\bar{n}(t) = \sum_{n=0}^{\infty} |C_n|^2 \{(n + k/2) - k/2 \cos^2 2\theta_n \\ - k/2 \sin^2 2\theta_n \cos(H_n t) \exp[-cH_n^2 t]\}.\end{aligned} \tag{2.32}$$

With the suppression of oscillation in the average photon number it reduces to a steady state value

$$\begin{aligned}\bar{n}(\infty) &= \sum_{n=0}^{\infty} |C_n|^2 \{(n + k/2) - k/2 \cos^2 2\theta_n \\ &= \bar{n}(0) + (k/2)\left[1 - \sum_{n=0}^{\infty} |C_n|^2 \cos^2 2\theta_n\right].\end{aligned} \tag{2.33}$$

To investigate the sub-Poissonian nature of the field we need to know the average of the square of the photon number in the field which can be expressed as

$$\begin{aligned}\overline{n^2}(t) = \sum_{n=0}^{\infty} |C_n|^2 \{(n^2 + nk + k^2/2) - (n + k^2/2) \cos^2 2\theta_n \\ - (n + k^2/2) \sin^2 2\theta_n \cos(H_n t) \exp[-cH_n^2 t]\}.\end{aligned} \tag{2.34}$$

The sub-Poissonian characteristics of the field is exemplified for a particular Hamiltonian of atom-field interaction which is presented in § 3.

From eq. (2.28) it is evident that atomic coherence decays in time and at sufficiently large times it goes to zero. This is also true for the coherence terms of the field, i.e.,

$$P_{nm}(\infty) = 0, \quad \text{for } n \neq m. \tag{2.35}$$

Quadrature squeezing properties of the field can also be calculated by using eq. (2.30). The degree of squeezing can be measured by the squeezing parameter,

$$S_i = \frac{\langle (\Delta X_i)^2 \rangle - |\langle [X_1, X_2] \rangle|/2}{|\langle [X_1, X_2] \rangle|/2}, \tag{2.36}$$

where X_i can be defined as

$$\begin{aligned}X_1 &= (1/2)(a e^{i\omega t} + a^\dagger e^{-i\omega t}) \\ X_2 &= (1/2i)(a e^{i\omega t} - a^\dagger e^{-i\omega t}).\end{aligned} \tag{2.37}$$

A quadrature is said to be squeezed if $S_i < 0$. In terms of the expectation values $a, a^\dagger$ of $S_{1,2}$ can be defined as

$$S_1 = 2\langle a^\dagger a\rangle + 2\mathrm{Re}\langle a^2 e^{2i\omega t}\rangle - 4(\mathrm{Re}\langle a e^{i\omega t}\rangle)^2$$
$$S_2 = 2\langle a^\dagger a\rangle - 2\mathrm{Re}\langle a^2 e^{2i\omega t}\rangle - 4(\mathrm{Im}\langle a e^{i\omega t}\rangle)^2. \quad (2.38)$$

It can be shown easily that at the steady state,

$$S_1 = S_2 = 2\langle a^\dagger a\rangle > 0. \quad (2.39)$$

So at the steady state the generalized JC model shows there is no squeezing in any quadrature.

Finally, we would like to point out that decoherence on the generalized JC model acts as a relaxation on the dressed state coherence only. But it does not affect the dressed state populations unlike the case of ordinary spontaneous emission [10] where both the dressed state population and coherences decay in time and subsequently all the energy and coherence of the atom and the field drain out of the system at the steady state.

2b. Transient dynamics of atom or field for ordinary JCM

Here we explain the features which appear in the transient dynamics of atom or field due to this particular nature of decoherence. We take the ordinary JCM as the candidate to describe it. As the decoherence mechanism affects individual subsystems, e.g., the atom or the field in an almost similar manner we concentrate only on the dynamics of the atomic properties.

The Hamiltonian for the ordinary JCM describing a two-level atom interacting with a single mode field confined in a cavity within RWA is given by

$$H = \hbar\omega a^\dagger a + (\hbar\omega_0/2)\sigma_z + \hbar g(a^\dagger\sigma_- + a\sigma_+), \quad (2.40)$$

where $a(a^\dagger)$ is the annihilation (creation) operator of the single mode cavity field with frequency ω and σ_+, σ_- and σ_z are the usual spin half operators for the two-state atom with characteristic frequency ω_0. g is the coupling strength between the atom and the field mode.

To explain the transient dynamics the following expressions are in order for ordinary JCM. Calculating at exact resonance, i.e., when $\delta = \omega - \omega_0$ becomes zero and taking $\hbar = 1$ we obtain the atomic inversion $W(t)$ as

$$W(t) = \sum_{n=0}^{\infty} |C_n|^2 \cos[2g\sqrt{(n+1)}t]\exp[-4cg^2(n+1)t]. \quad (2.41)$$

The field population matrix elements evolve as

$$P_{nn}(t) = |C_n|^2\{\cos^2[g\sqrt{(n+1)}t]\exp[-4cg^2(n+1)t] + (1/2)[1-\exp[-4cg^2(n+1)t]\} + |C_{n-1}|^2\{\cos^2[g\sqrt{n}t]\exp[-4cg^2 nt] + (1/2)[1-\exp[-4cg^2 nt]\}. \quad (2.42)$$

The atomic and field coherence, i.e., off-diagonal atomic and field density matrix elements can be evaluated as

$$\begin{aligned}\kappa_{ab} =& (C_n C^*_{n+1}/4) \sum_{n=0}^{\infty} \{\exp[A^+_{nn+1}t] - \exp[A^-_{nn+1}t] \\ &+ \exp[B^+_{nn+1}t] - \exp[B^-_{n\,n+1}t]\},\end{aligned} \tag{2.43}$$

and

$$\begin{aligned}P_{nm} =& (C_n C^*_m/4)\{\exp[A^+_{nm}t] + \exp[A^-_{nm}t] - \exp[B^+_{nm}t] + \exp[B^-_{nm}t]\} \\ &+ \text{terms with } \{n \to n-1 \text{ and } m \to m-1\},\end{aligned} \tag{2.44}$$

where

$$\begin{aligned}A^{\pm}_{nm} =& -i[\omega(n-m) \pm g\sqrt{(n+1)} \mp g\sqrt{(m+1)}] \\ &- c[\omega(n-m) \pm g\sqrt{(n+1)} \mp g\sqrt{(m+1)}]^2, \\ B^{\pm}_{nm} =& \mp i[\omega(n-m) + g\sqrt{(n+1)} + g\sqrt{(m+1)}] \\ &- c[\omega(n-m) + g\sqrt{(n+1)} + g\sqrt{(m+1)}]^2.\end{aligned}$$

An interesting aspect of the decoherence mechanism for ordinary JCM is that it affects the population and coherence properties of the individual subsystem in a different way. For the ordinary JCM, we have shown the collapse and revival dynamics of the atomic population for a parameter value of cg as 0.005, and photon number gets erased instantaneously. But the coherence properties of the atom or field decay at a slower rate. We show the time evolution of atomic population inversion and atomic coherence and the purity of the reduced atomic density in figures 1, 2 and 3 respectively. For numerical purpose, we assume $\omega/g = 2$. Note that $cg\hbar^2$ is a dimensionless quantity and here we assume $\hbar = 1$.

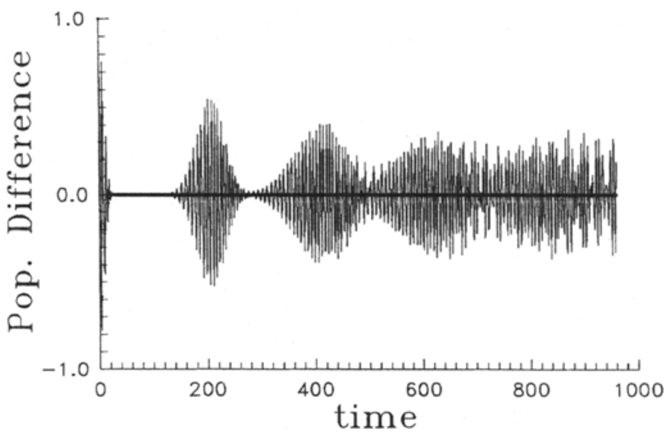

Figure 1. Population inversion $W(t)$ (eq. 2.41) is plotted with scaled time (gtx) when the field is initially in coherent state with $|x|^2 = 16$ (thinner line is pure JCM and thicker line is with decoherence ($cg\hbar^2 = 0.005$)).

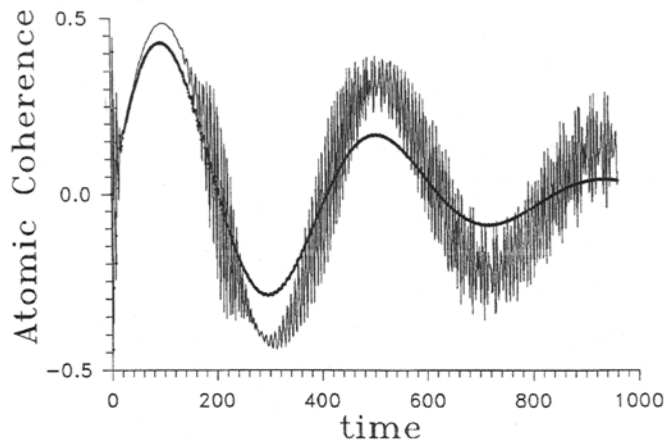

Figure 2. Atomic coherence κ_{ab} (eq. 2.43) is plotted with scaled time (gtx) when the field is initially in coherent state with $|x|^2 = 16$ (thinner line is pure JCM and thicker line with decoherence $cg\hbar^2 = 0.005$)).

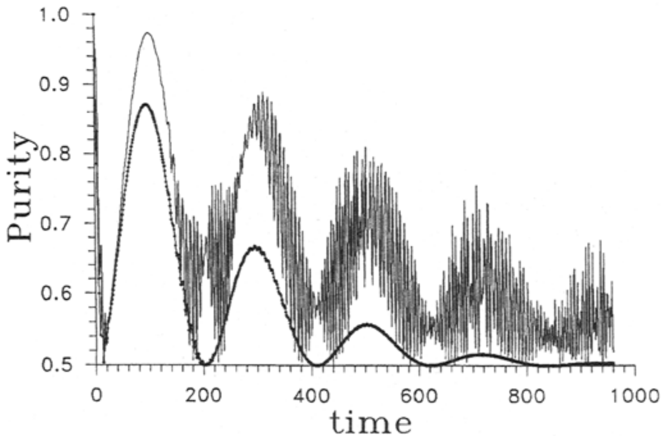

Figure 3. Purity of the atom $\mathrm{Tr}\,(\kappa^2)$ is plotted with scaled time (gtx) when the field is initially in coherent state with $|x|^2 = 16$ (thinner line is pure JCM and thicker line is with decoherence $cg = 0.005$)).

In figure 1 we show the time evolution of $W(t)$ in eq. (2.41) with the value of $cg = 0.005$ and $|x|^2 = 16$. It is seen that the revival of atomic inversion gets erased and a permanent collapse of population appears. The oscillation in atomic inversion and photon number is well studied for the simple JCM system [1]. The oscillation appears due to the coherence built up during the atom-field interaction and thus the individual atom or field population depends on the coherence of the dressed state levels. Thus when a decoherence is introduced in the dressed state the oscillation disappears very quickly.

We show the atomic coherence evolution eq. (2.42) in figure 2 to illustrate that the effect of decoherence appears more slowly compared to that in the population terms. In the long time limit the atomic or the field coherences vanish.

Gea–Banacloche [13] has recently given a recipe for the preparation of atomic superposed state in the middle of the first collapsed region for ordinary JCM. To obtain the macroscopically superposed state of an atom one has to stay in the pure state i.e., $\mathrm{Tr}(\kappa^2) = 1$ which is the necessary and sufficient condition for a pure state. This is because unlike the state vector, the density operator does not really describe an individual system but rather an ensemble of identically prepared atoms.

Keeping in view of Gea–Banacloche's observations we study how the decoherence affects the purity of the atom. This is displayed in figure 3. Here it is quite clear that although the atom is almost permanently collapsed for a low value of the decoherence parameter c, the purity of the atom oscillates between 1 (pure state) and 0.5 (maximally mixed ensemble). From figure (3) it is apparent that the purity does not get affected much although the collapse occurs in the inversion very rapidly. In the first half-collapse-time of JCM, the purity is not far from one; also it depends on the average initial photon number. In the long time the purity gets decayed sequentially after many Rabi cycles. The only effect which can be observed due to decoherence is that the profile of time evolution of purity gets smoothened over large oscillations due to the suppression of dressed state coherences. Thus we can conclude that the superposed state of the atom in a JCM system in the middle of the first collapse time is robust even under the influence of environmental pure decoherence on the atom-field composite system as long as the rate of decoherence is not large.

When the JCM system undergoes decoherence, two distinct interference effects arise. The first one is the ordinary interference of dressed state coherences in pure JCM which is dynamical in nature and is reversible, thereby giving recurrence of collapse and revival of the population and a transient oscillation in the coherence properties of atom and field over a slowly varying profile. The second, interference effect is due to the decay of coherence of dressed states which is irreversible in character. The mechanism of decoherence as introduced here affects the population and the coherence in different ways. From eqs (2.41), (2.42) and eqs (2.43), (2.44) it is clear that the decay rate in population and coherence of an individual subsystem depends on the time scale of oscillation in the free evolution of the dressed state of the JCM system. In the free evolution of population there is a fast oscillation of collapse and revival but in the coherence terms of an individual subsystem there is a slower time scale of oscillation along with a fast oscillation. Thus in the decay rate of coherence these two components will arise unlike the case of population where only one kind of decay rate will appear which is fast. So when the decoherence mechanism is introduced in JCM, it first kills the fast oscillation in population and coherence properties of the atom and field. This gives steady redistributed energy values of atom and field and a smooth profile of slow evolution of coherence terms. Then the decoherence mechanism acts slowly to wash out the coherence of individual subsystems and takes a few Rabi cycles to reach to their vanishing values in the steady state. The diminishing curve of atomic purity in figure 3 can be easily understood in terms of the number of Rabi cycles needed for the complete wash out of atomic coherence for a fixed value of the parameter c.

3. Nonlinear Jaynes–Cummings model

Here we consider a particular case of § 2(a), a Hamiltonian which describes a multiphoton interaction of atom and field with Kerr nonlinearity in the cavity. The Hamiltonian is given by

$$H = \hbar\omega a^\dagger a + \hbar q a^{\dagger 2} a^2 + (\hbar\omega_0/2)\sigma_z + \hbar g(a^{\dagger k}\sigma_- + a^k\sigma_+). \tag{3.1}$$

Here q is the nonlinearity parameter for the Kerr medium and the two-level atom is interacting with the field via a k-photon mediated process. For $k = 1$ and $q = 0$ this model reduces to the ordinary JCM in eq. (2.40).

With the help of §2(a) the various parameters can be identified as follows:

$$b_n = -\omega_0/2 + n\omega + qn(n-1), a_n = \omega_0/2 + n\omega + qn(n-1),$$

$$\Delta_n = \delta - q(2nk + k^2 - k), \quad \delta = \omega_0 - k\omega,$$

$$R_n = g\sqrt{\frac{(n+k)!}{n!}},$$

$$H_n^\pm = (n + k/2)\omega + q[n^2 - n + nk + k^2/2 - k/2] \pm \{(\Delta_n/2)^2 + R_n^2\}^{1/2}$$

and

$$\tan\theta_n = \frac{g[(n+k)!/n!]^{1/2}}{(\delta/2+q(nk+k^2/2-k/2)]+\{g^2(n+k)!/n! + [\delta/2 + q(nk + k^2/2 - k/2)]^2\}^2}. \tag{3.2}$$

For $k = 1$, this Hamiltonian is studied in detail in the context of micromaser [14], population trapping [15] and it shows beating [16] in population inversion instead of conventional collapse and revival due to its intensity dependent detuning [17]. For $\delta = 0$ and $q = g/2$ it shows an interesting behaviour [15] in this model due to the fact that the Rabi frequency in this case becomes $g(n/2+1)^{1/2}$ instead of $g(n+1)^{1/2}$ in conventional JCM.

Due to the intensity dependent detuning, here $\tan\theta_n$ is not equal to 1 even if $\delta = 0$. So the population inversion in the steady state in this case can be evaluated as

$$W(\infty) = \sum_{n=0}^{\infty} \cos^2 2\theta_n |C_n|^2. \tag{3.3}$$

So depending on the nonlinearity parameters q and k, atomic inversion can be saturated to a value higher than 0.5. Thus in the presence of nonlinearity, the transfer of energy from atom to field is less than that in the normal JCM.

The Mandel Q parameter which is defined as

$$Q = \frac{\overline{\Delta n^2} - \bar{n}}{\bar{n}} \tag{3.4}$$

can be obtained from the steady state values of $\bar{n}(\infty)$ and $\overline{n^2}(\infty)$ as

$$\bar{n}(\infty) = \sum_{n=0}^{\infty} |C_n|^2 \{(n + k/2) - k/2 \cos^2 2\theta_n\}, \tag{3.5}$$

$$\overline{n^2}(\infty) = \sum_{n=0}^{\infty} |C_n|^2 \{(n^2 + nk + k^2/2) - (n + k^2/2) \cos^2 2\theta_n\}. \tag{3.6}$$

So even if $q = 0$ and $\delta = 0$, $\cos 2\theta_n$ is equal to zero and thus

$$Q(\infty) = -\frac{1}{4\bar{n}(0) + k/2}. \tag{3.7}$$

If $q/g = 0.5, \delta = 0$ and $k = 1$, in this case also $\cos 2\theta_n = 0$ and the value of $Q(\infty)$ is the same as above.

Now if $\tan \theta_n < 1$, for $q \neq 0, \delta \neq 0, k \neq 1$, the steady state Q-parameter is always less than 0, which means that the sub-Poissonian nature is not erased.

4. More generalized JC models

Here we present a more generalized JCM which not only accounts for the various ordinary JCM of atom-field interactions as described in the earlier sections but also the models describing a class of spin-orbit interactions.

Here we write the total Hamiltonian of the spin-orbit system as

$$H = \sum_{n=n_0}^{n_m} b_n |n\, b\rangle\langle n\, b| + a_n |n\, a\rangle\langle n\, a| + \sum_{n=n_0}^{n_m - 1} R_n |n\, a\rangle\langle n + 1 b| + R_n^* |n + 1 b\rangle\langle n\, a|, \tag{4.1}$$

where $|a\rangle(|b\rangle)$ represents the upper (lower) spin state and n is the label for the angular momentum states. Unlike the ordinary JCM states $|n\rangle$ where n runs from 0 to ∞, for the angular momentum states n runs from $-l$ to $+l$ with l the orbital angular momentum. In this case $n_0 = -l$ and $n_m = +l$. For simplicity, we assume one quantum of energy is transferred at a time from spin to orbit states or vice versa. R_n is the coupling constant depending on the algebraic property of the operators, $\sum_{n=n_0}^{n_m-1} |n\rangle\langle n+1|, \sum_{n=n_0}^{n_m-1} |n+1\rangle\langle n|$ and $\sum_{n=n_0}^{n_m} |n\rangle\langle n|$.

Unlike the ordinary JCM, this Hamiltonian has two zero point energy states which do not couple with other states. These states are $|n_0\, b\rangle$ and $|n_m\, a\rangle$. These two states are diagonal in H as

$$H|n_0\, b\rangle = b_{n_0} |n_0\, b\rangle,$$

$$H|n_m\, a\rangle = a_{n_m} |n_m\, a\rangle. \tag{4.2}$$

For all other states with $n \neq n_0, n_m$, H can be diagonalized as

$$H|\psi_{n_1}^{\pm}\rangle = H_{n_1}^{\pm} |\psi_{n_1}^{\pm}\rangle, \tag{4.3}$$

where

$$H_{n_1}^{\pm} = \frac{b_{n_1+1} + a_{n_1}}{2} \pm \Omega_{n_1} \tag{4.4}$$

and

$$\Omega_{n_1}^2 = (\Delta_{n_1}/2)^2 + |R_{n_1}|^2; \tag{4.5}$$

with detuning

$$\Delta_{n_1} = a_{n_1} - b_{n_1+1}. \tag{4.6}$$

The eigenstates $|\psi_n^{\pm}\rangle$ are given by

$$\begin{aligned} |\psi_n^+\rangle &\equiv |+n_1\rangle = \cos\theta_{n_1}|b\,n_1+1\rangle + \sin\theta_{n_1}|a\,n_1\rangle, \\ |\psi_n^-\rangle &\equiv |-n_1\rangle = -\sin\theta_{n_1}|b\,n_1+1\rangle + \cos\theta_{n_1}|a\,n_1\rangle, \end{aligned} \tag{4.7}$$

with θ_{n_1} obtained from

$$\tan\theta_{n_1} = \frac{R_{n_1}}{\Delta_{n_1}/2 + \Omega_{n_1}}. \tag{4.8}$$

With these discussions in mind one can generalize the above Hamiltonian H for multistate spin systems instead of two state spin system. A more general Hamiltonian in that case can be written as

$$H = \sum_{s=s_0}^{s_m}\sum_{n=n_0}^{n_m} a_{ns}|n\,s\rangle\langle n\,s| + \sum_{s=s_0}^{s_m-k}\sum_{n=n_0}^{n_m-k} R_{ns}|n\,s+k\rangle\langle n+k\,s| + R_{ns}^*|n+k\,s\rangle\langle n\,s+k|, \tag{4.9}$$

where k quantum is exchanged between the spin-orbit interacting system and this Hamiltonian is also diagonalized in a similar way. Here we will not dwell on further applications of this Hamiltonian (4.9).

5. A hydrogen atom in presence of a magnetic field undergoing pure decoherence—effect on spin dynamics

Here we present a special case of spin-orbit interaction discussed in the previous section – a model of hydrogen atom in presence of a constant magnetic field [18, 19]. The Hamiltonian can be written as

$$H = H_0 + H_1, \tag{5.1}$$

where

$$H_0 = p^2/2m + V(r) \tag{5.2}$$

is the usual free part of hydrogen atom with $V(r)$ the central potential and p and m are the momentum and mass of the electron. The interaction term H_1 can be written as

$$\begin{aligned} H_1 &= \beta(L_z + \sigma_z) + \alpha\, L.S \\ &= \beta L_z + [(\alpha/2)L_z + \beta]\sigma_z + (\alpha/2)(L_-\sigma_+ + L_+\sigma_-), \end{aligned} \tag{5.3}$$

where β is the strength of the magnetic field applied in the z-direction and (L_z, L_+, L_-) and $(\sigma_z, \sigma_+, \sigma_-)$ are the usual orbital and spin angular momentum operators respectively. α is the spin-orbit coupling constant which depends on the central potential $V(r)$. We solve here only H_1 for a fixed value of l.

From the previous section one can immediately identify that

$$b_{n_l} = -\beta + \beta n_l - \alpha n_l/4, \tag{5.4}$$

and

$$a_{n_l} = \beta + \beta n_l + \alpha n_l/4. \tag{5.5}$$

Δ_{n_l} is defined as

$$\Delta_{n_l} = a_{n_l} - b_{n_l} = \beta + \alpha(2n_l + 1)/4 \tag{5.6}$$

and

$$H^{\pm}_{n_l} = \frac{\beta(2n_l+1)}{2} - \frac{\alpha}{8} \pm \frac{\alpha}{2}\sqrt{[(l-n_l)(l+n_l+1)]}, \tag{5.7}$$

where l is the orbital angular momentum quantum number, and θ_{n_l} is given by

$$\tan\theta_{n_l} = \frac{(\alpha/2)\sqrt{[(l-n_l)(l+n_l+1)]}}{\Delta_{n_l}/2 + \Omega_{n_l}}. \tag{5.8}$$

In this case $n_0 = -l$ and $n_{m_l} = +l$.

Using the above results one can obtain exactly the energy levels of any state of hydrogen atom in presence of a magnetic field by avoiding the approximations which are commonly made. One such approximation is the Zeeman approximation, where the magnetic field is assumed to be so weak that the terms with magnetic field can be treated as perturbation. The other approximation is the Paschen–Back limit when the magnetic field is so strong that the spin-orbit interaction is taken as perturbation.

Now we consider the Hamiltonian of a hydrogen atom in a magnetic field as the system Hamiltonian and assume a model of decoherence as in § 2(a) for this particular system. We discuss the dynamics of spin in presence of magnetic field and investigate the effect of pure decoherence on the spin motion of hydrogen atom in a magnetic field.

Using the same notations as used in § 2(a), the Liouville equation (2.11) for this Hamiltonian can be solved to obtain the spin inversion $W(t)$ as

$$\begin{aligned} W(t) =& (1/2)[(\rho_{1n_mn_m}(t) + \rho_{4n_mn_m}(t)) + \cos 2\theta_{n_m}(\rho_{1n_mn_m}(t) - \rho_{4n_mn_m}(t)) \\ & - \sin 2\theta_{n_m}(\rho_{2n_mn_m}(t) + \rho_{3n_mn_m}(t))] - \sum_{n_l=n_0}^{n_m} \cos 2\theta_{n_l}[(\rho_{1n_ln_l}(t) \\ & - \rho_{4n_ln_l}(t) - \sin 2\theta_{n_l}(\rho_{2n_ln_l}(t) + \rho_{3n_ln_l}(t))]; \end{aligned} \tag{5.9}$$

this expression is valid for any l.

We take the orbital angular momentum value $l = 1$, so the magnetic quantum number n_l can assume three values $-1, 0, +1$. The initial condition of the spin-orbit system is taken as

$$\psi(0) = (1/\sqrt{6})(|n_{+1}\rangle + |n_0\rangle + |n_{-1}\rangle)(|a\rangle + |b\rangle). \tag{5.10}$$

Now for $l = 1$, the initial condition can be rewritten in terms of the dressed state densities as follows:

$$\rho_{1n_l n_l} = (1 + \sin 2\theta_{n_l})/6,$$
$$\rho_{4n_l n_l} = (1 - \sin 2\theta_{n_l})/6,$$
$$\rho_{2n_l n_l} = (\cos 2\theta_{n_l})/6,$$
$$\rho_{3n_l n_l} = (\cos 2\theta_{n_l})/6. \quad (5.11)$$

Finally $W(t)$ can be expressed as

$$W(t) = \left[1/6 + (1/12)\sin 4\theta_{n_m} - (1/6)\sum_{n_l=n_0}^{n_m} \sin 4\theta_{n_l}\right] - (1/12)\sin 4\theta_{n_m}\exp(-cH_{n_m}^2 t)\cos(H_{n_m} t) + \sum_{n_l=n_0}^{n_m}(1/12)\sin 4\theta_{n_l}\exp(-cH_{n_l}^2 t)\cos(H_{n_l} t), \quad (5.12)$$

where H_{n_l} is defined as

$$H_{n_l} = H_{n_l}^{+} - H_{n_l}^{-}. \quad (5.13)$$

So for $c = 0$, that is when there is no decoherence, $W(t)$ has an oscillatory pattern composed of interference of $\cos(H_{n_l} t)$ terms and may show a collapse and revival type behavior as in atom-field JCM case. But when decoherence is present, i.e., $c \neq 0$, the spin inversion relaxes with time. At the steady state the value of $W(\infty)$ depends on the strength of the magnetic field and spin-orbit coupling strength which is apparent from eq. (5.12).

6. Summary and conclusion

We have investigated the effect of environment induced pure decoherence on the generalized Jaynes–Cummings model in the atom-field interaction case. It is shown that at the steady state, transient oscillations in atom and field observables are suppressed but all the nonclassical properties are not erased by the decoherence. For example, it is seen that an initial Poissonian field acquires a nonclassical character in the steady state. Steady state observables are shown to be model dependent. This is shown with an example of a multiphoton JCM in presence of a Kerr nonlinearity in the cavity.

It is also shown that pure decoherence affects the population and coherence properties of the individual subsystem in a different way. For ordinary JCM we have shown that for a small value of the parameter c, atomic population inversion collapses very fast but the purity and other phase related quantities of atom take a few Rabi cycles to reach their steady state. Thus the atomic superposition in the middle of the first-half-collapse time is robust even under the influence of environment induced pure decoherence.

We have studied the effect of decoherence on more generalized JC models which not only account for the atom-field interactions but also a class of spin-orbit interactions in the framework of JCM. As a particular example, the dynamics of spin is studied for the hydrogen atom in presence of a magnetic field undergoing decoherence. A more generalized JCM Hamiltonian and its solution are also proposed which can account for multiple spin states.

It is interesting to note that the pure decoherence can not erase all nonclassical features of atom and field unlike the case of spontaneous emission, where along with the decoherence, an energy dissipation guarantees the steady state with no nonclassical behaviour in either subsystem.

Nevertheless, the structure of the Liouville equation (2.11) has some general importance because various mechanisms of stochastic interactions with the system in the Born–Markov limit lead to this kind of equation. We hope that the generalized JCM undergoing decoherence may be worthwhile to investigate whenever a stochastic modulation of energy of the system is present.

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References

[1] See for example a recent review by, B W Shore and P L Knight, *J. Mod. Opt.* **40**, 1195 (1993) and references therein
[2] E A Hinds, in *Advances in atomic and molecular and optical physics* edited by D Bates and B Bederson (NY, Academic) **28**, p. 237
S Haroche and D Kleppner, *Phys. Today*, **42**, 24 (1989)
[3] A Tameshtit and J E Sipe, *Phys. Rev.* **A45**, 8280 (1992); **47**, 1697 (1993)
[4] J A Wheeler and W H Zurek, *Quantum theory of measurement.* (Princeton, 1986)
A Venugopalan, D Kumar and R Ghosh, *Physica* **A220**, 563 (1995)
[5] G J Milburn, *Phys. Rev.* **A44**, 5401 (1991); **47**, 2415 (1993)
[6] A Venugopalan, D Kumar and R Ghosh, *Physica* **A220**, 576 (1995)
[7] See, for example, a recent review on sub-Poissonian light by L Davidovich, *Rev. Mod. Phys.* **68**, 127 (1996) and references therein
[8] G J Milburn and D F Walls, *Phys. Rev.* **A28**, 2646 (1983)
[9] R Alicki. *Phys. Lett.* **A103**, 247 (1984)
[10] W H Louisell, *Quantum Statical properties of radiation* (NY, John Wiley, 1973)
G Gangopadhyay and D S Ray, in *Advances in multiphoton processes and spectroscopy* edited by S H Lin, A A Villaeys and Y Fujimura **8**, (1993)
[11] H Moya-Cessa, V Buzek, M S Kim and P L Knight, *Phys. Rev.* **A48**, 3900 (1993)
X Chen and L M Kuang, *Phys. Lett.* **A191**, 18 (1994)
[12] C Cohen-Tannoudji, J Dupont-Roc and G Grynberg, *Atom-Photon Interaction* (NY, Wiley, 1992)
[13] J Gea-Banacloche, *Phys. Rev. Lett.* **65**, 3385 (1990)
[14] B Deb and D S Ray, *Phys. Rev.* **A48**, 3191 (1993)
[15] A Bandyopadhyay and G Gangopadhyay, *J. Mod. Opt.* **43**, 487 (1996)
[16] S Du and C Gong, *Phys. Rev.* **A17**, 779 (1994)
[17] V Buzek and I Jex, *Opt. Commun.* **78**, 425 (1990)
[18] J J Sakurai, *Modern quantum mechanics* (Addison-Wesley, Reading, MA, 1985)
[19] S Yu, H Rauch and Y Zhang, *Phys. Rev.* **A52**, 2585 (1995)