PRAMANA
— journal of
physics

Vol. 49, No. 2,
August 1997
pp. 213–224

Inhomogeneous cosmological models with heat flux

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MS received 6 November 1995; revised 27 September 1996

Abstract. We present a general class of inhomogeneous cosmological models filled with non-thermalized perfect fluid by assuming that the background spacetime admits two space-like commuting Killing vectors and has separable metric coefficients. The singularity structure of these models depends on the choice of the parameters and the metric functions. A number of previously known perfect fluid models follow as particular cases of this general class. Physical and geometrical features of these models are studied and the general expression for temperature distribution is given.

Keywords. Inhomogeneous cosmology models; heat flux.

PACS Nos 04.20; 98.80

1. Introduction

The standard Friedman–Robertson Walker (FRW) cosmological model which prescribes a homogeneous and isotropic distribution for its matter content, has been quite successful in describing the present state of the universe. It is though realised that the homogeneous and isotropic character of the spacetime cannot be sustained at all scales especially for the early times. One of the main features of relativistic cosmology is the prediction of the big bang singularity in the finite past. This conviction arose out of the general result that under physically reasonable conditions of positivity of energy, causality, regularity etc., the initial singularity is inescapable in cosmology so long as we adhere to Einstein's equations and it can only be avoided by invoking quantum effects and/or modifying Einstein's theory.

Recently Senovilla [1] obtained a new class of exact solutions of Einstein's equations without big bang singularity, representing a cylindrically symmetric, inhomogeneous universe filled with perfect fluid which is smooth and regular everywhere satisfying energy and causality conditions. All the physical and geometrical invariants are finite and regular for whole of spacetime. Later Ruiz and Senovilla [2] have separated out a fairly large class of singularity free models through a comprehensive study of general cylindrically symmetric metric with separable functions of r and t as metric coefficients.

Dadhich, Tikekar and Patel [3] have established a link between the FRW models and the singularity free family by deducing the latter through a natural and simple inhomogenization and anisotropization of the former. The matter content of the above models has been a perfect fluid in thermal equilibrium. At very early times, matter in the universe is supposed to be in a highly dense and hot state. Subsequently consideration of dissipative effects in cosmology becomes highly significant. Dissipative effects in cosmology were considered in the context of large entropy per baryon and isotropy of microwave background radiation by Misner [4] and Caderni and Fabri [5]. Cosmological models with heat flux have been studied by several authors [6–17].

In this paper we have considered an inhomogeneous metric which admits two space-like commuting Killing vectors and has separable metric coefficients and obtained a general class of inhomogeneous solutions of Einstein's field equations with non-thermalized perfect fluid as the source term. Explicit exact solutions in a number of particular cases have been obtained. These particular solutions include the heat flux generalizations of the general class of inhomogeneous perfect fluid solutions of Ruiz and Senovilla [2].

2. Metric and field equations

We begin with the spacetime which admits two commuting space-time Killing vector fields that are hyper-surface orthogonal. It is well known that the metric of such spacetimes can be put in the generalized Einstein–Rosen form. It is further assumed that the metric coefficients are separable functions. Ruiz and Senovilla [2] take the metric in the form

$$\mathrm{d}s^2 = T^{2m}F^2(\mathrm{d}t^2 - H^2\mathrm{d}x^2) - T^{n+1}GP\mathrm{d}y^2 - T^{1-n}GP^{-1}\mathrm{d}z^2, \tag{1}$$

where $T = T(t)$ and F, G, P, H are functions of only x, m and n are constants. The freedom of choosing co-ordinates is used in having the same time dependence in the metric coefficients g_{11} and g_{44}. The co-ordinates are labelled as $x^1 = x, x^2 = y, x^3 = z$ and $x^4 = t$. The function $H(x)$ is introduced to facilitate the integration of Einstein field equations in terms of elementary functions. Note that the equation of state $p = k\rho$ has not been chosen *a priori*. Further one does not even have to assume time dependence to come from the single function $T(t)$, it has been shown [17] that it follows from the perfect fluid conditions.

We introduce the orthonormal tetrad

$$\theta^1 = T^m FH\mathrm{d}x, \quad \theta^2 = T^{(1+n)/2}\sqrt{GP}\mathrm{d}y,$$
$$\theta^3 = T^{(1-n)/2}\sqrt{G/P}\mathrm{d}z, \quad \theta^4 = T^m F\mathrm{d}t. \tag{2}$$

The non-zero components of the Ricci tensor for the metric (1) in the above tetrad frame have the explicit expressions

$$R_{11}T^{2m}F^2 = \frac{1}{H^2}\left[\frac{G''}{G} - \frac{1}{2}\frac{G'^2}{G^2} + \frac{1}{2}\frac{P'^2}{P^2} + \frac{F''}{F} - \frac{F'G'}{FG} - \frac{H'G'}{HG} - \frac{F'^2}{F^2} - \frac{F'H'}{FH}\right] - m\frac{\ddot{T}}{T}, \tag{3}$$

$$R_{22}T^{2m}F^2 = \frac{1}{H^2}\left[\frac{1}{2}\left(\frac{G''}{G} - \frac{P''}{P}\right) + \frac{1}{2}\frac{P'^2}{P^2} - \frac{1}{2}\frac{G'P'}{GP} - \frac{1}{2}\frac{H'G'}{HG} + \frac{1}{2}\frac{P'H'}{PH}\right] - \frac{1}{2}(1+n)\frac{\ddot{T}}{T}, \tag{4}$$

$$R_{33}T^{2m}F^2 = \frac{1}{H^2}\left[\frac{1}{2}\left(\frac{G''}{G} + \frac{P''}{P}\right) - \frac{1}{2}\frac{P'^2}{P^2} + \frac{1}{2}\frac{G'P'}{GP} - \frac{1}{2}\frac{H'G'}{HG} - \frac{1}{2}\frac{P'H'}{PH}\right] - \frac{1}{2}(1-n)\frac{\ddot{T}}{T}, \tag{5}$$

$$R_{44}T^{2m}F^2 = -\frac{1}{H^2}\left[\frac{F''}{F} - \frac{F'^2}{F^2} - \frac{F'H'}{FH} + \frac{F'G'}{FG}\right] + (m+1)\frac{\ddot{T}}{T} + \frac{1}{2}(n^2 - 1 - 4m)\frac{\dot{T}^2}{T^2}, \tag{6}$$

$$R_{14}T^{2m}F^2 = \frac{1}{2H}\frac{\dot{T}}{T}\left[(1-2m)\frac{G'}{G} + n\frac{P'}{P} - 2\frac{F'}{F}\right]. \tag{7}$$

Here and in what follows, a prime and a dot indicate derivative with respect to x and t respectively.

The matter content of the spacetime is assumed to be a non-thermalized perfect fluid described by the energy momentum tensor

$$T_{\alpha\beta} = (\rho + p)U_\alpha U_\beta - p g_{\alpha\beta} + (q_\alpha U_\beta + q_\beta U_\alpha),$$
$$U_\alpha U^\alpha = 1 \quad U_\alpha q^\alpha = 0, \tag{8}$$

where ρ and p denote the energy density and fluid pressure of matter distribution. U^α denotes unit time-like flow vector of fluid and q^α denotes the space-like heat flow vector orthogonal to U^α. The components of these vectors in the tetrad frame (2) are found to be

$$U^\alpha = (0,0,0,1), \quad q_\alpha = (q,0,0,0) \tag{9}$$

where q is a function of co-ordinates governed by the Einstein field equations

$$R_{\alpha\beta} = -8\pi[(\rho + p)U_\alpha U_\beta - \tfrac{1}{2}g_{\alpha\beta}(\rho - p)] - 8\pi(q_\alpha U_\beta + q_\beta U_\alpha). \tag{10}$$

Equations (8)–(10) imply the following relations

$$R_{11} = R_{22} = R_{33} \tag{11}$$

$$8\pi\rho = -\tfrac{1}{2}(3R_{22} + R_{44}) \tag{12}$$

$$8\pi p = \tfrac{1}{2}(R_{22} - R_{44}) \tag{13}$$

$$8\pi q = -R_{14}. \tag{14}$$

Subsequently it is desired that $R_{14} \neq 0$ so that spacetime (1) can sustain presence of heat flow. We choose the function $F(x)$ in the metric (1) as

$$F^2 = G^{2d}P^{2\lambda}, \tag{15}$$

where d and λ are constants. The equations (11) written out explicitly read

$$n\frac{\ddot{T}}{T}=\frac{1}{H^2}\left[\frac{P''}{P}-\frac{P'^2}{P^2}+\frac{G'P'}{GP}-\frac{P'H'}{PH}\right] \tag{16}$$

$$\frac{\ddot{T}}{T}(1-n-2m)=\frac{1}{H^2}\left[-\frac{G''}{G}-\frac{P''}{P}+\frac{G'^2}{G^2}-\frac{2F''}{F}-\frac{G'P'}{GP}+\frac{2F'G'}{FG}\right.$$
$$\left.+\frac{H'G'}{HG}+\frac{P'H'}{PH}+\frac{2F'^2}{F^2}+2\frac{F'H'}{FH}\right]. \tag{17}$$

The left hand sides of these equations are functions of time only while the right hand sides are functions of x only. Therefore both the sides of respective equations must be equal to separation constants (different for different equations). Thus we write

$$\frac{\ddot{T}}{T}=\varepsilon a^2,\ \varepsilon=0,\ \pm 1,\ a=\text{constant} \tag{18}$$

implying

$$T(t)=A\cosh(at)+B\sinh(at),\ \varepsilon=1$$
$$T(t)=At+B,\ \varepsilon=0 \tag{19}$$
$$T(t)=A\cos(at)+B\sin(at),\ \varepsilon=-1,$$

where A and B are constants of integration. On using (15) and (18), equations (16) and (17) take the forms

$$\alpha'+\alpha\beta-\alpha\frac{H'}{H}=\varepsilon n a^2 H^2 \tag{20}$$

and

$$(2d+1)\beta'-2d\beta^2-(2d+1)\beta\frac{H'}{H}+\alpha^2-4\lambda\alpha\beta=(2m-1-2n\lambda)\varepsilon a^2 H^2, \tag{21}$$

where

$$\alpha=\frac{P'}{P},\ \beta=\frac{G'}{G}. \tag{22}$$

The physical variables ρ, p, q in the light of equations (15)–(20) have explicit expressions

$$32\pi\rho T^{2m}F^2=\varepsilon a^2\left[2m+2-n^2+2\lambda n-\frac{(2d-3)}{(2d+1)}(1-2m+2n\lambda)\right]$$
$$+(4m+1-n^2)\left(\frac{\dot{T}^2}{T^2}-\varepsilon a^2\right)+\frac{(2d-3)}{(2d+1)}\cdot\frac{Z}{H^2}, \tag{23}$$

$$32\pi p T^{2m}F^2=\varepsilon a^2(4m-n^2-3)+(4m+1-n^2)\left(\frac{\dot{T}^2}{T^2}-\varepsilon a^2\right)+\frac{Z}{H^2}, \tag{24}$$

$$16\pi q T^{2m}F^2=-\frac{\dot{T}}{TH}\left[(1-2m-2d)\beta+(n-2\lambda)\alpha\right], \tag{25}$$

where

$$Z = (1 + 4d)\beta^2 - \alpha^2 + 4\lambda\alpha\beta. \tag{26}$$

In the above, it is assumed that $2d + 1 \neq 0$. From equations (23) and (24) it easily follows that

$$\rho = \frac{2d-3}{2d+1}p + \frac{1}{16\pi T^{2m}F^2}\left[\frac{2(4m+1-n^2)}{2d+1}\left(\frac{\dot{T}^2}{T^2} - \varepsilon a^2\right) + \frac{\varepsilon a^2(2d+2m-1+2\lambda n - n^2)}{2(2d+1)}\right]. \tag{27}$$

Accordingly the matter distribution will satisfy an equation of state $\rho = kp$ subject to one of the following conditions:

(i) $\dot{T}^2/T^2 = \varepsilon a^2, 2d + 2m - 1 + 2\lambda n - n^2 = 0$,
(ii) $4m + 1 = n^2, 2d + 2m - 1 + 2\lambda n - n^2 = 0$,
(iii) $\varepsilon = 0, n^2 = 4m + 1$,

where $k = (2d-3)/(2d+1) \neq 1$. However, the parameters can be chosen appropriately so that the matter distribution satisfies equation of state $\rho = p$ corresponding to stiff fluid.

The parameter q which implies the presence of heat flow with matter will vanish if any one of the following conditions is fulfilled:

$$\begin{aligned} &\text{(i) } 2d = 1 - 2m, n = 2\lambda \\ &\text{(ii) } \alpha = 0, \beta = 0 \\ &\text{(iii) } \beta = 0, n = 2\lambda \\ &\text{(iv) } \alpha = 0, 2d = 1 - 2m. \end{aligned} \tag{28}$$

Accordingly the set up developed above is used to obtain heat flow generalization of the general class of inhomogeneous perfect fluid solutions of Ruiz and Senovilla when $\alpha \neq 0$, $\beta \neq 0$, and $2d \neq 1 - 2m$ or $n \neq 2\lambda$.

We give below the expressions for the expansion scalar θ, the shear scalar σ and the acceleration vector f_i describing the kinematical behaviour of the velocity field U^α given by (9)

$$\begin{aligned} \theta &= \frac{(m+1)\dot{T}}{T^{m+1}F}, \\ \sigma^2 &= \frac{\{(1-2m)^2 + 3n^2\}\dot{T}^2}{6T^{2m+2}F^2}, \\ f_i &= (-d\beta - \lambda\alpha, 0, 0, 0). \end{aligned} \tag{29}$$

The ratio of σ to θ is a constant. This implies that there is no possibility that the models may get isotropized at some later time. With $2m = 1$ and $n = 0$, the set up can provide cylindrically symmetric models describing distribution of non-thermalized shear-free perfect fluid.

The heat flux vector q_i is governed by the phenomenological expression for heat conduction

$$q_i = \psi(\mathscr{T}_{,k} + \mathscr{T}f_k)(\delta_i^k - U^kU_i),$$

where ψ is the thermal conductivity and $\mathcal{T}$ is the temperature. If thermal conductivity ψ is a function of time alone, then the above equation can be formally integrated to give

$$\mathcal{T} = -\frac{F\dot{T}}{16\pi\psi T^{m+1}}\int F^{-2}\{(1-2m-2d)\alpha+(n-2\lambda)\beta\}\,\mathrm{d}x + l_0(t)F, \tag{30}$$

where $l_0(t)$ is an arbitrary function of time.

In the next section explicit solutions of the differential equations (20) and (21) in certain special cases are obtained.

3. Explicit solutions

We have not made any attempt to obtain the general solution of the system of equations (20) and (21). It is observed that these equations admit solutions in terms of elementary functions in certain special cases. We discuss briefly these solutions omitting the details of calculations.

Case 1. The simplest family of solutions is characterized by $\varepsilon = 0$ implying

$$T(t) = At + B. \tag{31}$$

In this case (20)–(22) can be integrated completely if $4\lambda^2 + 4d + 1 \geqslant 0$ and the general solution is obtained in the form

$$P = (N + C^{\gamma})^{l_1}, G = P^{l_2}C^{l_3}, H = MC'P^{l_4}, \tag{32}$$

where C is an arbitrary function of x, M and N are arbitrary constants and

$$l_2 = (4d+1)^{-1}(-2\lambda \pm \sqrt{4\lambda^2+4d+1}), l_3 = -(2d+1)/2d$$
$$l_4 = (1+2d)^{-1}[-4\lambda-(1+6d)l_2], l_1 = (1+2d)[2(1+4d)l_2+4\lambda]^{-1}$$
$$\gamma = (1+4d)/4d. \tag{33}$$

The physical variables are given by

$$16\pi\rho T^{2m}F^2 = \frac{A^2(4m+1-n^2)}{2(At+B)^2} - \frac{(2d+1)(4d+1)(2d-3)}{16d^2M^2P^{2l_4}C^2}[C^{\gamma}P^{-1/l_1}-2], \tag{34}$$

$$16\pi p T^{2m}F^2 = \frac{A^2(4m+1-n^2)}{2(At+B)^2} - \frac{(2d+1)^2(4d+1)}{16d^2M^2P^{2l_4}C^2}[C^{\gamma}P^{-1/l_1}-2], \tag{35}$$

$$8\pi q T^{2m}F^2 = -\frac{A(1+2d)}{2MP^{l_4}C(At+B)d}\left[\frac{\{n-2\lambda+l_2(1-2m-2d)\}(1+4d)}{2\{2(1+4d)l_2+4\lambda\}} - (1-2m-2d)\right]. \tag{36}$$

It is assumed that $d \neq 0, 1+2d \neq 0, 1+4d \neq 0$. These cases are dealt with separately. With $2d \neq 1-2m$ or $n \neq 2\lambda$, the above solution is a heat flux generalization of the Case 1, solution of Ruiz and Senovilla [2]. When $n = 3, m = 2, N = -1, C(x) = (1+x^2)^{3/5}$,

$d = -\lambda \neq -3/2$, the above solution represents heat flux counterpart of the radiation universe discussed by Davidson[18].

The solutions of equations (20)–(22) for the exceptional cases $d = 0$, $d = -1/2$, $d = -\frac{1}{4}$ and $4\lambda^2 + 4d + 1 = 0$ are given below:

Case $d = 0$: $P = (N + C)^{l_1}$, $G = P^{l_2} C^{-1}$, $H = MC' P^{l_4} C^{-1}$ where M, N are arbitrary constants and

$$l_1 = l_2/(1 + l_2^2), l_4 = -4\lambda - l_2, l_2^2 = \tfrac{1}{2}[-1 - 4\lambda + \sqrt{(1-4\lambda)^2 + 4}]$$

with $\lambda < 1/4$.

Case $d = -\frac{1}{2}$: $P = A_1 G^l$, $G = (2\lambda \pm \sqrt{4\lambda^2 - 1}) K_1 C + K_2$, $H = C'$,

$l(2\lambda \pm \sqrt{4\lambda^2 - 1}) = 1$, where A_1, K_1, K_2 are constants and $\lambda^2 \geqslant 1/4$.

Case $d = -\frac{1}{4}$: $P = (ax + b)^{1/8\lambda}$, $G = e^{-ax} P^{1/4\lambda}$, $H = e^{-ax} P^{(1-32\lambda^2)/4\lambda}$. Here a and b are arbitrary constants.

Case $4\lambda^2 + 4d + 1 = 0$: $P = \exp(ax^{8\lambda^2/(1+4\lambda^2)})$ $G = P^{1/2\lambda}(ax)^{(1-4\lambda^2)/(1+4\lambda^2)}$, $H = P^{1/2\lambda}$.

In the absence of heat flux the solutions corresponding to the cases $d = -1/4$, $d = 0$ and $4\lambda^2 + 4d + 1 = 0$ respectively reduce to the fluid solutions corresponding to the cases $m = 3/4$, $m = 1/2$ and $4m = n^2 + 3$ discussed by Ruiz and Senovilla. The case $d = -1/2$ is a new case.

Case 2. For $\varepsilon \neq 0$, there are several integrable cases. We discuss a solution obtained with $\varepsilon = 1$ in brief. The solution has

$$H = 1, P = e^{-l_1 \mu a x} f^{l_1}, G = e^{\mu a x} f,$$

$$f = A_1 e^{max} + A_2 e^{-max}, l_1 = \sqrt{-2d}, m^2 = \frac{n + l_1 \mu^2}{l_1}$$

$$\mu^2 = \frac{1}{4(1+2d)}\left[2(2m-1) - \frac{n}{\sqrt{-2d}}(3 + 10d)\right], \lambda^2 = -(1+6d)/32d. \quad (37)$$

Here A_1, A_2 are arbitrary constants and $-\frac{1}{6} < d < 0$. The physical variables corresponding to this solution are found to be

$$32\pi\rho T^{2m} F^2 = a^2\left[2m + 2 - n^2 + \frac{\sqrt{1+6d}}{2\sqrt{-2d}} n - \frac{(2d-3)}{(2d+1)}\left\{1 - 2m + 2n\frac{\sqrt{1+6d}}{2\sqrt{-2d}}\right\}\right] + (4m + 1 - n^2)\left(\frac{\dot{T}^2}{T^2} - a^2\right)$$
$$+ \frac{(2d-3)}{(2d+1)}\left[a^2\mu^2\{1 + 6d - \sqrt{1+6d}\} + \frac{f'^2}{f^2}\{1 + 6d + \sqrt{1+6d}\} + 2a\mu\frac{f'}{f}(1+2d)\right], \quad (38)$$

$$32\pi p T^{2m} F^2 = a^2(4m - n^2 - 3) + (4m + 1 - n^2)\left(\frac{\dot{T}^2}{T^2} - a^2\right)$$
$$+ \left[a^2\mu^2\{1 + 6d - \sqrt{1 + 6d}\} + \frac{f'^2}{f^2}\{1 + 6d + \sqrt{1 + 6d}\} \right.$$
$$\left. + 2a\mu\frac{f'}{f}(1 + 2d) \right], \tag{39}$$

$$16\pi q T^{2m} F^2 = -\frac{\dot{T}}{T}\left[\frac{f'}{f}\left\{1 - 2m - 2d + \sqrt{-2d}\left(n - \frac{\sqrt{1 + 6d}}{2\sqrt{-2d}}\right)\right\}\right.$$
$$\left. + \mu a\left\{1 - 2m - 2d - \sqrt{-2d}\left(n - \frac{\sqrt{1 + 6d}}{2\sqrt{-2d}}\right)\right\}\right]. \tag{40}$$

This solution is the heat flow generalization of the perfect fluid, case 2-solution obtained by Ruiz and Senovilla.

Case 3. This case is characterized by

$$\varepsilon = \pm 1,\ 1 + 4d + 4\lambda^2 > 0,\ d \neq -\tfrac{1}{2},\ d \neq +\tfrac{1}{4}$$
$$\frac{n(1 + 2d)(1 - 4d)}{2m + 2n\lambda - 1 + 8n\lambda d} = 2\lambda \pm \sqrt{1 + 4d + 4\lambda^2}. \tag{41}$$

An explicit solution obtained in this case is

$$H = 1,\ P = f^{l_1},\ G = f f'^{l_2} \tag{42}$$

where $l_1 = 2\lambda \pm \sqrt{1 + 4d + 4\lambda^2}$, $l_2 = (1 + 2d)/2d$ and $f''/f = -(2\varepsilon n a^2 d/(2\lambda \pm \sqrt{1 + 4d + 4\lambda^2}))$. The physical parameters ρ, p and q are given by

$$32\pi \rho T^{2m} F^2 = \varepsilon a^2\left[2m + 2 - n^2 + 2\lambda n - \frac{2d - 3}{2d + 1}\{1 - 2m + 2n\lambda\right.$$
$$\left. + 4ndl_2(l_1 - 4\lambda)\}\right] + (4m + 1 - n^2)\left(\frac{\dot{T}^2}{T^2} - \varepsilon a^2\right)$$
$$+ \frac{2d - 3}{2d + 1}\frac{l_2^2}{l_1^2}\cdot 4n^2 a^4 d^2\left(\frac{f}{f'}\right)^2, \tag{43}$$

$$32\pi p T^{2m} F^2 = \varepsilon a^2[4m - n^2 - 3 - 4ndl_2)(l_1 - 4\lambda)]$$
$$+ (4m + 1 - n^2)\left(\frac{\dot{T}^2}{T^2} - \varepsilon a^2\right) + 4n^2 a^4 d^2 \frac{l_2^2}{l_1^2}\left(\frac{f}{f'}\right)^2, \tag{44}$$

$$16\pi q T^{2m} F^2 = -\frac{\dot{T}}{T}\left(\frac{f}{f'}\right)\left[(1 - 2m - 2d + l_1 n - 2\lambda l_1)\frac{f'^2}{f^2}\right.$$
$$\left. - \frac{2l_2}{l_1}(1 - 2m - 2d)\varepsilon n a^2 d\right]. \tag{45}$$

This solution is the heat flow generalization of the perfect fluid case-3-solution given by Ruiz and Senovilla. The function f will be in the form of trigonometric or hyperbolic function depending on the value of ε.

Case 4. Here we will obtain the heat flow generalization of the case-4 fluid solution of Ruiz and Senovilla characterized by the relation $2m = 1 + n$. The metric variables of this solution are given by

$$H = \frac{MC'}{P},\ G = PC^{(-1+2d)/2d},\ P^2 = -2\varepsilon nda^2M^2C^2 + NC^{(4d+1)/2d} - Q > 0 \tag{46}$$

together with

$$2m = n + 1,\ d + \lambda = 0. \tag{47}$$

The density, pressure and heat flow parameter are given as

$$32\pi\rho T^{2m}F^2 = \varepsilon a^2\left[4(1-m^2) - \frac{(2m-1)}{2d}(2d-3)(2d+1)\right] + 4m(2-m)\left(\frac{\dot{T}^2}{T^2} - \varepsilon a^2\right) - \frac{(2d+1)(4d+1)(2d-3)Q}{4d^2}, \tag{48}$$

$$32\pi pT^{2m}F^2 = -\varepsilon a^2\left[4(1-m^2) + \frac{n}{2d}(2d+1)^2\right] + 4m(2-m)\left(\frac{\dot{T}^2}{T^2} - \varepsilon a^2\right) - \frac{(2d+1)^2(4d+1)Q}{4d^2}, \tag{49}$$

$$32\pi qT^{2m}F^2 = (2d+1)(1-2m-2d)P\dot{T}/MTdC. \tag{50}$$

The metric of the spacetime of this class of solutions has well defined cylindrical symmetry and it contains a large subclass of singularity free cosmological models with heat flow [17].

Case 5. In this case we shall obtain solutions of equations (20)–(22) corresponding to $\varepsilon = -1$. Two particular types of solutions are obtained in this case, characterized by relations $\lambda + d = 0$ and $n + 1 = 2m$.

The first type of solutions has metric variables

$$H = 1,\ P = f^{-4d}f',\ G = ff',\ \frac{f''}{f} = \frac{na^2}{8d} < 0. \tag{51}$$

The physical variables have the explicit expressions

$$32\pi\rho T^{2m}F^2 = -4a^2(1-m^2) + 4m(2-m)\left(\frac{\dot{T}^2}{T^2} + a^2\right) + 4(1+2d)(2d-3)\frac{na^2}{8d}, \tag{52}$$

$$32\pi p T^{2m} F^2 = 4a^2(1-m^2) + 4m(2-m)\left(\frac{\dot{T}^2}{T^2} + a^2\right) + 4(1+2d)^2 \frac{na^2}{8d}, \tag{53}$$

$$8\pi q T^{2m} F^2 = -(1+2d)(1-2m-2d)\frac{\dot{T}}{T}\frac{f'}{f}. \tag{54}$$

The other type of solutions has

$$H = 1,\ G = f^{-(1+2d)/2d} f',\ P = f',\ \frac{f''}{f} = 2na^2 d < 0. \tag{55}$$

The physical variables have the explicit expressions

$$32\pi \rho T^{2m} F^2 = -4a^2(1-m^2) + 4m(2-m)\left(\frac{\dot{T}^2}{T^2} + a^2\right) + (2d-3)(2d+1)\left[\frac{1+4d}{4d^2}\frac{f'^2}{f^2} - 2(2m-1)a^2\right], \tag{56}$$

$$32\pi p T^{2m} F^2 = 4a^2(1-m^2) + 4m(2-m)\left(\frac{\dot{T}^2}{T^2} + a^2\right) + (1+2d)^2\left[\frac{1+4d}{4d^2}\frac{f'^2}{f^2} - 2(2m-1)a^2\right], \tag{57}$$

$$8\pi q T^{2m} F^2 = \frac{(1+2d)}{4d}(1-2m-2d)\frac{\dot{T}}{T}\frac{f'}{f}. \tag{58}$$

The former type of solution includes a heat flow generalization of the cylindrically symmetric stiff fluid solution of Davidson [19]. When the heat flux is switched off by stipulating $2d = 1 - 2m$, the solutions reduce to inhomogeneous cylindrically symmetric perfect fluid solutions other than those obtained by Ruiz and Senovilla.

In the next section we state in brief the physical and geometrical features of the solutions reported above and deduce the singularity-free subfamily of solutions with heat flow.

4. Discussion

The families of solutions obtained in § 3 possess a very rich singularity structure similar to the class of solutions of Ruiz and Senovilla. It is observed that the presence of heat flux does not give rise to any qualitative change in the singularity structure of the Ruiz–Senovilla class of fluid solutions. That is singularity character of models remain undisturbed whether singular or non-singular. There are solutions with big bang singularity only, solutions with big bang and time-like singularity, solutions with time-like singularity only, solutions with both time-like and space-like singularities and singularity free solutions. Inclusion of heat flux in the non-singular models was considered [17] and shown that it could be achieved for $p = k\rho$ as well as without it.

However a number of solutions admit a linear equation of state $p = k\rho$ as in the case of Ruiz–Senovilla class of solutions. In each case of solutions the expression for temperature distribution can be obtained by using the respective explicit expressions for metric variables P, G and T in equation (30).

The subfamily of solutions (46), (47) of case-4 defined by

$$\varepsilon = 1, n \geqslant 3, T(t) = \cosh(at), C(x) = \cosh(kax), \tag{59}$$

where k is a real constant, provides singularity free models with heat flux. The line element of the spacetime for these models can be expressed in the form

$$\begin{aligned} ds^2 = {} & \cosh^{2m}(at)[\cosh(kar)]^{-(1+2d)}\left[dt^2 - \frac{M^2k^2a^2}{P^2}\sinh^2(kar)dr^2\right] \\ & - [\cosh(at)]^{2-2m}[\cosh(kar)]^{-(1+2d)/2d}dz^2 \\ & - [\cosh(at)]^{2m}[\cosh(kar)]^{-(1+2d)/2d}\cdot P^2 d\phi^2, \end{aligned} \tag{60}$$

where

$$P^2 = 2(1-2m)da^2M^2\cosh^2(kar) + N[\cosh(kar)]^{(4d+1)/2d} - Q. \tag{61}$$

Here the co-ordinates x and y are replaced by r and ϕ. The range of co-ordinates is taken to be $-\infty < T, z < \infty, 0 \leqslant r < \infty, 0 \leqslant \phi \leqslant 2\pi$. The density, pressure and heat flow parameter for this class of solutions have the following expressions

$$\begin{aligned} & 32\pi\rho[\cosh(at)]^{2m}[\cosh(kar)]^{-(1+2d)} \\ & \qquad = a^2\left[4(1-m^2) - \frac{(2m-1)}{2d}(2d-3)(2d+1)\right] \\ & \qquad + 4m(m-2)a^2\operatorname{sech}^2(at) \\ & \qquad - \frac{(2d+1)(4d+1)(2d-3)Q}{4d^2}, \end{aligned} \tag{62}$$

$$\begin{aligned} 32\pi p[\cosh(at)]^{2m}[\cosh(kar)]^{-(1+2d)} = {} & -a^2\left[4(1-m^2) + \frac{n}{2d}(2d+1)^2\right] \\ & + 4m(m-2)\operatorname{sech}^2(at) \\ & - \frac{(2d+1)^2(4d+1)}{4d^2}Q, \end{aligned} \tag{63}$$

$$\begin{aligned} & 32\pi q[\cosh(at)]^{2m}[\cosh(kar)]^{-(1+2d)} \\ & \qquad = \frac{(2d+1)(1-2m-2d)}{Md}\cdot a\tanh(at)\cdot P\operatorname{sech}(kar). \end{aligned} \tag{64}$$

This family represents a general solution which includes the cylindrically symmetric inhomogeneous singularity free fluid solutions with heat flux obtained earlier [17].

Acknowledgement

LKP and RT thank IUCAA for hospitality and the facilities provided for the work.

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