PRAMANA
—journal of
physics

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Vol. 49, No. 1,
July 1997
pp. 147–154

Rotation curves of galaxies: Missing mass or missing physics

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Abstract. The rotation curves of galaxies are modelled using very special properties of an hydrodynamically turbulent fluid possessing helicity fluctuations. The development of correlations among these fluctuations leads to the formation of organized structures characterized by a new flat branch of the spatial energy spectrum in addition to the well known Kolmogorov spectrum. It is proposed that the flat nature of the rotation curves of galaxies may be a result of the energy cascading processes occuring in turbulent galactic atmospheres. Thus, in this model, there is no need of invoking dark matter to account for the flat rotation curves of galaxies.

Keywords. Rotation curves; galaxies; turbulence; inverse cascade.

PACS Nos **95.30; 98.30; 98.52**

1. Introduction

We observe coherent structures and correlated motions delineating well defined patterns on all spatial and temporal scales. Some of the examples are: a convection cell, a tornado, a cyclone, convective flows on stellar surfaces, the Red spot on Jupiter, spiral patterns in galaxies, clusters of galaxies and perhaps - ourselves. The belief is that at the roots of this diversity lie the unique and transcendental properties of non-equilibrium systems. Such systems exhibit organization of matter and motion through the formation of dissipative structures. A stable system near equilibrium, when disturbed bounces back to the initial state. But a system driven sufficiently far from equilibrium may become unstable, reach a bifurcation point after which it attains a completely new identity, a dissipative structure, named so because it needs more energy to sustain itself than the initial configuration. Associated with these bifurcations is the breaking of symmetry, i.e. one starts with symmetric equations and ends up with asymmetric solutions, one of which the system chooses to obey. Sea shells often show a preferential chirality! That means, we need representations of dynamics, which are not invariant with respect to time inversion. Such representations have been found for highly unstable systems [1]. Thus, structure formation takes place in an intrinsically random, irreversible, unsymmetric non-equilibrium systems. The essential conditions for the formation of dissipative structures, thus, are: a macroscopic system far from equilibrium, with correlated fluctuations that can maintain themselves by deriving from their environment. Far from equilibrium conditions exist everywhere in the universe, the big bang being the ultimate example. Many structures like clusters of galaxies are suggested to form and exist in non-equilibrium circumstances [2]. Attempts to fathom them in equilibrium have lead us to darkness; to an unending search and an unquenchable need for dark matter! Here, we show that the flat rotation curves of galaxies can be successfully modeled if the recently discovered properties of a turbulent medium are included in addition to the standard gravitational effects [3]. Fluid turbulence is a complex subject. Even a genius like Heisenberg seems to have said that he hoped, before he died, someone would explain quantum mechanics to him, but after he died, he hoped, God, would explain turbulence to him [4]. Simply put, turbulence is a random state of fluid motions on many different spatial and temporal scales exchanging energy among themselves. The spatial and temporal scales are constrained by boundaries, buoyancy and dissipation. The problem of turbulence is addressed in two ways: (i) the Kolmogorov approach in which the statistically stationary states are studied using dimensional arguments and (ii) the Navier–Stokes way, in which, one hopes that the solutions of the Navier–Stokes equations would comply with the predictions of the Kolmogorov approach [5, 6]. Here, we describe the Kolmogorov approach.

2. The Kolmogorov approach

It is well known that the energy cascades from large spatial scales to small spatial scales in a homogeneous and isotropic three dimensional fluid turbulence and the energy spectrum is given as $E(K)\alpha K^{-5/3}$. But, what if the assumptions of homogeneity and isotropy are dropped? In two-dimensional turbulence, the energy

$$E = \int \frac{1}{2}\rho V^2 d^3r \tag{1}$$

and the enstrophy

$$U = \int (\nabla \times \mathbf{V})^2 d^3r \tag{2}$$

are the two quadratic inviscid invariants for vanishing normal component of velocity $\mathbf{V}$ i.e. for rigid or periodic boundaries. The Kolmogorovic arguments give energy spectrum in the inertial range, corresponding to the energy invariant as

$$E(K) \propto K^{-5/3} \tag{3}$$

and corresponding to the enstrophy invariant as

$$E(K) \propto K^{-3} \tag{4}$$

It has been verified by several means that the energy cascades from small scales to large scales according to the spectrum given by equation (4) and from large scales to small scales according to the spectrum given by equation (3). Thus a 2-D system admits an inverse cascade of energy. This is also true for a quasi 2-D system i.e. the one with the vertical component of velocity and spatial scale much smaller than their horizontal components [7, 8]. But the question is: is inverse cascade of energy possible in a 3-D system? Since, the real world is 3-dimensional! The recent developments in 3-D turbulence point to the existence of inverse cascade under well defined conditions. Using the standard fluid equations

$$\frac{d\mathbf{V}}{dt} = -\frac{1}{\rho}\nabla p + \mathbf{F}, \tag{5}$$
$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{V}) = 0,$$
$$p = p(\rho), \quad \mathbf{F} = -\nabla\varphi,$$

it can be shown that the system supports another invariant called helicity H defined as

$$H(t) = \int \mathbf{V}\cdot(\nabla\times\mathbf{V})\mathrm{d}^3r \tag{6}$$

under vanishing normal component of the vorticity $\vec{\omega} = \nabla\times\mathbf{V}$. Turbulence with $H \neq 0$ lacks reflectional symmetry. For a reflectionally symmetric system, $H = 0$, but it may happen that the higher moments of helicity distribution are non-zero and affect the statistics of the flow [9]. The moments of helicity distribution are defined as

$$H_n = \lim_{V_L\to\infty}\frac{1}{V_L}\Sigma_i[h^{(i)}]^n,$$

where

$$h^i = \int_{V_i}(\mathbf{V}\cdot\nabla\times\mathbf{V})\mathrm{d}^3r_i \tag{7}$$

and V_i is a small volume element of the total volume V_L. For a random distribution, $H_1 = 0$ but all even moments are finite and in particular $H_2 \neq 0$ i.e. the fluctuations about the mean have constant variance. Thus, a new invariant I is defined as

$$I = \int\langle(\mathbf{V}_1\cdot\nabla\times\mathbf{V}_1)(\mathbf{V}_2\cdot\nabla\times\mathbf{V}_2)\rangle\mathrm{d}^3r \simeq A\int E^2(K)\mathrm{d}K \tag{8}$$

which describes helicity–helicity correlations and can be expressed in terms of the energy spectrum $E(K)$ for a quasi-normal distribution of helicities. Here $E = \int E(K)\mathrm{d}K$ is the total energy density.

In the inertial range for the energy invariant we have

$$(KV_K)V_K^2 = \epsilon = V_0^2\tau,$$

where K = wavenumber, V_0 = the initial rms velocity on small scales, τ = the duration for which this energy is available, V_K = velocity in Fourier space and ϵ = average energy transfer rate per unit mass (ergs $\mathrm{g}^{-1}\,\mathrm{s}^{-1}$). This, combined with $KE(K) = V_K^2$ yields the well-known Kolmogorov spectrum

$$E(K) = \epsilon^{2/3}K^{-5/3}. \tag{9}$$

It would be appropriate to comment on ϵ here. Kolmogorov (1941) conjectured that in a quasi-steady state there should be a stationary flow of energy in the K space from the source to the sink. Thus the energy transfer rate per unit mass should be a constant and be equal to the dissipation rate at the sink. Although numerous experiments have confirmed that ϵ is a strongly fluctuating quantity, surprisingly there is no experimental evidence indicating a deviation from the Kolmogorov spectrum [10].

The value of ϵ for the galaxy has been estimated to be of the order of 8×10^{-3} ergs g^{-1} s^{-1} by considering the various sources (such as supernovae, stellar winds, etc) which contribute to the turbulence energetics. In the same vein τ is calculated to be 3×10^7 yr [11].

From equation (5), we find total energy $E = \int E(K)\mathrm{d}K$ or

$$E(l) = \epsilon^{2/3} l^{2/3} \quad (\text{for } K \simeq 1/l).$$

The corresponding velocity field may be described as

$$V(l) = (l_z \epsilon)^{1/3} (l/l_z)^{1/3} \tag{10}$$

for some normalizing length, l_z. Similarly, in the inertial range for the I-invariant, we have

$$(KV_K)[KE^2(K)] = \epsilon' = I_0/\tau, \tag{11}$$

where ϵ' = the average mean square helicity density exchange rate between the scales. Combining this with

$$KE(K) = V_K^2 \tag{12}$$

gives

$$E(K) = (I_0/\tau)^{2/5} K^{-1}.$$

Or in real space,

$$E(l) = (I_0/\tau)^{2/5} \ln(l/l_z) \quad \text{for } l > l_z. \tag{13}$$

Here, the normalizing length l_z can be used to make the transition from one inertial law (eq. (10)) to the other (eq. (13)). The velocity field in this range is described as

$$V(l) = (\epsilon^2 l_z \tau)^{1/5} [\surd \ln(l/l_z)], \tag{14}$$

where

$$I_0 = V_0^4 l_z$$

which follows from eqs (11) and (12).

3. Modeling of rotation curves

The issue of the flat rotation curves of galaxies and the need for dark matter is described very precisely in figure 1 [12]. The flat nature of the orbital motion in galaxies is accounted through the relationship $(mV^2)/2 = GMm/R$ by assuming that $M \propto R$ and therefore velocity V = constant. Since the mass $M \propto R$ has no luminosity associated with it, it is known as dark matter. We present an alternative explanation of a flat rotation curve, by resorting to some properties of turbulence described in the previous section.

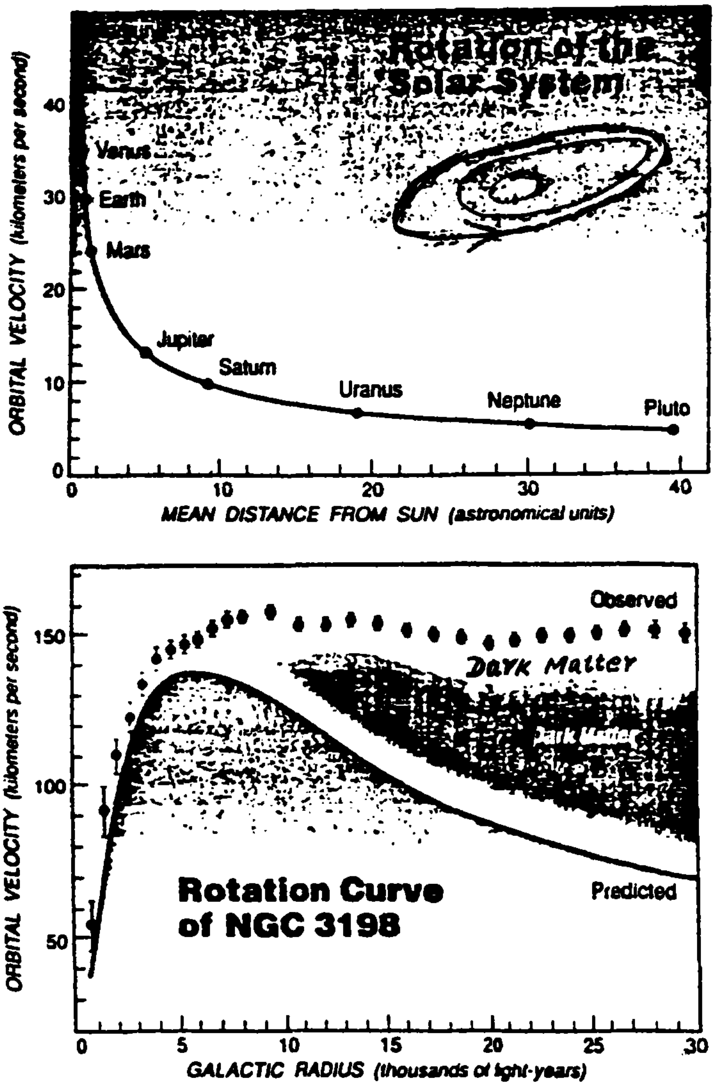

Figure 1. The case for dark matter in spiral galaxies. Top: The orbital velocities of the planets (dots) decrease with distance from the Sun exactly as predicted by Newtonian gravitation (line), assuming a system dominated by one solar mass at its center. Bottom: The cosmos is not as well behaved on galactic scales. Here a graph of orbital velocity versus radius has been computed for NGC 3198, a spiral galaxy in Ursa Major, assuming that the distribution of light serves as a good indicator of the distribution of mass. The failure of the observed velocities (dots) to match the predicted ones is striking and points to an unseen component of dark matter in the galaxy. Courtesy the author.

The complete energy spectrum in a helically turbulent medium derived in Krishan [8] and Krishan and Sivaram [13] is reproduced here in figure 2. We have modeled the rotation curves of 21 galaxies observed by Amram *et al* [14], using the Kolmogorov

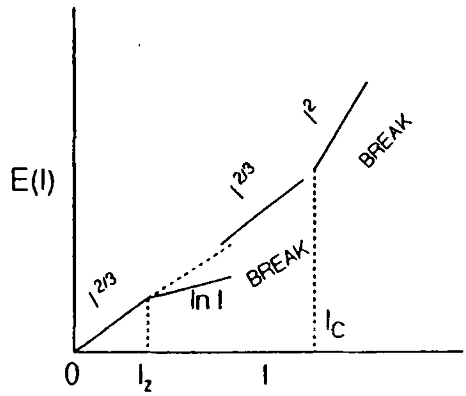

Figure 2. Turbulent energy spectrum. l_z, normalizing length; l_c, break due to Coriolis force [15].

branch $[V(l) \propto l^{1/3}]$ and the flat branch $[V(l) \propto \sqrt{\ln l}]$. We propose a law of velocities which is of the type

$$V(l) = Al + Bl^{1/3} \tag{15}$$

in the inner, i.e., $l \leq l_z$, and

$$V(l) = Cl^{-1/2} + D\sqrt{\ln(l/l_z)} \tag{16}$$

in the outer regions, i.e., $l \geq l_z$ of a galaxy, where A, B, C and D are the coefficients to be determined from the fits, with the observed velocity fields.

The first terms on the right-hand side of eqs (15) and (16) correspond to rigid rotation and gravity, respectively; therefore

$$A = \omega,$$

the angular velocity of a galaxy, and

$$C = \sqrt{GM},$$

where G is the universal gravitational constant, refers to the mass of a galaxy. The second terms on the right-hand side of eqs (15) and (16) are due to the turbulence cascading so that

$$B = \epsilon^{1/3}$$

and

$$D = (\epsilon^2 l_z \tau)^{1/5}.$$

By a judicious choice of l_z we can estimate $V_0, \tau, \epsilon, \omega$, and mass M of a galaxy. Some of the modeled rotation curves are shown in figure 3.

Our model gives typical values of the various quantities as

$$V_0 \approx 100\,\mathrm{km\,s^{-1}},$$
$$\tau \approx 10^7\,\mathrm{yr},$$

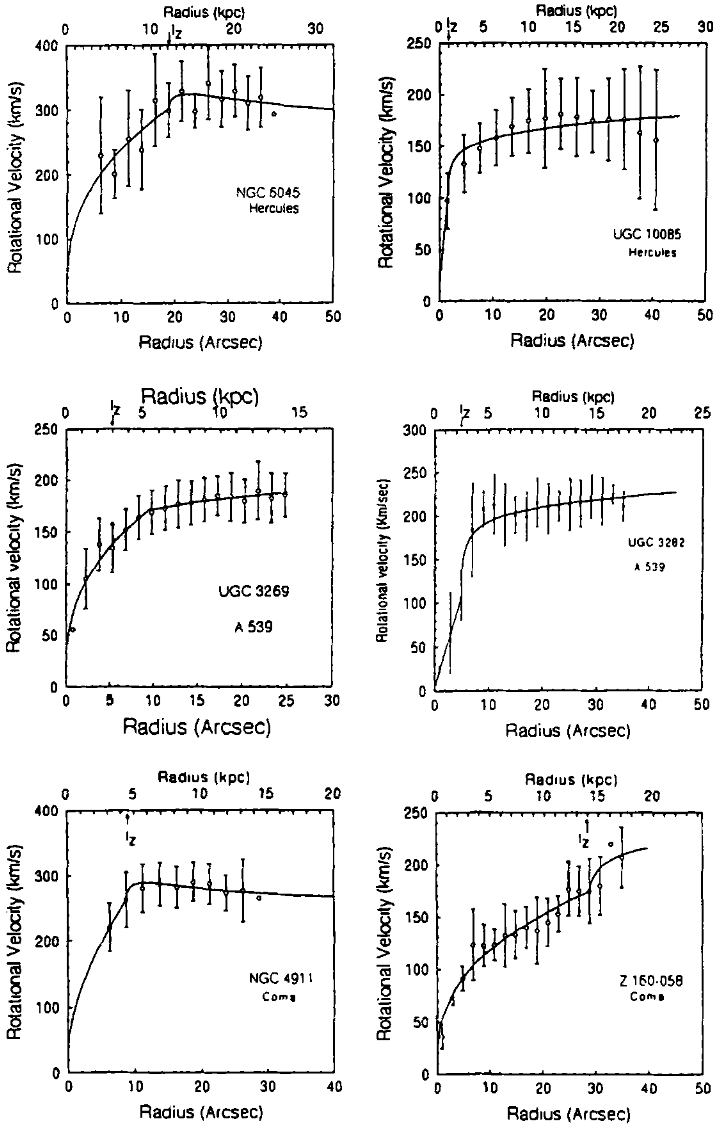

Figure 3. Rotation curves of the galaxies.

$$\epsilon \approx 10^{-2}\,\mathrm{ergs\,g^{-1}\,s^{-1}},$$

$$\omega \approx 10^{-16}\,\mathrm{s^{-1}},$$

$$\mathrm{Mass} \approx 10^{10}\,\mathrm{M_{\odot}}.$$

One must note that we did not have to choose any abnormal values of l_z for obtaining the best fits and it lies in the range 2–10 kpc. This tells us that on scales smaller than l_z, the turbulence is isotropic and on the scales equal to and large than l_z the turbulence becomes more and more anisotropic facilitating the inverse cascade of energy.

4. Conclusions

The velocity–radius relation has been derived using Kolmogorov arguments. We find that the matter at a large radius exhibits a balance of hydrodynamic forces, i.e., dynamical pressure, and Reynolds stress produced by the forced small-scale flow without the necessity of invoking a gravitational force, generated out of a mass distribution of the type $M \propto R$. In other words, our system is hydrodynamically bound.

We also find that ϵ and τ values for each of the galaxies obtained are almost of the same order as that quoted for our galaxy [11]. Therefore, it appears possible to model the observed rotation curves of galaxies by suitably combining the effects of rigid rotation, gravity, and turbulence. The validity of the 'turbulence model' can be further substantiated by confronting it with the observations of the velocity fields on the larger scales-like clusters and superclusters. It is intriguing that the energy spectrum (figure 1) at the largest scales i.e. $E(L) \propto L^2$ or $V(L) \propto L$ resembles the Hubble flow. Is the Hubble flow, a result of an inverse cascade of energy in a helically turbulent universe?

References

[1] I Prigogine, in *The World of Physics* edited by J H Weaver, vol. 2, p. 689 (1987)
[2] G L Verschuur, *Comment Ap.* **15**, 189 (1991)
[3] R D Prabhu and V Krishan, *Ap. J.* **428**, 483 (1994)
[4] F H Shu, in *The Physics of Astrophysics*, 2 (1991)
[5] V Krishan, *Indian J. Phys.* **B66**, 569 (1992)
[6] V Krishan, *MNRAS* **264**, 257 (1993)
[7] A Hasegawa, *Adv. Phys.* **34**, 1 (1985)
[8] V Krishan, in *Solar Interior and Atmosphere* edited by A N Cox, W C Livingston and M S Matthews (The Univ. of Arizona Press, Tuscon, 1991a) p. 1029
[9] H K Moffat and A Tsinober, *Ann. Rev. Fluid Mech.* **24**, 281 (1992)
[10] E Levich, *Phys. Rep.* **151**, 129 (1987)
E Levich, and E Tzvetkov, *Phys. Rep.* **128**, 1 (1985)
[11] A A Ruzmaikin, A M Shukurov and D D Sokoloff, *Magnetic fields of galaxies* (Dordrecht: Kluwer, 1988), p. 150
[12] The author regrets her inability to find the reference of the published paper from which figure 2 has been reproduced here
[13] V Krishan and C Sivaram, *MNRAS* **250**, 157 (1991)
[14] P Amram, E Le Coarer, M Marcelin, C Balkowski, W T Sullivan III and V Cayatte, *Astronomy & Astrophys. Suppl. Ser.* **94**, 175 (1992)
[15] V Krishan, *MNRAS* **250**, 50 (1991b)
[16] A N Kolmogorov, *Dokl. Akad. Nauk. SSSR* **30**, 299 (1941)
[17] E Levich and E Tzvetkov, *Phys. Rep.* **128**, 1 (1985)