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CP symmetry and its violation in fundamental interactions

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Abstract. The concept of discrete symmetries in classical and quantum physics is reviewed. An account is given of *CP* violation observed in the *K*-meson system and of other experiments where *CP* symmetry has been tested. The present theoretical ideas on *CP* violation within the standard model, and problems needing extension of the model are described. Finally, ideas and experimental approaches to *CP* violation beyond the standard model are reviewed in brief.

Keywords. Discrete symmetries; CP violation; standard model.

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1. The discrete symmetries *C*, *P* and *T*

Continuous symmetries, dependent on parameters which can be varied continuously, have their own significance in physics because they lead to conservation laws. Discrete symmetries, on the other hand, depend on a discrete set of parameters, and do not lead to conservation laws. They are nevertheless important in the study of the laws of physics. The simplest of discrete symmetries one can imagine are the space and time inversion symmetries, generally called parity (P) and time reversal (T). In the context of the special theory of relativity, these symmetries get linked with one more discrete symmetry, viz., particle-antiparticle interchange, usually called charge conjugation (C). C has basically nothing to do with space-time and is a kind of 'internal' symmetry. C, P and T thus play an important role in particle interactions at high energies, which are generally described using relativistic quantum field theory.

Of the symmetries, C, P and T, not all have the same significance for all fundamental interactions. Even though at one time C, P and T were all believed to be good symmetries, it has been known now for a long time that C and P are violated by weak interactions. Not only that, even the lower symmetry CP corresponding to a combination of C and P, has been found to be definitely violated in the K-meson system, but nowhere else. According to current ideas of cosmology, the universe started off with equal amounts of matter and anti-matter, and to understand the predominance of matter over anti-matter at the present time, a violation of CP at a fundamental level in the early universe is essential. However, C, P and T are all good symmetries of strong, electromagnetic and gravitational interactions as we know them today.

The combination of all three, viz., CPT, has a special significance. This combined symmetry is guaranteed to hold for all local interactions which also possess relativistic invariance by the so-called CPT theorem. This has a number of consequences for properties of antiparticles. For example, it implies the equality of masses and lifetimes of particles and antiparticles, and requires their charges to be equal and opposite in sign. In addition, the CPT theorem implies that CP invariance and T invariance are equivalent, as also CP violation and T violation. Because of this interconnection, very often no distinction is made between CP and T. While CP makes sense only at high energies where antiparticles can be produced, it can be tested even at lower energies through tests of T invariance.

2. Discrete symmetries in classical and quantum mechanics

In classical mechanics, the definitions of physical quantities like energy, momentum, angular momentum, etc., decide their transformation properties under the space-time symmetries P and T. Thus momenta change sign under P and T, energy is invariant under P and T, angular momentum changes sign under T, but not under P. Depending on the nature of the forces, one can then determine whether or not the system is invariant under P and T.

For example, if the dynamics is determined by laws of electrodynamics, both the particle equations of motion as well as Maxwell's field equations are invariant under P and T, provided the fields are assigned appropriate transformation properties.

Normally, C does not enter the classical domain. However, one could define it as an operation which changes the charge of a particle, leaving other attributes the same. In that case classical electrodynamics is invariant under C, provided the fields change sign under C.

In quantum mechanics, symmetries are implemented by means of unitary (or anti-unitary) operators which commute with the Hamiltonian H, according to Wigner's theorem. P is represented by a unitary operator which satisfies $U_P^2 = 1$, and therefore has eigenvalues ± 1. Thus in a theory with parity invariance the eigenstates of H can be chosen to be a state with a definite parity, called even or odd according as the eigenvalue of P is $+1$ or -1.

The operator for T has to be anti-unitary rather than unitary as can be seen from the requirement that the time-evolution equation be form-invariant under T. The evolution equation

$$i \frac{\partial}{\partial t} |\psi(t)\rangle = H |\psi(t)\rangle \tag{1}$$

when acted on by the time-reversal operator U_T gives

$$i \frac{\partial}{\partial(-t)} |\psi(-t)\rangle = H |\psi(-t)\rangle, \tag{2}$$

where

$$U_T |\psi(t)\rangle = |\psi(-t)\rangle, \tag{3}$$

provided U_T satisfies

$$[U_T, H] = 0 \tag{4}$$

and

$$U_T i = -i U_T. \tag{5}$$

Thus U_T has to be antilinear. This fact has many important consequences, as we shall see later.

While C has no place in non-relativistic quantum mechanics, it arises naturally in relativistic quantum mechanics, particularly, in the Dirac hole theory for antiparticles. In Dirac's interpretation, antiparticles are hole states in a sea of completely filled negative-energy states. This description does afford an operation of charge conjugation:

$$\psi(\mathbf{x},t) \to U_C\psi(\mathbf{x},t) = C\overline{\psi}^T(\mathbf{x},t), \quad (\overline{\psi} = \psi^\dagger\gamma^0), \tag{6}$$

where C is a matrix, and $\psi(\mathbf{x},t)$ is a column vector solving the Dirac equation. However, the real solution of the problem of negative-energy states involves the use of second quantization, and lies in the realm of quantum field theory.

In the quantum field-theoretic description of the discrete symmetries C, P and T, all fields undergo a unitary transformation.

$$\begin{aligned} \psi(\mathbf{x},t) &\to U_C^{-1}\psi(\mathbf{x},t)U_C = \eta_C C\overline{\psi}^T(\mathbf{x},t) \\ \psi(\mathbf{x},t) &\to U_P^{-1}\psi(\mathbf{x},t)U_P = \eta_P P\psi(-\mathbf{x},t) \\ \psi(\mathbf{x},t) &\to U_T^{-1}\psi(\mathbf{x},t)U_C = \eta_T T\psi(\mathbf{x},-t). \end{aligned} \tag{7}$$

Here C, P, T on the right-hand side are matrices, and η_C, η_P, η_T are phase factors, reflecting the inherent phase-arbitrariness in the definition of states in quantum mechanics. These phases may be chosen arbitrarily. The above illustration is for a spin-1/2 Dirac field, but other fields would have similar transformation properties. Invariance of a theory under the discrete symmetries means invariance of the action

$$S = \int \mathrm{d}^4x\mathcal{L}(\mathbf{x},t), \tag{8}$$

where $\mathcal{L}(\mathbf{x},t)$ is the Lagrangian density.

The phase factors appearing in the transformations can be determined from experiment, at least relative to one another, provided one assumes the corresponding invariance to hold. These phase factors are given the name 'intrinsic' C, P or T, as the case may be.

In simple theories, invariance under *CP* is simple to verify. However, in complicated interacting theories with several terms, the phases η_P, η_C have to be chosen carefully to ensure invariance of the action. Only in the case when no consistent choice of phases exists for which the action is invariant can one say that theory violates *CP*.

As for the combined transformation *CPT*, it is found that there is always a consistent choice of phases for *CPT* invariance so long as the theory is local and Lorentz invariant. This is guaranteed by the *CPT* theorem due to Pauli, Lüders and Schwinger [1].

3. CP violation in nature

To look at a bit of history, it was believed for a long time that nature respects all the discrete invariances C, P and T. So much so, that paradoxical situations arose because of this prejudice. An example is the famous τ-θ puzzle, which came about in the early 1950s, when the strange mesons τ and θ were found to have identical properties except for the fact that they decayed into two different final states:

$$\tau \longrightarrow \pi^+\pi^-\pi^0$$
$$\theta \longrightarrow \pi^+\pi^-. \tag{9}$$

This meant that the intrinsic parity assignments of τ and θ had to be opposite, for the pions were known to have parity -1, and the decays were expected to proceed with zero orbital angular momentum. The other explanation, which was considered heretical was that τ and θ were the same particle K, but parity was violated in these weak decays, and therefore no definite intrinsic parity could be assigned to K. This suggestion of parity violation in weak interaction proposed by Lee and Yang [2] led to the experiment of Wu *et al* [3] which confirmed parity violation in the radioactive β decay of ^{60}Co.

The experiment consisted in looking at the distribution of decay electrons from a polarized ^{60}Co-source. A forward-backward asymmetry relative to the spin direction of ^{60}Co nuclei was an indication of parity violation, since the observable $\mathbf{s}_N \cdot \mathbf{p}_e$ changes sign under $P(\mathbf{s}_N \rightarrow \mathbf{s}_N,\ \mathbf{p}_e \rightarrow -\mathbf{p}_e)$, and should have zero average value if P is conserved.

Further evidence for P violation came from the experiment of Culligan *et al* [4] who showed the $e^+(e^-)$ had helicity $+\frac{1}{2}(-\frac{1}{2})$ in the decays

$$\mu^{\pm} \longrightarrow e^{\pm} + \nu + \bar{\nu}. \tag{10}$$

Since the initial state was unpolarized, this was evidence for P violation, since helicity changes sign under parity.

Eventually, the $V - A$ theory of weak interactions was formulated by Marshak and Sudarshan [5], and by Feynman and Gell-Mann [6], which incorporated the maximal parity violation observed experimentally. This made use of the current×current form of the interaction:

$$\begin{aligned} \mathcal{L}_W &= \frac{G_F}{\sqrt{2}} J_{w\mu} J_W^{\mu\dagger}; \\ J_{W\mu} &= \sum_i \bar{\nu}_i \gamma_\mu (1-\gamma_5) l_i + \sum_i \bar{u}_i \gamma_u (1-\gamma_5) d_i, \end{aligned} \tag{11}$$

where the sum is over generations of quarks and leptons. (In writing this equation, mixing between generations has been neglected for simplicity). This form of the interaction also violated C, but in such a way that CP was left invariant.

For the experiments, this implied that if the ^{60}Co experiment could be repeated with anti-^{60}Co, the forward-backward asymmetry would have been exactly opposite in sign to that observed in ^{60}Co, so that

$$\langle \mathbf{s}_N \cdot \mathbf{p}_e \rangle + \langle \mathbf{s}_{\bar{N}} \cdot \mathbf{p}_{\bar{e}} \rangle = 0 \tag{12}$$

as required by CP invariance. Similarly, since the helicities of $e^{\pm}$ in $\mu^{\pm}$ decay are equal and opposite, CP is not violated.

The first evidence for CP violation was found in 1964 by Christenson *et al* [7]. They found that a definite state (the long-lived neutral K meson) can decay into two different CP eigenstates: the predominantly $CP = -1$ 3π state, and a very small (0.2%) admixture of the pure $CP = +1$ 2π state. Since the intrinsic CP assignments of the pions were fixed from strong interaction production processes, this meant that the weak-interaction decay process does not conserve CP (direct CP violation) or that K_L is not an eigenstate of CP (indirect CP violation), or both. In fact, K_L, which is an eigenstate of the Hamiltonian, is a linear combination of the states K^0 and $\overline{K}^0$ produced by strong interaction:

$$|K_L\rangle = a|K_1\rangle + b|K_2\rangle; \tag{13}$$

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) \tag{14}$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle), \tag{15}$$

where K_1 and K_2 are CP eigenstates. Hence even in the latter case, the Hamiltonian is not invariant under CP.

Direct CP violation is accompanied by a change in strangeness by 1 unit ($\Delta S = 1$), whereas CP violation in K^0–$\overline{K}^0$ mixing corresponds to $\Delta S = 2$. Thus, it is possible that only indirect CP violation exists. Alternatively, if only a $\Delta S = 1$ CP-violating term appears in the Hamiltonian, a $\Delta S = 2$ CP violation can be induced in second order of perturbation theory. Theories permitting only $\Delta S = 2$ CP violation are called superweak theories. Experiments can in principle check for direct CP violation.

Present experiments have only seen definite evidence for indirect CP violation [8]:

$$\frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_L \to \text{all})} = (2.03 \pm 0.04) \cdot 10^{-3},$$

$$\frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_L \to \text{all})} = (9.14 \pm 0.34) \cdot 10^{-4}. \tag{16}$$

Usually what is presented is [8]

$$|\eta_{00}| = \left|\frac{A(K_L \to 2\pi^0)}{A(K_S \to 2\pi^0)}\right| = (2.259 \pm 0.023) \times 10^{-3},$$

$$|\eta_{+-}| = \left|\frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}\right| = (2.269 \pm 0.023) \times 10^{-3}. \tag{17}$$

For direct CP violation to be present, $|\eta_{00}| \neq |\eta_{+-}|$ for which there is no evidence.

In addition to this CP-violating charge asymmetry has been observed in semileptonic K decay. The K-meson system is the only place where CP violation has been observed experimentally. However, CP violation has been looked for in other experiments as well, which have only provided upper limits. Other possible experiments in which CP violation has been tested are as follows.

3.1 *Electric dipole moments*

One of the important consequences of CP (or rather T) invariance is the vanishing of electric dipole moments of particles. For an elementary particle, the only vector to which a dipole moment, which is itself a vector, can be proportional to is its spin. Thus $\mathbf{d} = \alpha\mathbf{s}$. This interaction energy of the electric dipole moment with an electric field is

$$H_{\text{e.d.}} = -\mathbf{d} \cdot \mathbf{E} = -\alpha\mathbf{s} \cdot \mathbf{E}. \tag{18}$$

Since s changes sign under T, while E does not, this interaction violates T. In the relativistic form,

$$H_{\text{e.d.}} = i\alpha \int d^3x \bar{\psi}\sigma_{\mu\nu}\gamma^5\psi F^{\mu\nu}, \quad (F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu) \tag{19}$$

which can be shown to violate CP, P and T. A generalization can be made to a 'weak' dipole interaction

$$H_{\text{w.d.}} = i\tilde{\alpha} \int d^3x \bar{\psi}\sigma_{\mu\nu}\gamma^5\psi(\partial^\mu Z^\nu - \partial^\nu Z^\mu) \tag{20}$$

which also violates CP, in addition to P and T.

Electric dipole moments of various particles have been looked for without success, giving only upper bounds [8]. Weak dipole moments of leptons have also been tested for in experiments at the Z resonance at LEP and upper bounds have been obtained [9].

3.2 *Detailed balance in nuclear reactions*

T invariance predicts equal amplitudes for forward and backward reactions. This has been tested in several reactions, as for example,

$$^{24}\text{Mg} + \alpha \rightleftarrows {}^{27}\text{Al} + p. \tag{21}$$

So far no violation has been observed, and the experiments give upper bounds on effective T-violating parameters.

3.3 *Spin and momentum correlations*

Triple-products of spin and momenta, $\mathbf{p}_1 \times \mathbf{p}_2 \cdot \mathbf{p}_3$ or $\mathbf{s}_1 \times \mathbf{s}_2 \cdot \mathbf{p}_3$ etc. are odd under T. Hence naively, their observation would signal violation of T. However, this is true only if final-state interactions are sufficiently weak and can be neglected. This is because a T operation interchanges initial and final states, in addition to changing the sign of momenta and angular momenta. Forward and backward amplitudes are equal in magnitude only if final-state interactions (giving rise to so-called 'strong phases') can be neglected. This is the case, for example, with the decay $n \to pe^-\bar{\nu}$, where triple products have been looked for unsuccessfully.

Alternatively, in cases when the incoming state is a CP eigenstate, the observation of a momentum or spin correlation which is CP odd, rather than T odd, would unambigously signal CP violation.

4. Theoretical understanding of CP violation

Most current experimental results are explained quite remarkably well by the so-called standard model, developed by Glashow, Salam and Weinberg. This theory is based on local gauge invariance under the group $SU(2)_L \times U(1) \times SU(3)_C$. Under this symmetry the spin-1/2 quarks and leptons are arranged in left-handed doublets of weak isospin group $SU(2)_L$ and right-handed singlets. The quarks are triplets under $SU(3)_C$

representing 'colour' transformations. At present there are three known generations of quarks and leptons, which amounts to a replication of the left-handed doublets thrice. Thus, there are three 'up'-type quarks corresponding to a value $+\frac{1}{2}$ for the third component of weak isospin (called u, c and t quarks), and three 'down'-type quarks corresponding to $-\frac{1}{2}$ for the third component of weak isospin (called d, s and b quarks). The other basic ingredients are the 12 spin-1 gauge bosons γ, $W^{\pm}$ (with a mass of 80.2 GeV), Z^0 (with a mass of 91.1 GeV), and G_a $(a = 1, 2, \ldots, 8)$, and a spin-0 massive Higgs boson which is not yet seen. The photon and the gluons are massless. The constituents of the standard model are displayed below:

$$\begin{aligned}
&\text{Quarks: } \begin{pmatrix} u_{i\alpha} \\ d_{i\alpha} \end{pmatrix}_L \begin{matrix} (u_{i\alpha})_R \\ (d_{i\alpha})_R \end{matrix}, \\
&\text{Leptons: } \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L (e_i)_R, \\
&\text{Gauge bosons: } \gamma,\ W^{\pm}(80.2\,\text{GeV}),\ Z^0(91.1\,\text{GeV}), G_a, \\
&\text{Higgs boson: } H^0(> 58\,\text{GeV}).
\end{aligned} \tag{22}$$

Both the generation index i and the colour index α run from 1 to 3, and the colour index a for gluons runs from 1 to 8. Quarks, leptons and the massive gauge bosons and Higgs boson get their masses by the mechanism of spontaneous symmetry breaking. This also produces a mixing among different generations. As a result, the charged-current weak interactions, which are mediated by $W^{\pm}$, are described by a complex, unitary, 3×3 matrix, called the Cabibbo–Kobayashi–Maskawa (CKM) matrix. This matrix is totally arbitrary in the theory, and is only determined from experiment. It can be described by 4 parameters, 3 of which are like Euler angles $(\theta_1, \theta_2, \theta_3)$ describing rotations among the 3 generations, while the fourth one is a phase (δ) making some of the matrix elements complex. This complex phase is the cause of CP violation in the couplings of quarks to $W^{\pm}$. The smallest number of generations for which the phase can exist is three. (There would be no CP violation in a two-generation standard model).

The mixing gives rise to charged-current interactions of the quarks represented by the interaction Lagrangian

$$\mathcal{L}_{\text{c.c.}} = \frac{g}{2\sqrt{2}} \bar{u}_{i\alpha} \gamma^{\mu}(1 - \gamma^5) W_{\mu}^{+} d_{j\alpha} V_{ij} + \text{h.c.}, \tag{23}$$

where i, j represent the flavour indices, and V_{ij} is the ijth element of the CKM matrix.

How does this show up in the K-meson system? Indirect CP violation arises because of the phase in the amplitude for $K^0 \leftrightarrow \overline{K}^0$ transition. The quark structure of K^0 in $d\bar{s}$ and that of $\overline{K}^0$ is $\bar{d}s$. The amplitude for mixing of K^0 with $\overline{K}^0$ arises in the standard model from a fourth-order box diagram with exchange of a W^+W^- pair. The intermediate quark states involved are combinations of all the u quarks (up, charm and top), taken two at a time. The resulting amplitude is a sum of terms, the ij term being proportional to $V_{id}V_{is}^{*}V_{js}^{*}V_{jd}$. This complex quantity leads to a phase in the K^0–$\overline{K}^0$ mixing matrix, leading to indirect CP violation. For, then, one finds on diagonalizing the matrix, that K_L is not a CP eigenstate, but a linear combination of K_1 and K_2. Thus K_L contains an admixture of the CP even state K_1, permitting it to decay into 2π even if CP is conserved in the decay.

The magnitude of CP violation is determined by the quantity

$$J = \sin^2\theta_1 \sin\theta_2 \sin\theta_3 \cos\theta_1 \cos\theta_2 \cos\theta_3 \sin\delta \lesssim 3.5 \times 10^{-5} \sin\delta, \tag{24}$$

where $\theta_1, \theta_2, \theta_3$ are constrained by other experiments.

Direct CP violation also arises in the standard model through the so-called penguin diagrams, which are second-order in the weak interaction coupling. The amplitudes for K^0 and $\overline{K}^0$ decaying into 2π are relatively complex. Hence even if there is no CP violation in the mixing, so that $K_L = (|K^0\rangle - |\overline{K}^0\rangle)/\sqrt{2}$, K_L could still decay into 2π, because the matrix elements for $K^0 \to 2\pi$ and $\overline{K}^0 \to 2\pi$ are different.

Thus, the standard model can explain both direct and indirect CP violation. However, precise comparison with experiment is made difficult because the bound-state problem: The quark wave functions in the $K, \bar{K}$ are not known well enough.

The standard model predicts, similarly, CP violation in the B-meson system, i.e. mesons involving the b quark. This is expected to be somewhat larger, because the combination of mixing angles appearing in the B system are less constrained. This is seen from the so-called unitarity triangle of the CKM matrix. Unitarity gives the relations

$$\begin{aligned}
&\sum_{i=u,c,t} V_{id}V^*_{is} = 0 \quad \text{(strange } K \text{ mesons)},\\
&\sum_{i=u,c,t} V_{is}V^*_{ib} = 0 \quad \text{(strange } B_s \text{ mesons)},\\
&\sum_{i=u,c,t} V_{id}V^*_{ib} = 0 \quad \text{(nonstrange } B_d \text{ mesons)}.
\end{aligned} \tag{25}$$

Each relation represents a triangle in the complex plane, since the sum of 3 complex numbers vanish. The angles of the triangles are a measure of CP violation, the magnitudes of individual terms representing decay probabilities.

In the first two cases, one of the sides is very small compared to the other two, hence the triangles collapse to straight lines. In the third case all sides are comparable, though not large. Thus, CP violation is sizeable. However, B_d decay probabilities are low, and hence larger samples are needed. A substantial number of these will be produced at B factories, and it is hoped that CP violation would be observable.

The standard model predicts effects in all other systems which are too small to be detected in conceivable experiments. For example, quark dipole moments vanish at the two-loop level [10], and lepton dipole moments vanish at the three-loop level [11]. The estimates for dipole moments are:

$$\begin{aligned}
d_n &\approx 10^{-33} e\,\text{cm},\\
d_e &\approx 10^{-38} e\,\text{cm}.
\end{aligned} \tag{26}$$

Thus any other effects seen experimentally will be because of physics beyond the standard model.

If SM agrees well with experiments, is there need to go beyond SM? Quite apart from the usual reason given, that there are too many parameters in SM, there seem to be other reasons why SM is not adequate for CP violation. These are, the resolution of the strong

CP problem, and the understanding of the baryon-antibaryon asymmetry of the universe, or the so called problem of electroweak baryogenesis.

5. The strong CP problem

The vacuum of the strong interaction sector of the standard model, governed by the dynamics of the $SU(3)_C$ group, has a complicated structure: There are topologically distinct field configurations possible, all of which are degenerate. These distinct vacua $|n\rangle$ are characterized by a 'winding number' n, which takes all integral values. These vacua are not invariant under certain $SU(3)$ gauge transformations which change n. The correct gauge-invariant vacuum can only be a linear superposition of these vacua:

$$|\theta\rangle = \sum_n e^{-i\theta n}|n\rangle. \tag{27}$$

Each value of θ labels a different theory. For a given value of θ, if one calculates Green's functions, one finds that one has to sum over paths which connect the vacuum $|n\rangle$ to $|n+\nu\rangle$, for different values of ν. This sum amounts to adding a term

$$S_\theta = \theta\nu \equiv \theta \int d^4x \, \frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{a\mu\nu} \tag{28}$$

to the action. This term is odd under *CP* and *P*, and leads to 'strong' *CP* violation. It can therefore contribute to the neutron dipole moment, and can therefore be constrained by the strong experimental limit on the dipole moment. This requires θ to be smaller than about 10^{-9}. One can ask how such a small parameter arises. There is no answer, and this is one problem. But this is not all.

Suppose one assumes that for some reasons θ is small, there is still a problem – there is another contribution which changes θ. The axial vector $U(1)_A$ current

$$j^\mu_A = \sum_{i=1}^{N_f} \bar{q}\gamma^\mu\gamma_5 q \tag{29}$$

has an anomalous divergence

$$\partial_\mu j^\mu_A = 2N_f\left(\frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\right). \tag{30}$$

As a result one can show that under a chiral rotation of quarks through an angle α, the value of θ changes by $2N_f\alpha$.

Now, in defining the physical quark states in the theory, one has to make a change of basis which amounts to a chiral $U(1)$ rotation through an angle $\alpha = \arg\det M$, where M is the quark mass matrix, arising from spontaneous breaking of the electroweak gauge symmetry. Thus the observable value of θ is $\bar{\theta} = \theta + 2N_f(\arg\det M)$. This makes the problem worse, as now $\bar{\theta}$ has to be $\leq 10^{-9}$. Hence θ has to be tuned to lie close to $-2N_f(\arg\det M)$ to within 1 part in 10^9!

Most solutions proposed to the strong *CP* problem are beyond SM. The most popular one requires the introduction of spontaneously broken global $U(1)_{PQ}$ symmetry, as proposed by Peccei and Quinn in 1977 [12]. These theories have a light spin-0 particle known as the axion, which has not been seen. There are theories which make the axion 'invisible' in laboratory experiments [13], but such theories are replete with cosmological and astrophysical implications which will really test the theory [14]. In particular, the axion is a candidate for dark matter.

6. Electroweak baryogenesis

It was pointed out by Sakharov in 1967 [15] that if the present baryon asymmetry of the universe were to be understood in terms of fundamental processes occurring in the early universe which started with zero net baryon number, then three requirements had to be met. Firstly, there had to be baryon number (B) violating processes. Secondly, these processes had to violate C and CP. Thirdly, the processes had to be out of equilibrium at some epoch. C violation is needed to produce a difference in baryon and anti-baryon production, whereas CP (or T) violation ensures that the forward and backward processes proceeded at different rates. If the B production processes were to be in equilibrium, any excess produced would eventually be washed out by the reverse process. Inequilibrium during the Hubble expansion guarantees that this probability of the reverse process is smaller.

Grand unified theories which unify electroweak and strong interactions at a high unification energy scale satisfy all these criteria at temperatures around 10^{15} GeV, and can in principle generate baryon excess [16]. However, it was realized that during the electroweak phase transition at $T \approx 10^2$ GeV, non-perturbative anomalous B-violating processes can wash out whatever excess was produced. These processes can also regenerate a baryon asymmetry [17]. However, indications are that the SM CP violation cannot give a sufficient baryon number [18]. This seems to call for a new mechanism of CP violation.

Alternative scenarios for baryogenesis, as for example, through lepton-number generation at an intermediate scale, have been proposed [19]. These nevertheless involve new sources of leptonic CP violation.

7. CP violation in extensions of SM

In general, extra particles can give rise to more complex couplings and sources of CP violation. This is true in models with extra scalars, extra fermions, or both. An example of such a model is the minimal supersymmetric standard model.

An interesting possibility is spontaneous CP violation. In this case CP is imposed on the Lagrangian, and CP violation arises because a scalar field has a complex vacuum expectation value on minimization of the potential. This requires a minimum of two doublets. However, to prevent unwanted Yukawa couplings a certain discrete symmetry is imposed. Then spontaneous symmetry breaking needs three doublets at the least.

Spontaneous CP violation is generally associated with the domain wall problem in cosmology. As the universe cools below the temperature corresponding to the scale of CP breaking, domains of different CP phases form. The surface energy density in the walls separating the domains is sizeable. Also, because domain walls are two-dimensional, the rate of total energy decrease is only like the temperature T. Eventually the energy in the domain walls far exceeds the closure density ($\rho_0 \approx 10^{-46}\,\mathrm{GeV}^4$) of the universe:

$$\rho_{\mathrm{wall}} \sim \langle\phi\rangle^3 T \approx 10^{-7}\,\mathrm{GeV}^4. \tag{31}$$

A solution is that the *CP*-breaking transition occurs before an inflationary period. In that case, domain walls are irrelevant since we live in one domain which has got enormously stretched during inflation.

Another possibility often pursued is that *CP* is violated by 'soft' terms of mass dimension less than 4 in the real scalar field part of the Lagrangian. In that case the effect in other sectors of the theory are small and calculable.

8. A model-independent approach

Without reference to specific models, it is possible to study *CP*-violating experimental effects in terms of an effective Lagrangian [20]. In this relatively model-independent approach, one writes down all possible *CP*-violating couplings relevant to the experiment and consistent with Lorentz invariance. In case of fermion pair production in e^+e^- for example, these effective couplings will have electric and weak dipole moment terms. One could then parametrize possible experimental effects in terms of coefficients of these effective *CP*-violating couplings. Furthermore, the parameters could themselves be determined from various candidate theories. This would also enable confrontation of these theories with experiment.

Using an effective Lagrangian, one can make predictions for various experimental signatures of *CP* violation. Two special types of effects are event asymmetries and *CP*-odd correlations. An asymmetry of the type

$$\mathcal{A} = \frac{P(A \to B) - P(A^c \to B^c)}{P(A \to B) + P(A^c \to B^c)}, \tag{32}$$

where P denotes a transition probability, and A^c and B^c denote respectively states *CP* conjugate to A and B, would be a measure of *CP* violation. $\mathcal{A}$ has the advantage of being dimensionless and with absolute value less than or equal to 1. A *CP*-odd correlation is the expectation value of an observable which is odd under the *CP* transformation. While *CP*-odd correlations may not be as convenient as asymmetries described above, they may be easier to determine experimentally.

9. Conclusions

CP as an approximate symmetry of fundamental interactions is violated in weak interactions. The standard model can explain the observed *CP* violation in K mesons and make predictions for observable *CP* violation in the B-meson system. Any other *CP* violation would lie in the domain of extensions of the standard model. The current theoretical ideas on the solution of the strong *CP* problem and baryogenesis seem to need some extension of the standard model. Quite apart from this, it would be pragmatic to keep an open mind and use model-independent approaches to derive constraints on possible *CP*-violating couplings from experiment.

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