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Violation of signal locality and unitarity in a merger of quantum mechanics and general relativity

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Abstract. It is shown that a violation of signal locality and unitarity occur in a particular merger of quantum mechanics and general relativity.

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In spite of much effort that has been spent over decades, it has not been possible to formulate a completely satisfactory quantum theory of gravity. This has led some to suspect that quantum theory and the general theory of relativity are incompatible in their present forms [1]. In the absence of a satisfactory quantum theory of gravity, it has been traditional to use a semi-classical theory in which the metric of space-time is determined by the local expectation value of the stress-energy-momentum tensor of quantized matter via the classical field equations of Einstein:

$$G_{\mu\nu} = 8\pi\langle T_{\mu\nu}(x)\rangle. \tag{1}$$

Although such a semi-classical theory is expected to be reliable far away from the Planck scale, Kibble has emphasized that it implies a non-linear modification of quantum theory [2]. He has also suggested a Gedanken experiment which shows that its predictions are inconsistent with those of quantum theory which should automatically incorporate gravity if one takes the view that the universe is wholly quantum and that classical Einsteinian gravity is only an approximate description of phenomena that emerge in certain situations. Let me describe a slight variation of Kibble's Gedanken experiment. Imagine a device like a Stern–Gerlach apparatus that splits the wavefunction ψ of an incoming particle (whose mass M is sufficiently large for the purpose in hand) into a superposition of two orthogonal pieces that are eventually registered by one of two detectors D_1 and D_2 localized at x_1 and x_2. The final wavefunction of the combined system (particle plus the detectors) is then

$$\Phi = \frac{1}{\sqrt{2}}[\psi_1(x)D_1(x) + \psi_2(x)D_2(x)]. \tag{2}$$

A small test mass m is placed midway between the two detectors. Every time a detector fires, m is attracted towards it according to quantum theory. According to traditional semi-classical theory, however, the metric due to the particle wavefunction ψ would be determined by

$$\tfrac{1}{2}[\langle T_{\mu\nu}(x_1)\rangle + \langle T_{\mu\nu}(x_2)\rangle] \tag{3}$$

which does not change when a detector fires, and so m does not move at all.

Recently a proposal has been made [3] for a quasiclassical theory of gravity in which one has to solve the coupled equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu}(\psi), \quad T^{\mu\nu}_{;\nu} = 0, \tag{4}$$

$$i\hbar\partial_{;t}\psi = H(-i\hbar\partial_{;x})\psi, \tag{5}$$

where the metric is supposed to be quasiclassical and the matter part first quantized. The key feature is that the classical variables become correlated to the state of the quantum variables. If the quantum state is a superposition of different states, the classical variables need not have a definite value but may take a distribution of values depending on the quantum state. Effectively, the quantum variables act like a source of noise on the classical evolution. These variables are then not entirely classical in nature but may more properly be called *quasiclassical*. In contrast to the traditional semi-classical approach (eq. (1)), this approach maintains the linearity of quantum mechanics through the coupled equations (4) and (5) while allowing the values of the quasiclassical variables to depend on the quantum state. This provides a means of computing the backreaction that quantum variables have on the evolution of classical variables without having to make a full semi-classical analysis. If this approach extends to quantum field theory, it has potentially far reaching consequences. By keeping the gravitational field quasiclassical, it reopens the debate on the necessity to quantize gravity [4]. Alternatively, when viewed as an approximation to a fully quantized theory of gravity far away from the Planck scale, it improves on conventional quantum field theory in curved space-time by taking into account the backreaction of quantized matter fields on the classical space-time background. Such effects are expected to be crucial for a complete understanding of information loss in black hole evaporation [5].

The consistency of this approach with linear quantum mechanics can be checked in the case of Kibble's Gedanken experiment discussed above. Since the metric due to the matter wavefunction Φ is determined in this approach by

$$\tfrac{1}{2}[T_{\mu\nu}(x_1)|D_1|^2 + T_{\mu\nu}(x_2)|D_2|^2] \tag{6}$$

rather than by (3), each time a detector fires, Φ 'collapses' to only one of the terms in (2), and so the mass m moves. This is a general consequence of the formalism of quantum mechanics and is independent of interpretations. For example, even the many-worlds interpretation in which there is no collapse in the universe as a whole would imply this for every branch into which the universe splits. This means that the metric changes each time a detector fires, and m is attracted towards it, which is consistent with quantum theory.

The question arises as to whether consistency with linear quantum mechanics can be demonstrated most generally. I will now show that there is at least one case in which an inconsistency arises. This happens because of collapse-induced non-locality. Although quantum mechanics implies Bell-type non-locality through non-local quantum correlations that cannot be reproduced in any local hidden variable theory, it is consistent with *signal locality*, i.e., the prohibition of superluminal signals at the statistical level, even though the notion of wavefunction collapse as an actual physical transition from a pure to a mixed state implies non-local influences at the level of individual events [6]. In a realist interpretation of these events such non-local influences constitute a definite violation of the principle of relativity.

Consider the two-particle position and momentum correlated entangled state

$$|\psi\rangle = \frac{1}{\sqrt{2}}[\,|A\rangle|B\rangle\delta^{(3)}(x_A - x_B - d(t)) + |A'\rangle|B'\rangle\delta^{(3)}(x_{A'} - x_{B'} - d(t))], \quad (7)$$

where $|A\rangle$ and $|A'\rangle$ are two possible orthogonal position eigenstates of particle A moving to the left (say) of the source, and $|B\rangle$ and $|B'\rangle$ are the corresponding correlated position eigenstates of particle B moving to the right; $d(t)$ is the distance between the particles at the time t determined by their relative velocity. The density operator $\rho = |\psi\rangle\langle\psi|$ is non-diagonal, and one can calculate the reduced density operator of one of the particles (A or B) by tracing over the states of the other. For example,

$$\rho_B = \mathrm{Tr}_A\rho = |B\rangle\langle B| + |B'\rangle\langle B'|, \quad \mathrm{Tr}\rho_B = 1. \quad (8)$$

It is the orthogonality of the two possible states $|A\rangle$ and $|A'\rangle$ that results in ρ_B being diagonal. This implies that effectively ρ_B describes an incoherent mixture of $|B\rangle$ and $|B'\rangle$, and that these do not interfere with each other even though the two-particle state $|\psi\rangle$ remains entangled. This is because when the system is prepared in the state $|\psi\rangle$, one could determine which path particle B takes by placing detectors in the beams A and A', and this determination does not disturb particle B in any mechanical sense, as emphasized by Einstein, Podolsky and Rosen.

According to Feynman [7], 'if an experiment is performed which is capable of determining whether one or another alternative is actually taken, the probability of the event is the sum of the probabilities for each alternative. The interference is lost'. This rule can be extended in the spirit of Einstein, Podolsky and Rosen by replacing 'is performed' in Feynman's rule by 'could be performed without disturbing the system'. This is a novel feature of two-particle interferometry [8]. It has the important corollary that a localized measurement on A cannot affect ρ_B (the trace over the possible states of A having already been taken in computing it), and hence the expectation value of all local, hermitean operators $\hat{O}$ in the Hilbert space of B, given by

$$\langle\hat{O}\rangle = \mathrm{Tr}\,\rho_B\hat{O}, \quad (9)$$

remains unchanged when such a measurement is made. This guarantees that no superluminal signal can be sent utilizing entangled states and collapse-induced non-locality.

The situation is different if gravitational effects are included in the way suggested in [3]. When a localized measurement is made on A by placing detectors at x_A and $x_{A'}$, the state of the combined system of the two particles A and B and the detectors can be written as

$$|\Phi\rangle = \frac{1}{\sqrt{2}}[|A\rangle|B\rangle|D_A\rangle\delta^{(3)}(x_A - x_B - d(t)) + |A'\rangle|B'\rangle|D_{A'}\rangle\delta^{(3)}(x_{A'} - x_{B'} - d(t))], \quad (10)$$

where $|D_A\rangle$ and $|D_{A'}\rangle$ are the states of the two detectors coupled to $|A\rangle$ at x_A and $|A'\rangle$ at $x_{A'}$ respectively. On completion of the measurement, 'collapse' or state vector reduction occurs, i.e., only one of the two terms in (10) survives. If it is the first term, the particle B is instantaneously projected to its state $|B\rangle$ localized at x_B; if it is the second term, it is projected to its state $|B'\rangle$ localized at $x_{B'}$. This localization of B is represented by a change of its wavefunction from the entangled ψ to $\phi_B(x) = \langle x|B\rangle = \delta^{(3)}(x - x_B)$ or $\phi_{B'}(x) = \langle x|B'\rangle = \delta^{(3)}(x - x_{B'})$. The local stress–energy–momentum density $T_{\mu\nu}(x)$ of B (which is not an observable) must therefore change from

$$\tfrac{1}{2}[\psi^*(x)\hat{T}_{\mu\nu}\psi_B + \psi^*_{B'}(x)\hat{T}_{\mu\nu}\psi_{B'}(x)] \tag{11}$$

before the measurement to $\phi^*_B\hat{T}_{\mu\nu}\phi_B(x)$ or $\phi^*_{B'}(x)\hat{T}_{\mu\nu}\phi_{B'}(x)$ after the measurement, resulting in an instantaneous and non-local change of the metric field around x_B or $x_{B'}$ to a Schwarzchild form via eq. (4) [9]. Although such a change may be quantitatively insignificant and for all practical purposes ignored in many cases, in a theory in which the gravitational field is regarded as being fundamentally classical or quasiclassical, it would already imply a *non-local* and *real factual*, and therefore in principle observable, change at the level of individual events that contradicts relativity theory.

Even in the alternative scenario in which the quasiclassical gravitational field is regarded as an approximation to a fully quantum theory of gravity, this non-local change of metric due to 'collapse' has an unacceptable consequence. 'Collapse' or state vector reduction is a non-causal, non-linear and non-unitary change during which the normal dynamical law of evolution (linear and unitary (eq. (5)) remains suspended. Once it has occurred, normal dynamical evolution resumes but with a difference in the presence of gravity—with the covariant derivatives in eq. (5) changed non-causally by the 'collapse'-induced metric change. Consequently, the new states $|\mathcal{B}\rangle$ and $|\mathcal{B}'\rangle$ that are solutions to eq. (5) with the changed metric must be related to the old ones $|B\rangle$ and $|B'\rangle$ by a non-unitary operator $\mathcal{C}$, i.e., $|\mathcal{B}\rangle = \mathcal{C}|B\rangle$, $|\mathcal{B}'\rangle = \mathcal{C}|B'\rangle$ with $\mathcal{C}^\dagger \neq \mathcal{C}^{-1}$. I must emphasize that $\mathcal{C}$ is *not* to be confused with the projection operator for collapse. Then the new density operator for B has the property

$$\mathrm{Tr}\,\rho'_B = \mathrm{Tr}(|\mathcal{B}\rangle\langle\mathcal{B}| + |\mathcal{B}'\rangle\langle\mathcal{B}'|) \tag{12}$$

$$= \mathrm{Tr}\,\mathcal{C}\rho_B\mathcal{C}^\dagger \neq 1. \tag{13}$$

Consequently, the expectation values of all observables change, unitarity is violated and superluminal signalling becomes possible. It is clearly only gravity (as formulated in [3]) that is capable of exposing this unacceptable and ugly head of 'collapse'. This inconsistency, if true, can only be removed by changing standard measurement theory to make it consistent with relativistic invariance. Although 'retarded collapse' (the effect of collapse propagating within the light-cone) looks like an attractive possibility in this context, it is incompatible with quantum mechanics because it is equivalent to a local model and therefore compatible with Bell's inequalities which are violated by quantum mechanics.

This violation of signal locality, a fundamental tenet of relativity theory, is a feature of the proposal in [3] that is, however, not shared by the traditional semi-classical approach because of the use of the local expectation value of the stress-energy-momentum tensor in eq. (1), which is an observable and does not change with measurements. But, as Kibble has shown through his Gedanken experiment, this very feature makes it inconsistent with quantum theory.

All this tends to confirm the suspicion that the two pillars of modern physics, quantum theory and general relativity, are probably incompatible in their present forms.

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