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Weak decay constants of light and heavy pseudoscalar mesons

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Abstract. We investigate the weak leptonic decays of light and heavy pseudoscalar mesons in a relativistic quark model of independent quarks. We perform a static calculation of the decay constant f_M purely on grounds of simplicity. In order to minimize the possible uncertainty in the static calculation, we estimate the ratios of the decay constants which are found to be in good agreement, in the heavy flavor sector, with the predictions of other models available in the literature and existing experimental data. However, there is a noticeable discrepancy in the current prediction for pion decay constant which demonstrates the inherent limitations of the static approximation in the study of non-strange light mesons.

Keywords. Pseudoscalar mesons; quark model; decay constants; static approximation.

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1. Introduction

Knowledge of the pseudoscalar decay constants f_M has been realized to be quite useful in extracting information on the fundamental quantities of the standard model such as the CKM-matrix elements and the top quark mass as well as on the rare B-meson decays. In addition, the decay constants govern the strength of several leptonic, semileptonic and non-leptonic processes. Because of their theoretical and phenomenological importance, there has been several attempts to have reliable estimations of these decay constants within the framework of various suitable schemes.

A theoretical estimation of the decay constants for the decays of pseudoscalar mesons of the type $(M \to l\bar{\nu}_l)$ requires a rigorous field theoretic formulation of the quark–antiquark annihilation inside the meson-bound-state; which like many other low-energy phenomena, cannot be studied in a straight-forward manner by the first-principle-application of QCD–the underlying theory of strong interaction between quarks and gluons at the structural level of hadrons. Therefore, various phenomenological models [1–16] have been proposed to study the weak leptonic decays of light as well as heavy pseudoscalar mesons giving widely different predictions on the decay constants. While predictions of various models in this sector are not consistent, the experiments have so far made limited progress. In view of this, Barik and Dash [17] investigated the weak leptonic decays of light as well as heavy pseudoscalar mesons in a field-theoretic framework of independent quark model with a scalar-vector harmonic potential [17–21]. The model predictions [17] are not only found to agree remarkably well with the experimental data [22] for f_π and f_K in the light flavor sector, but also consistent with the findings of many other model calculations giving

$$f_{B_c} > f_{B_s} > f_B;\ f_{D_s} > f_D;\ f_D > f_B;\ f_\pi > f_B.$$

In this work [17], the quark–antiquark annihilation inside the meson-bound-state is treated field theoretically with the construction of the decaying meson state as appropriate momentum-wave-packets reflecting the respective constituent quark–antiquark momentum distribution amplitudes obtained from the bound quark eigen-mode of the model.

In the present work we would like to tackle the same problem in a straight-forward manner in the same model [17–21] through a different approach without incorporating rigorously the mesonic core-state as a momentum wave-packet. Instead one can use the possible expansion of the quark–antiquark field operators appearing in the effective interaction Hamiltonian density that describes the S-matrix operator for the decay process, by a linear combination of the complete set of quark eigen-modes available in the model. The quark annihilation and the antiquark creation operator in the corresponding flavor and eigen-modes provide the appropriate co-efficients of the expansion. The decaying pseudoscalar meson can be considered in the ground state by the usual spin flavor $SU(6)$-expression with the quark–antiquark creation operators corresponding to the lowest eigen-modes. The transition amplitudes for the decay would then involve essentially the lowest eigen-modes of the quark–antiquark. Such an approach effectively taken in a static limit, which makes the calculation quite straight-forward and tractable, has been applied successfully in the study of the weak radiative decays of the charmed pseudoscalar mesons [19] and the radiative decays of light and heavy mesons [20, 21]. The results so obtained are found to be reasonably satisfactory except in the decays involving the light mesons especially the pion. Going beyond the static limit and adding some momentum dependence due to recoil effect in a more realistic calculation [20], the predictions in the light meson (especially the pion)-decays can get significantly improved. However, the predictions in heavier meson decays remain almost the same as obtained from the static calculation [19]. Therefore, a calculation of the individual decay constants in weak decays like $(M \to l\bar{\nu}_l)$, in a 'static'-approximation may not be adequate enough. Nevertheless we do believe that a reliable estimation can still be made for the ratio of the weak decay constants corresponding to the different mesons particularly in heavy sector. With this contention in mind we would prefer to revisit the weak leptonic decays of light and heavy pseudoscalar mesons in the present model employing a 'static'-approximation purely on grounds of simplicity.

This paper is organized in the following manner. In §2 we provide some relevant conventions and consequences of the present model. We describe the transition matrix elements and derive the partial decay width with correct kinematic factors from which we extract the expression for the pseudoscalar decay constants in §3. Section 4 embodies our results and discussion.

2. Model framework

According to this model, a meson is pictured as a color singlet assembly of a quark and an antiquark independently confined by an average flavor independent potential of the form [17–21]

$$U(r) = \tfrac{1}{2}(1 + \gamma^0)(ar^2 + V_0), \quad a > 0 \tag{1}$$

which represents phenomenologically the confining interaction expected to be generated by the non-perturbative multigluon mechanism. The possible quark–gluon interaction at short distance originating from one gluon exchange and the quark–pion interaction required in the non-strange light-flavor sector to preserve chiral symmetry are presumed to be residual interactions compared to the dominant confining part. Although this residual interactions treated perturbatively in the model are crucial in generating meson-mass-splittings [23, 24], their role in the decays of mesons are considered less significant. Therefore, to a first approximation, it is believed that the zeroth order quark dynamics inside the meson-core generated by the confining part of interaction, can provide an adequate description of the meson decay process. In such a picture, the independent quark Lagrangian density in zeroth order is

$$\mathscr{L}_q^0(x) = \bar{\psi}_q(x)\left[\frac{i}{2}\gamma^\mu \overleftrightarrow{\partial}_\mu - m_q - U(r)\right]\psi_q(x). \tag{2}$$

The ensuing Dirac equation with $E'_q = (E_q - V_0/2)$, $m'_q = (m_q + V_0/2)$, $\lambda_q = (E'_q + m'_q)$ and $r_{0q} = (a\lambda_q)^{-1/4}$ admits static solution of positive and negative energy in zeroth order. Corresponding to the ground state mesons, these solutions can be obtained in the form

$$\begin{aligned}\phi_{q\lambda}^{(+)}(\mathbf{r}) &= \frac{1}{\sqrt{4\pi}}\begin{pmatrix} i g_q(r)/r \\ \boldsymbol{\sigma}\cdot\hat{r} f_q(r)/r\end{pmatrix}\chi_\lambda, \\ \phi_{q\lambda}^{(-)}(\mathbf{r}) &= \frac{1}{\sqrt{4\pi}}\begin{pmatrix} i(\boldsymbol{\sigma}\cdot\hat{r}) f_q(r)/r \\ g_q(r)/r\end{pmatrix}\tilde{\chi}_\lambda.\end{aligned} \tag{3}$$

Here the two component spinors χ_λ and $\tilde{\chi}_\lambda$ stand for

$$\chi_\uparrow = \begin{pmatrix}1\\0\end{pmatrix}, \chi_\downarrow = \begin{pmatrix}0\\1\end{pmatrix} \quad \text{and} \quad \tilde{\chi}_\uparrow = \begin{pmatrix}0\\-i\end{pmatrix}, \tilde{\chi}_\downarrow = \begin{pmatrix}i\\0\end{pmatrix}$$

respectively. The reduced radial parts in the upper and lower component solutions corresponding to the quark-flavor 'q' are

$$\begin{aligned} g_q(r) &= \mathcal{N}_q\left(\frac{r}{r_{0q}}\right)\exp(-r^2/2r_{0q}^2), \\ f_q(r) &= -\frac{\mathcal{N}_q}{\lambda_q r_{0q}}\left(\frac{r}{r_{0q}}\right)^2 \exp(-r^2/2r_{0q}^2),\end{aligned} \tag{4}$$

where the normalization factor $\mathcal{N}_q$ is given by

$$\mathcal{N}_q^2 = \frac{8\lambda_q}{\sqrt{\pi} r_{0q}} \frac{1}{(3E'_q + m'_q)}. \tag{5}$$

The quark binding energy of zeroth order in the meson ground-state is derivable from the bound-state condition

$$\sqrt{\frac{\lambda_q}{a}}(E'_q - m'_q) = 3. \tag{6}$$

This provides a brief outline of the model and its conventions which can now be adopted to re-investigate the weak leptonic decays of pseudo-scalar mesons [12–17] using a simplified approach based on static-approximation.

3. Weak decay constant

In this section we consider the weak leptonic decays of charged pseudoscalar mesons such as $\pi^\pm, K^\pm, D^\pm, D_s^\pm, B^\pm$ and $B_c^\pm$ using the model convention described in the previous section in the static approximation. Assuming that the main contribution to the weak leptonic decay comes from annihilation of the bound quark–antiquark inside the meson into a single virtual W-boson which ultimately disintegrates into a leptonic pair $(l\bar{\nu}_l)$, we can illustrate it by the corresponding Feynman diagram in figure 1 from which S-matrix element is effectively written as

$$S_{fi} = -i\frac{G_F}{\sqrt{2}}\mathscr{V}_\theta \int d^4x\, T[\langle l(\mathbf{k}_l, \lambda_l)\nu(\mathbf{k}_\nu, \lambda_\nu)|\bar{\psi}_\nu(x)\gamma^\mu(1-\gamma_5)\psi_l(x)$$

$$\times \sum_{\mathrm{q}} \bar{\psi}_{\mathrm{q}_1}(x)\gamma_\mu(1-\gamma_5)\psi_{\mathrm{q}_2}(x)|M\rangle]. \tag{7}$$

Here $\mathscr{V}_\theta = \cos\theta_c(\sin\theta_c)$ with θ_c being Cabibbo angle. The usual lepton field operator and quark field operator are taken respectively in the form

$$\psi_l(x) = \sum_{\lambda_l} \frac{1}{(2\pi)^{3/2}} \int \frac{d\mathbf{k}_l}{\sqrt{2E_l}} [\hat{d}_l(\mathbf{k}_l, \lambda_l)u(\mathbf{k}_l, \lambda_l)\exp(-ik_l x)$$

$$+ \hat{\tilde{d}}_l^\dagger(\mathbf{k}_l, \lambda_l)v(\mathbf{k}_l, \lambda_l)\exp(ik_l x)] \tag{8}$$

and

$$\psi_{\mathrm{q}}(x) = \sum_\xi [\hat{b}_{\mathrm{q}\xi}\phi^{(+)}_{\mathrm{q}\xi}(r)\exp(-iE_{\mathrm{q}\xi}t) + \hat{\tilde{b}}^\dagger_{\mathrm{q}\xi}\phi^{(-)}_{\mathrm{q}\xi}(r)\exp(iE_{\mathrm{q}\xi}t)], \tag{9}$$

where 'q' stands for quark flavor, ξ for the set of Dirac quantum numbers specifying all possible eigenmodes; $\hat{b}_{\mathrm{q}\xi}$ and $\hat{\tilde{b}}^\dagger_{\mathrm{q}\xi}$ are quark annihilation and antiquark creation operators; $\phi^{(+)}_{\mathrm{q}\xi}(\mathbf{r})$ and $\phi^{(-)}_{\mathrm{q}\xi}(\mathbf{r})$ are the quark and antiquark wave functions with quark binding energy $E_{\mathrm{q}\xi}$ corresponding to the eigen-modes 'ξ'.

The quark field operators $\psi_{\mathrm{q}}(x)$ appearing in the effective interaction Hamiltonian density that describes the S-matrix element in (7) finds its expansion as a linear combination of all possible quark eigen-modes $\phi^{(+)}_{\mathrm{q}\xi}(\mathbf{r})$ and $\phi^{(-)}_{\mathrm{q}\xi}(\mathbf{r})$ available in the

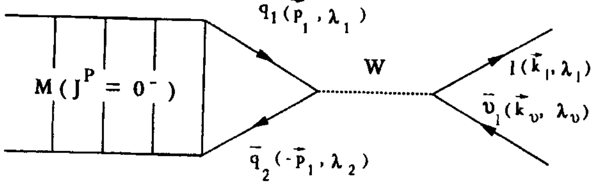

Figure 1. One boson contribution to the weak leptonic decays of light and heavy pseudoscalar mesons.

model with quark annihilation and antiquark creation operators ($\hat{b}_{q\xi}$ and $\hat{\tilde{b}}^{\dagger}_{q\xi}$) as appropriate co-efficient of expansion. Since the decaying meson is considered in its ground state in the present calculation based on a 'static' approximation, the relevent S-matrix element here, in the long run, would essentially acquire contribution of only the lowest eigen-modes present in the field expansion of (9). Therefore to avoid unnecessary complication, we treat the quark field expansion effectively in terms of the lowest eigen-modes as given in (3) corresponding to the ground-state meson without affecting the final result in any way. With this contribution, the leptonic and hadronic parts can be simplified separately in the vacuum insertion technique to find the effective S-matrix element in the form

$$S_{fi} = \frac{-i}{(2\pi)^2}\frac{G_F}{\sqrt{2}}\mathscr{V}_\theta\,\delta(E_l + E_\nu - M) \times l^\mu(\mathbf{k}_l, \mathbf{k}_\nu;\, \lambda_l, \lambda_\nu)\, h_\mu \tag{10}$$

where the leptonic and hadronic amplitude parts are

$$l^\mu(\mathbf{k}_l, \mathbf{k}_\nu; \lambda_l, \lambda_\nu) = \frac{\bar{u}_l(\mathbf{k}_l, \lambda_l)\gamma^\mu(1-\gamma_5)v_\nu(\mathbf{k}_\nu, \lambda_\nu)}{\sqrt{4E_l E_\nu}} \tag{11}$$

and

$$h_\mu = \int \mathrm{d}\mathbf{r} \exp[i(\mathbf{k}_l + \mathbf{k}_\nu)\cdot\mathbf{r}]\Big\langle 0\Big| \sum_{q,m} \hat{\tilde{b}}_{q_1, m_1} \hat{b}_{q_2, m_2} \bar{\phi}^{(-)}_{q_1, m_1}(\mathbf{r})\gamma_\mu$$

$$\times (1-\gamma_5)\phi^{(+)}_{q_2, m_2}(\mathbf{r})\Big| M\Big\rangle. \tag{12}$$

It must be noted that there is an obvious difficulty encountered here in realizing the required energy conservation at the quark-boson vertex; since the sum total of the kinetic energy of the constituent quark and antiquark in the process in not equal to the mass–energy of the decaying pseudoscalar meson considered in its rest frame. This is a common feature with all the phenomenological models based on leading order calculation. In the absence of any rigorous field theoretic formulation of the bound quark–antiquark annihilation inside the meson, we assume that the differential amount of energy is somehow made available to the W-boson when the quark–antiquark pair annihilation takes place with the disappearance of meson-bound-state. In doing so the sum total of quark–antiquark energy ($E_q + E_{\bar{q}}$) is replaced by the meson rest-mass M in the argument of the energy delta function.

In calculating the S-matrix element from (10), the decaying pseudoscalar meson M in its ground state is described here by the usual spin-flavor $SU(6)$-expressions with the quark and antiquark creation operators corresponding to the lowest eigen-mode. Then in the flavor $SU(2)$-symmetry ($m_u = m_d \neq m_s$), the hadronic part h_μ is obtained in the form

$$h_\mu = \sqrt{\frac{3}{2}}\int \mathrm{d}\mathbf{r} \exp[i(\mathbf{k}_l + \mathbf{k}_\nu)\cdot\mathbf{r}][\bar{\phi}^{(-)}_{q\downarrow}(r)\gamma_\mu(1-\gamma^5)\phi^{(+)}_{q\uparrow}$$

$$- \bar{\phi}^{(-)}_{q\uparrow}(r)\gamma_\mu(1-\gamma^5)\phi^{(+)}_{q\downarrow}], \tag{13}$$

where the factor $\sqrt{3}$ is effectively due to the color-singlet configuration of the meson M.

With $L^{\mu\nu} = l^{\mu} l^{\nu+}$ and $H_{\mu\nu} = h_{\mu} h_{\nu}^{\dagger}$, the weak leptonic decay width can be calculated from the general expression

$$\Gamma(M \to l\nu) = \sum_{\lambda_l, \lambda_\nu} \int \frac{|S_{fi}|^2}{T} \frac{V \mathrm{d}\mathbf{k}_l}{(2\pi)^3} \frac{V \mathrm{d}\mathbf{k}_\nu}{(2\pi)^3} \tag{14}$$

in the form

$$\Gamma(M \to l\nu) = \frac{3G_F^2 \mathscr{V}_\theta^{2}}{2(2\pi)^5} \int \mathrm{d}\mathbf{k}_l \mathrm{d}\mathbf{k}_\nu \delta(E_l + E_\nu - M) L^{\mu\nu} H_{\mu\nu}. \tag{15}$$

The contribution of the space-like part of the hadronic amplitude, in fact, vanishes in the angular integration and the only non-vanishing contribution to S_{fi} is effectively derived from its time-like component h_0 which, in terms of reduced radial parts of the upper and lower component solutions $g_q(r)$ and $f_q(r)$, is obtained as

$$h_0 = \frac{\sqrt{2}}{4\pi} \int \mathrm{d}\mathbf{r} \exp[i(\mathbf{k}_l + \mathbf{k}_\nu)\cdot\mathbf{r}] \left[\frac{f_{q_1}(r) f_{q_2}(r) - g_{q_1}(r) g_{q_2}(r)}{r}\right]^2. \tag{16}$$

It is then straight forward to find the effective hadronic as well as leptonic tensor in the respective forms as

$$H_{00} = \left| \frac{\sqrt{2}}{4\pi} \int \mathrm{d}\mathbf{r} \exp[i(\mathbf{k}_l + \mathbf{k}_\nu)\cdot\mathbf{r}] \left[\frac{f_{q_1}(r) f_{q_2}(r) - g_{q_1}(r) g_{q_2}(r)}{r}\right]^2 \right|^2 \tag{17}$$

and

$$L^{00} = \frac{1}{4E_l E_\nu} \mathrm{Tr}[(\not{k}_l + m_l)\gamma^0(1 - \gamma_5)(\not{k}_\nu + m_\nu)\gamma^0(1 - \gamma_5)]. \tag{18}$$

It is convenient to integrate the quantity in (15) by a transformation of co-ordinates with the substitution of relative momentum $\mathbf{p}$ and centre of mass momentum $\mathbf{P}$ as

$$\mathbf{p} = \tfrac{1}{2}(\mathbf{k}_l - \mathbf{k}_\nu); \quad \mathbf{P} = (\mathbf{k}_l + \mathbf{k}_\nu). \tag{19}$$

Of course, $\mathbf{P}$ is constrained by energy conservation; but because momentum conservation is not automatically guaranteed while describing the meson state in terms of its basic constituents (quark–antiquark) bound inside the meson, the final state leptons need not have $|\mathbf{P}| = |\mathbf{k}_l + \mathbf{k}_\nu| = 0$. We must, however, hope that most of $\Gamma(M \to l\nu)$ comes from $|\mathbf{P}|$ small compared to M if this model calculation is to have any resemblance to the physical decays. We, therefore, proceed by setting $|\mathbf{P}| = 0$ in l^{μ} as well as in the energy delta function $\delta(E_f - E_i)$ only; but not in h_μ, thus preserving the typical bound-state-characteristics of the constituent quark and antiquark. Calculating the leptonic part and hadronic part in (15) appropriately in the above expression after integration; one obtains

$$\Gamma(M \to l\nu) = \frac{3G_F^2 \mathscr{V}_\theta^{2}}{2(2\pi)^5} \mathscr{L}^{00} \mathscr{H}_{00} \tag{20}$$

with

$$\mathscr{L}^{00} = 2\pi m_l^2 \left(1 - \frac{m_l^2}{M^2}\right)^2 \tag{21}$$

and

$$\mathscr{H}_{00}=4\pi^2\int_0^\infty \mathrm{d}r\left[\frac{f_{q_1}(r)f_{q_2}(r)-g_{q_1}(r)g_{q_2}(r)}{r}\right]^2. \tag{22}$$

Then using the explicit forms of $f_q(r)$ and $g_q(r)$ given in (4), the hadronic part $\mathscr{H}_{00}$ can be integrated out in a closed form so as to finally obtain the decay width in the usual form as

$$\Gamma(M\to l\nu)=\frac{G_F^2\mathscr{V}_\theta^2}{8\pi}Mm_l^2\left(1-\frac{m_l^2}{M^2}\right)^2 f_M^2. \tag{23}$$

Here the weak leptonic decay constant f_M has been extracted out, after realizing the correct kinematic factors in (23). Thus in terms of the model quantities the weak decay constant f_M is realized here in the form

$$f_M^2=\frac{3}{4\sqrt{\pi}M}\left(\frac{\mathscr{N}_{q_1}\mathscr{N}_{q_2}}{r_{0q_1}r_{0q_2}}\right)^2\left(\frac{r_{0q_1}^2r_{0q_2}^2}{r_{0q_1}^2+r_{0q_2}^2}\right)^{3/2}$$
$$\times\left[\frac{15}{4\lambda_{q_1}^2\lambda_{q_2}^2(r_{0q_1}^2+r_{0q_2}^2)^2}-\frac{3}{\lambda_{q_1}\lambda_{q_2}(r_{0q_1}^2+r_{0q_2}^2)}+1\right]. \tag{24}$$

Hence f_M for the specific pseudoscalar meson decay can be calculated from (24) with the relevant input-parameters available from the constituent level description of the meson. Although we extract here the expression for the weak decay constant f_M through the calculation of the corresponding decay width, it is always possible to obtain it directly from the hadronic part of the transition matrix element. Therefore, we can use (24) as a general expression for the decay constant even for the neutral pseudoscalar meson such as B_s^0.

4. Results and discussion

In this section we evaluate the decay constants for the weak leptonic decays of pseudoscalar mesons such as $\pi^\pm, K^\pm, D^\pm, D_s^\pm, B^\pm, B_s^0$ and $B_c^\pm$ from the expression in (24). The input parameters that are primarily required are the potential parameters (a, V_0) of the model and the quark masses $(m_u=m_d, m_s, m_c, m_b)$. The potential parameters (a, V_0) are given according to ref. [17–21] as

$$(a, V_0)\equiv(0{\cdot}0171626\,\mathrm{GeV}^3,\ -0{\cdot}1375\,\mathrm{GeV}) \tag{25}$$

and the quark masses [17–21] are taken from the model in its earlier applications demonstrating its success in wide-ranging hadronic phenomena. The meson masses appearing in the expression for f_M in (24) are taken to be the experimental meson masses [22]. However in the decay of the heavy meson $B_c^\pm$, the corresponding experimental data for the mass M_{Bc} is not yet available and therefore we take our model mass as $M_{Bc}=6{\cdot}2642$ GeV [17]. The model dynamics provides other relevant quantities such as E_q, λ_q, r_{0q} and $\mathscr{N}_q$ etc. which are given in table 1.

Table 2 provides our result for the decay constants in comparison with those of some previous calculations [6–8, 10, 11, 14, 17]; along with the experimental data. The predictions of almost all the model calculations available in the literature except a few

Table 1. The quark mass m_q and corresponding quark binding energy E_q along with other relevant model solutions for the potential parameters $(a, V_0) \equiv (0{\cdot}017166\,\text{GeV}^3, -0{\cdot}1375\,\text{GeV})$.

Quark 'q'	m_q (GeV)	E_q (GeV)	λ_q (GeV)	r_{0q} (GeV)$^{-1}$	$\mathcal{N}_q$ (GeV)$^{-1/2}$
u	0·07875	0·47125	0·55	3·20806	0·68901
d	0·07875	0·47125	0·55	3·20806	0·68901
s	0·31575	0·59100	0·90675	2·83114	0·80581
c	1·49276	1·57951	3·07227	2·08674	1·02147
b	4·77659	4·76633	9·54292	1·57185	1·19425

such as ref. [2, 16] indicate that f_B is smaller than both f_π and f_D. The present calculation not only confirms this but also predicts $f_K = 148$ MeV in reasonable agreement with the experimental value. The prediction on f_D and f_{D_s} are also found to be within the experimental error limit [22]. This model calculation in the static limit, like many others [8, 10, 13, 14] also concludes that

$$f_{B_c} > f_{B_s} > f_B;\ f_{D_s} > f_D;\ f_D > f_B;\ f_\pi > f_B.$$

It is, in fact, more reliable to evaluate the ratio R of the decay constants which minimizes the possible model constraints in such calculation. In ref. [17] these ratios as calculated from this model beyond static approximation have been compared with those of several other model calculations providing an overall agreement. In table 3 we provide a comparison between these ratios obtained in the static limit here and those of ref. [3, 4, 6, 8, 10, 11, 13–15, 17], to find that the ratios obtained from both approaches agree each other very well in the decays of heavy-flavored mesons. This vindicates our earlier contention that in the study of heavier meson decays whether one adopts an approach in the static limit or beyond, the final results are almost the same.

However a noticeable discrepancy is observed here in the pionic sector; predicting $f_\pi \simeq 215$ MeV. Though such a prediction on f_π is comparable to that of Hay and Ulehla [$f_\pi = 195$ MeV] and even better than that of Gunduc, Hay and Walters [$f_\pi = 408{\cdot}18$ MeV] in their bag model calculations [12] based on static cavity approximation, it is certainly an over-estimation compared to the previous prediction [17] of this model as well as the experimental value [22]. The failure of the present approach in describing the weak decay of light meson especially the most illusive pion is therefore ascribed to the inherent limitation of the static calculation in this sector. Moreover the quark model description of the pion is that of $q\bar{q}$ bound-state which is different from the pion of PCAC. Admittedly though there exists an apparent dichotomy between the $q\bar{q}$-structure and the Goldstone-boson facet of the pion, a somewhat blended picture [23, 25] can not be totally ruled out. Thus no conclusive remark has been given with regard to real dynamics of the pion constituents consisting of the lightest flavor (up and down) quark–antiquark. Until a clear understanding of such intriguing nature of pion is available through some more complete theoretical formulation, such a simplistic approach as the present one is perhaps quite inadequate to describe the real dynamics involved in this sector and hence the discrepancy in the predicted value for f_π. This also vindicates our contention that the static calculation is not entirely trustworthy in

Table 2. Decay constants of pseudoscalar meson in MeV in comparison with the predictions of other models and the experiment.

Model	f_π	f_K	f_D	f_{D_s}	f_B	f_{B_s}	f_{B_c}
Expt [22]	131·73 ± 0·15	160·6 ± 1·3	< 310	—	—	—	—
Present work	215·2	148·1	108·9	134·6	75·8	96·1	134·9
Relativized quark [8] set 2	60	101	109	129	71	91	133
Relativized quark [8] set 3	79	138	131	175	83	119	204
Potential [8] set 2[b]	100	153	149	160	96	111	141
Potential [17]	138	157	161	205	122	154	221
Potential [14]	139	176	150	210	125	175	425
Bag [10]	178	182	146	166	98	—	—
Bag [11]	—	—	172	196	149	170	255
Lattice [6]	—	—	174 ± 53	234 ± 72	105 ± 34	155 ± 75	—
Lattice [7]	141 ± 21	155 ± 21	282 ± 18	—	183 ± 28	—	—

Table 3. Ratios of the decay constants in comparison with the prediction of other models.

Model	f_B/f_D	f_{B_s}/f_{D_s}	f_{B_s}/f_B	f_{D_s}/f_D
Present work	0·70	0·71	1·27	1·24
Relativized quark [8] set 2	0·65	0·71	1·28	1·18
Relativized quark [8] set 3	0·63	0·68	1·43	1·34
Potential [8] set 2^b	0·64	0·69	1·16	1·07
Potential [17]	0·75	0·76	1·27	1·27
Potential [13]	0·63	0·67	1·26	1·19
Potential [14]	0·83	0·83	1·40	1·40
Factorization [15]	0·68	0·68	1·25	1·25
Bag [10]	0·66	—	—	1·12
Bag [11]	0·87	0·87	1·13	1·14
Sum Rule [3]	1·10	0·92	1·07	1·25
Lattice [4]	0·62	0·70	1·07	1·11
Lattice [6]	0·60	0·66	1·48	1·34

describing the decays of light flavored meson especially the pion. Obviously the ratio f_K/f_π is found to be under-estimated compared to that in ref. [17] and the experiment [22].

On the whole, the predictions of the present model calculations in the static limit in the decays of heavy mesons are in reasonable agreement with those of our previous calculation [17] beyond static limit justifying the applicability of the static calculation in the heavy flavor sector. On the other hand, the quantitative discrepancy observed in the decay of light meson such as the pion is due to the inherent limitation of static approximation in the light (especially the non-strange light)-flavor sector. This has been overcome quite satisfactorily in extending the model application to this sector beyond the static limit [17]. Thus within the working approximation, the present model [17-21], adopting a static calculation on the grounds of simplicity alone, provides a suitable and straight-forward frame-work to study the weak leptonic decays of heavy pseudoscalar mesons.

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