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Chaos in intrinsic motion of nuclei and its role in dissipative collective dynamics

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Abstract. We shall discuss the role of chaotic intrinsic motion in dissipative dynamics of the collective coordinates for nuclear systems. Using the formalism of linear response theory, it will be shown that the dissipation in adiabatic collective motion depends on the degree of chaos in the intrinsic dynamics of a system. This gives rise to a shape dependent dissipation rate for collective coordinates when the intrinsic motion is described by the independent particle model in a nucleus. The shape dependent chaos parameter measuring the degree of chaos in the intrinsic dynamics of the nuclear system will be obtained using the interpolating Brody distribution of nearest neighbour spacings in the single particle energy spectrum. A similar shape dependence is also found to be essential for phenomenological dissipation rates used in fission dynamics calculations.

Keywords. One-body dissipation; nuclear friction; chaos.

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1. Introduction

The chaotic aspects of nuclear dynamics have invoked considerable interest [1–11] during the last decade. The nucleus is a particularly suitable candidate to study chaos because of its rich dynamics displaying both single-particle and collective features, interplay between microscopic and macroscopic degrees of freedom and the possibility of perturbing such a system with varying degrees of violence in a collision. It is worthwhile to clarify at this point that though the nucleus being a many body system of interacting fermions is expected to be fully chaotic, it is always not necessarily so. It may be recalled here that the fluctuation properties of low energy nuclear spectra follow those of eigenvalues of Gaussian Orthonormal Ensemble (GOE) of random matrices [12,13], which were subsequently found [14] to characterize the fluctuation statistics of eigenvalues of systems with nonintegrable classical motion. Fluctuation analyses of rotational spectra were performed [15] to gain insight into the structure of hot rotating nuclei. On the other hand, both chaotic and regular aspects were also found [16] in different regimes of nuclear spectra. Since many nuclear properties can be understood in terms of a relatively few degrees of freedom, the domain of the nuclear dynamics relevent for these properties can display chaotic features in varying degrees. In particular, the short range nature of the nucleon-nucleon interaction makes the mean field description of the nucleus a very good approximation. In its simplest form, independent particles moving in a potential well is known [17] to explain a number of nuclear bulk properties quite

successfully. Thus in the mean field regime, the nuclear dynamics will be regular or chaotic depending on the nature of the mean field. While the single particle motion would be completely regular in a spherical potential, it would be more and more chaotic as the potential is progressively deformed [18], reaching the full chaotic limit for a strongly deformed nuclear shape. In the present article, we shall concern ourselves with the study of chaos in the nuclear single particle dynamics and its role in damping of nuclear collective motions.

Dissipation occurs in nuclear dynamics when the kinetic energy associated with the collective coordinates are irreversibly transferred to the intrinsic (non-collective) degrees of freedom of a nucleus. Such situations arise in the decay of the giant resonances, fission and in collisions between two heavy nuclei. Since collisions provide the opportunity to study the dissipation or damping phenomena under controlled conditions, the early motivation for developing theoretical models for dissipation in nuclei came from damped low energy heavy ion collisions [19]. Gross [20] pioneered the concept of one-body mechanism of dissipation in which the nucleons bouncing off the moving boundary of the time dependent average one-body potential soak energy from the collective motion resulting in a damping force in the collective degrees of freedom. Blocki and his collaborators [21] subsequently derived a simple expression namely the "wall formula" for one body dissipation. The wall formula was also obtained from a formal theory of one body nuclear dissipation based on classical linear response techniques [22]. The wall formula, in conjunction with the "window formula" [21] (to account for the role of nucleon exchange through the neck in a dinuclear system), was found to be quite successful in reproducing a large volume of experimental data of damped heavy ion collisions [23], fusion [24] and fission [21,25].

One of the important assumptions of the wall formula concerns the randomization of the particle motion. It is assumed [21] that the successive collisions of a nucleon with the one body potential give rise to a velocity distribution which is completely random. In other words, a complete mixing in the classical phase space of the particle motion is required. This is normally satisfied for one body potentials whose shapes are rather irregular. It was early realised [21,22] that any deviation from this randomization assumption would give rise to a reduced strength of the wall formula. This happens because the energy transferred to a particle from the time-dependent wall could be partly reversible if the motion of the particle is not completely random. In the present paper, we shall re-examine the classical wall formula in order to distinguish between the reversible and irreversible energy transfers. We shall identify the irreversible energy transfer as the true one-body dissipation. We shall further show that the original wall formula can be modified to describe the irreversible energy transfer. This modification would be obtained in the form of a scaling factor which would account for the degree of irregularity of the system. The modified wall formula thus obtained would be applicable to systems in which the particle motions are not fully randomized. In order to achieve the above, we shall exploit the characteristic features of the phase space of particle motion.

Recent progresses [26] in understanding the nature of dynamical systems have demonstrated that even simple systems possess a rich phase space structure and classical dynamics can be idealized for the two extreme cases of either fully regular (integrable) or fully chaotic (nonintegrable) motions. However, most of the systems of practical interest fall in between the above limiting cases and one has to often deal with "mixed" systems

which display both regular and chaotic features. Arvieu *et al* [3] considered the classical motion of a particle moving in a deformed potential and pointed out the importance of the topology of phase space to characterise such motions. Heiss *et al* [27] studied single particle motion in axially deformed nuclei both classically and quantum mechanically. Considering shapes with quadrupole and higher multipole deformations, they succeeded in explaining the distinguishing features of shell structure in prolate and oblate shapes in terms of the underlying classical dynamics. In their detailed numerical study of classical particles in vibrating cavities of various shapes, Blocki *et al* [9] showed a strong correlation between chaos in classical phase space and the efficiency of energy transfer from the collective to the intrinsic motion. In particular, it was demonstrated in Ref. [9] that while the energy trasfer is much smaller than the wall formula limit in a cavity undergoing quadrupole vibration, it reaches the wall formula limit for higher multipole vibrations. Similar conclusions were also reached when the single particle motion was treated quantum mechanically [10]. As mentioned earlier, these essentially confirm the contention that the wall formula is valid when the intrinsic dynamics is fully chaotic.

In the present paper, we shall review some of our works [8, 18, 28, 29] aimed at a modification of the wall formula in order to make it amenable to mixed systems. It would be argued that while the energy transfer to the particle motion described by the regular part of the classical phase space is reversible, energy flows irreversibly to the particle motion which belongs to the chaotic phase space. We shall subsequently make use of a measure for the degree of chaos or nonintegrability for mixed systems which is defined as the relative volume of the phase space that belongs to the chaotic trajectories. We shall incorporate this measure of nonintegrability into the linear response formulation of the one body dissipation [22]. We would obtain a "scaled wall formula" which would be applicable for any system between a fully regular (no dissipation) and a fully chaotic one (original wall formula dissipation). Finally, we would present some numerical results to illustrate the effect of deformation on one body dissipation. We would further point out that a deformation dependent friction is found to be essential in fission dynamics calculations [30, 31] and the nature of this deformation dependence is very similar to our present theoretical prediction.

In the following sections, we shall first briefly review the classical linear response theory for one body dissipation, developed earlier by Koonin and Randrup [22], and then describe its modification [29] to obtain the scaled wall formula in § 3. In § 4, the scaling parameter or the measure of chaos will be numerically obtained for different deformed shapes. Finally, § 5 will contain a summary and discussions of our results.

2. One body dissipation revisited

We shall first briefly review the classical linear response theory of one body dissipation developed earlier by Koonin and Randrup [22]. The independent particle model of the nucleus will be considered here in which the motion of the individual nucleons in a time-dependent nuclear single particle potential is governed by the one body Hamiltonian $H(\mathbf{r}, \mathbf{p}; t)$. Here $\mathbf{r}$ and $\mathbf{p}$ denote the nucleon position and momentum respectively whereas t denotes the time. The Hamiltonian H may be split into a time independent part H_0 and a

perturbation H_1, arising from the time dependence of H:

$$H(\mathbf{r},\mathbf{p};t) = H_0(\mathbf{r},\mathbf{p}) + H_1(\mathbf{r},\mathbf{p};t) \tag{2.1}$$

with $H_1(\mathbf{r},\mathbf{p};t=0) = 0$.

Under the linear response approximations which require the validity of a perturbative treatment and the assumption that the relaxation time of the intrinsic motion is short in comparison with the time scale for collective motion, the rate of energy dissipation from the collective motion can be expressed as,

$$\dot{Q} = -\int \mathrm{d}\mathbf{r} \int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3} \left(\int_0^\infty \mathrm{d}t' \dot{H}_1(\mathbf{R}_0(\mathbf{r},\mathbf{p};t'),\mathbf{P}_0(\mathbf{r},\mathbf{p};t');t) \right) \dot{H}_1(\mathbf{r},\mathbf{p};t) \times \frac{\partial f_0(\mathbf{r},\mathbf{p})}{\partial H_0} \tag{2.2}$$

This equation corresponds to a physical picture in which a particle originating from a point $(\mathbf{r},\mathbf{p})$ in phase space contributes a dissipation rate equal to the product of the initial impulse received $\dot{H}_1(t)$ and the sum of all impulses received subsequently along its entire (unperturbed) trajectory $(\mathbf{R_0},\mathbf{P_0})$. In the above expression, f_0 is the single particle phase space distribution function governed by the unperturbed Hamiltonian H_0 and the factor $\partial f_0/\partial H_0$ ensures that for a Fermi-Dirac distribution only the particles near the Fermi surface contributes.

The assumption of a adiabatic collective motion required to arrive at Eq. (2.2) was critically examined in ref. [32] where the wall formula was obtained as the low-frequency limit of the one-body RPA damping. It was observed that the relaxation of the ideal adiabatic requirement to realistic collective speeds reduces the damping by 20%–30% Therefore the above eq. (2.2) shall give the leading contribution to one-body damping even when the collective and intrinsic time scales become comparable.

Considering a leptodermous system in which the nuclear potential is uniform throughout the volume but rises steeply at the surface, the time integral in eq. (2.2) can be written as a sum of the impulses received by a particle during its successive encounters with the nuclear surface along its unperturbed trajectory. Separating the contribution of the first impulse given to a particle near its point of origin at $t' = 0$ (local part) from those arising out of the successive reflections from other regions of the nuclear surface (non-local part), Koonin and Randrup obtained the energy damping rate as,

$$\dot{Q} = \dot{Q}_{\mathrm{local}} + \dot{Q}_{\mathrm{non-local}} \tag{2.3}$$

where

$$\dot{Q}_{\mathrm{local}} = \frac{\rho\bar{v}}{4\pi} \int u^2(a) \left(\int \mathrm{d}^2\Omega_a \right) \mathrm{d}^2 a \tag{2.4}$$

and

$$\dot{Q}_{\mathrm{non-local}} = \frac{2\rho\bar{v}}{\pi} \int u(a) \left(\int \mathrm{d}^2\Omega_a \cos^2\theta_a \sum_b u(b) \cos\theta_b \right) \mathrm{d}^2 a \tag{2.5}$$

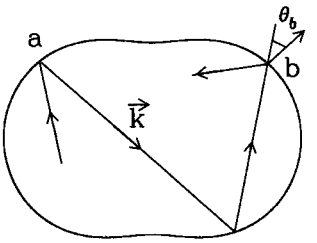

Figure 1. A typical trajectory in a deformed cavity.

In the above equations, we have followed the notations of ref. [22] where k is the momentum of a particle as it leaves the surface point 'a' at an angle Ω_a, $u(a)$ is the normal component of the surface velocity at 'a' and θ_b is the angle between the trajectory and the normal at a point 'b' on the surface (figure 1). The summation is over all the subsequent reflecting points 'b' along the trajectory after the first encounter at 'a'. ρ and $\bar{v}$ are respectively the nuclear mass density and the average nucleon speed inside the nucleus.

For regular systems such as a spherical cavity, it was shown in ref. [22] that the non-local contribution exactly cancels the local part. This essentially corresponds to an energy transfer which is completely reversible. Thus the energy gained by a particle from the wall is eventually fed back to the wall when the particle motion is regular. The regularity in particle motion ensures complete reversibility under adiabatic conditions. An integrable system thus becomes completely non-dissipative in this picture. On the other hand, the non-local part becomes zero for a system which is sufficiently irregular. This can also be seen from the eq. (2.5) where the surface velocity components at successive reflection points are completely random for a cavity of highly irregular shape and therefore cancel out in the summation. This in turn corresponds to a completely irreversible energy transfer. The dissipation rate is thus solely governed by the local term for an irregular system which now reduces to the wall formula,

$$\dot{Q}_{\text{wall}} = \rho\bar{v} \int u^2(a) \mathrm{d}^2 a \tag{2.6}$$

It is to be noted further that the energy damping is also given by the wall formula irrespective of the system being regular or chaotic when the time available for damping is short compared to the interval between successive collisions of a particle with the wall. Such situations may arise when other competing processes reduce the interaction time during which the one-body damping is important. If this interaction time is too short for any possible transfer of particle energy to the wall after the first collision, the net energy transfer rate would be given by the local term or equivalently, the original wall formula.

3. One body dissipation in mixed systems

We have considered in the above the one-body dissipation rates in completely regular or fully chaotic systems. However, most of the physical systems of interest do not belong to

either of the two limits of full regularity or complete chaos. The dynamics of such systems display both the characteristic features of regularity (tori) and chaos (fog) in the classical phase space. To obtain the dissipation rate in such mixed systems, we shall proceed as follows.

We shall first distinguish between the regular and chaotic trajectories which contribute to the Eqs. (2.4) and (2.5) above. Originating from a given point on the cavity wall and moving in a given direction, a regular trajectory closes smoothly in phase space. On the other hand, another trajectory leaving the same point but in a different direction could be a chaotic one which does not close in the phase space. Let Ω_{1a} be the solid angle which contains all the chaotic trajectories originating from the surface point 'a' while Ω_{2a} is the corresponding solid angle for the regular trajectories ($\Omega_{1a} + \Omega_{2a} = 4\pi$). It may be noted here that each of these solid angles could be composed of disjoint components. The Eq. (2.3) then can be written as,

$$\dot{Q} = \frac{\rho\bar{v}}{4\pi}\int u^2(a)\left(\int_{\Omega_{1a}} \mathrm{d}^2\Omega_a\right)\mathrm{d}^2a + \frac{2\rho\bar{v}}{\pi}\int u^2(a) \times \left(\int_{\Omega_{1a}} \mathrm{d}^2\Omega_a \cos^2\theta_a \sum_b u(b)\cos\theta_b\right)\mathrm{d}^2a + \frac{\rho\bar{v}}{4\pi}\int u^2(a)\left(\int_{\Omega_{2a}} \mathrm{d}^2\Omega_a\right)\mathrm{d}^2a + \frac{2\rho\bar{v}}{\pi}\int u^2(a)\left(\int_{\Omega_{2a}} \mathrm{d}^2\Omega_a \cos^2\theta_a \sum_b u(b)\cos\theta_b\right)\mathrm{d}^2a \tag{3.1}$$

The last two terms on the right hand side of this equation represent the local and non-local contributions to the dissipation due to the trajectories which are regular. We shall assume at this point that the net dissipation due to the regular trajectories will be zero, similar to the spherical system whose all the trajectories are regular [22]. Though the non-dissipative nature of a regular system has been demonstrated only for a spherical system and a general proof for any regular system is not available to the best of our knowledge, the latter can be well anticipated on physical grounds and has been numerically confirmed [9] for several integrable systems. The zero dissipation should therefore be a generic property of the phase space of integrable systems and we expect it to hold for that part of the phase space of a mixed system which is similar to that of an integrable system.

We shall next consider the second term on the right side of the Eq. (3.1) which represents the non-local contribution of the chaotic trajectories. We immediately note that this term should also vanish due to the random nature of the surface velocity components at successive reflecting points as has been mentioned in the earlier section. We shall therefore be left with only the local term arising from the chaotic trajectories and obtain,

$$\dot{Q} = \rho\bar{v}\int \tilde{\mu}(a)u^2(a)\mathrm{d}^2a \tag{3.2}$$

where

$$\tilde{\mu}(a) = \frac{\Omega_{1a}}{4\pi}$$

and can be defined as the fraction of the trajectories leaving a surface point 'a' which are chaotic. Since we do not expect any correlation between $\tilde{\mu}(a)$ and $u^2(a)$, the above

equation can be further reduced to,

$$\dot{Q} = \bar{\mu}\dot{Q}_{\text{wall}} \tag{3.3}$$

where

$$\bar{\mu} = \frac{\int \tilde{\mu}(a)\mathrm{d}^2 a}{S}$$

and S is the total surface area. Thus $\bar{\mu}$ is the average fraction of the trajectories which are chaotic when the sampling is done uniformly over the surface.

We have thus arrived at a scaled version of the wall formula where the dissipation rate depends on the degree of chaos in the single particle motion. For a completely irregular system, $\bar{\mu}$ is 1 and we recover the full wall formula whereas for a system with integrable single particle dynamics, there would be no chaotic trajectories ($\bar{\mu} = 0$) and hence no dissipation. For mixed systems, it will be necesary to calculate $\bar{\mu}$ separately to obtain the dissipation.

It is of interest to note here that a quantity (chaoticity/chaos parameter) defined in a similar fashion to $\bar{\mu}$ is often used in the literature [33, 34] to express the degree of irregularity in the dynamics of a system. This parameter, to be denoted henceforth as μ, is defined as the fraction of the phase space occupied by the chaotic trajectories. For a three-dimensional billiard, this essentially reduces to a uniform volume sampling to find the fraction of the trajectories which are chaotic. In what follows, we shall approximate $\bar{\mu}$ by μ to simplify our calculation. In order to justify this approximation, we first note that both the volume and surface samplings give the same limiting values of 0 and 1 respectively for fully regular or completely chaotic systems. Also, both the volume and surface averages would be equal for systems in which the chaotic trajectories are uniformly distributed. Moreover, as a regular cavity is deformed progressivly towards more irregular shapes, both μ and $\bar{\mu}$ also change monotonically from 0 to 1. We therefore do not anticipate any drastic difference between these two measures for mixed systems. However, it would still be of interest to make a precise comparison between μ and $\bar{\mu}$ which we plan to do in future.

4. Chaos parameter from energy spectrum

We have so far considered the chaos parameter μ (or $\bar{\mu}$) as a classically defined quantity which can be calculated by sampling trajectories in the classical phase space. However for the present purpose, we shall obtain the chaos parameter from a measure of the fluctuations of the single particle energy spectrum of the corresponding quantum system. This will be essentially done for two reasons. Firstly, the measure of chaos from quantum spectra is known [35] to be equivalent to the measure from classical phase space and secondly, the ready accessability of the quantum calculation to us. Thus the quantum calculation would give us a measure of chaos in the corresponding classical system. It may be noted that though a quantal formulation of dissipation has been obtained [36] earlier, the generic features of chaos in the quantum spectra has not been incorporated in such a formulation as yet.

For a regular or classically integrable system, it has been proved [37] that in the semiclassical limit, successive energy levels of the corresponding quantum hamiltonian

arrive randomly. This results in a Poisson distribution $P(s)$ for the spacings (s) between the nearest neighbours in the energy spectra. For generic irregular systems with time-reversal symmetry, it has been conjectured [14] that spectral fluctuations are universally reproduced by the Gaussian Orthonormal Ensemble (GOE). For mixed systems, it has been argued [35] that in the semiclassical limit a spectrum should consist of regular and irregular parts that are associated with the classical regular and irregular regions in phase space. This was subsequently confirmed by Bohigas *et al* [38] in their detailed work on manifestation of classical phase structure in quantum systems. Assuming that irregular and regular regions yield energy level sequences with GOE and Poisson spacing distributions, respectively, and that the whole spectrum is generated by a statistically independent superposition of those sequences with respective weight factors μ and $(1-\mu)$, Berry and Robnik [39] obtained a semiclassical formula for $P(s)$ that smoothly interpolated between the Poisson ($\mu = 0$) and GOE ($\mu = 1$) distributions. One can thus extract a value of μ by a least-square fit of this parametrised $P(s)$ to the level-spacing distribution of a given spectrum. However, it has been subsequently pointed out [40] that the Berry–Robnik distribution converges extremely slowly to the semiclassical limit. It was further shown [40] that the phenomenological Brody distribution [41]

$$P_{\text{Brody}}(\mu, s) = as^{\mu}\mathrm{e}^{-bs^{\mu+1}} \tag{4.1}$$

with $a = (\mu+1)b$ and $b = [\Gamma(1+\frac{1}{1+\mu})]^{1+\mu}$ is more appropriate in the near semiclassical regime.

Before we close this section, it is worthwhile to point out that several authors have discussed [42,43] other approaches to obtain smooth interpolation between Poisson and GOE distributions. French *et al* [42] considered increasing symmetry breaking in a given Hamiltonian as the driving mechanism to push the spacing distribution from Poisson to GOE. On the other hand, Lenz *et al* [43] proposed a random matrix ensemble displaying transition from Poisson to GOE spectra. However, an interpolation from Poisson to GOE based on classical phase space arguments is more appropriate for the present purpose. We shall therefore use Brody parametrisation in the present work.

4.1 *Chaos parameter for a two centre shell model*

We have earlier studied the spectral statistics of single particle motion in two centre shell model potentials [8,28]. The total Hamiltonian of a single particle in the combined field of two axially symmetric harmonic oscillators centred at $z_1 < 0$ and $z_2 > 0$ (the position of the barrier is at the origin) joined smoothly by a neck (figure 2) and with a spin-orbit potential is given as:

$$H = T + V + V_{SO} \tag{4.2}$$

where T is the kinetic energy, V denotes the sum of the oscillator and neck potentials and V_{SO} is the spin-orbit interaction. The details of the potentials are given in ref. [28]. The above potential is essentially characterised by the neck parameter $\epsilon = U_2/U_1$ and the separation between the two centres $R = z_2 - z_1$.

A large number of levels (≈ 150) were calculated in Ref. [28] and the best-fit Brody parameter was extracted over a range of values of the separation R. The system

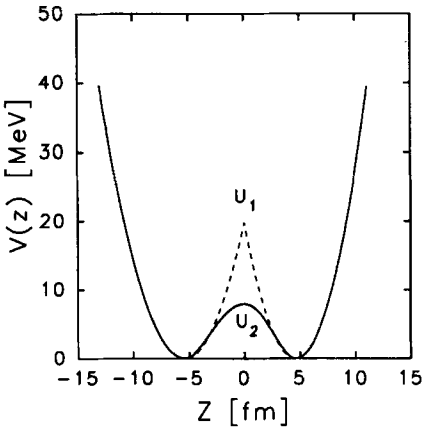

Figure 2. The two centre oscillator potential along the z-axis. The barrier heights without (dotted line) and with (full line) the neck potentials are U_1 and U_2, respectively.

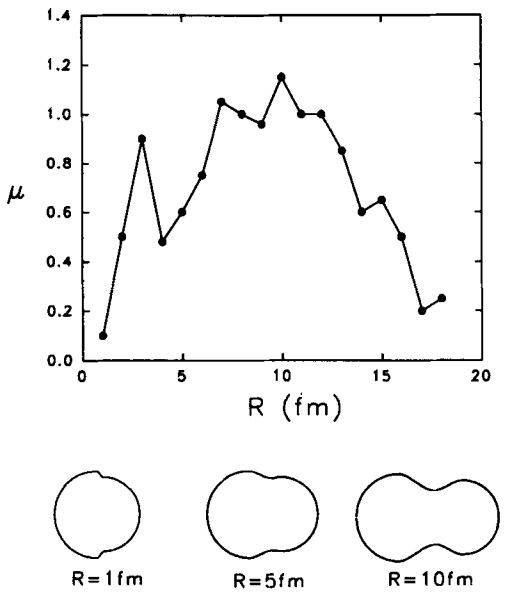

Figure 3. The variation of the chaos parameter μ with separation R for two centre shell potential. Dots (•) denote the calculated values and the lines are to guide the eye. Lower part shows the evolution of the profile of the equipotential surface. The neck parameter is held fixed at $\epsilon = 0.8$ for the cases above.

considered was of mass number $A = 260$ with an asymptotic mass asymmetry of 1.6. Figure 3 shows the variation of the chaos parameter with R. It is observed in this figure that the system is almost regular for near spherical shapes ($R \approx 0$) as expected. With increasing R, the single particle motion becomes increasingly chaotic as the system evolves towards a dinuclear complex. At still larger separations, the neck eventually snaps and two independent spherical nuclei emerge each supporting a regular single

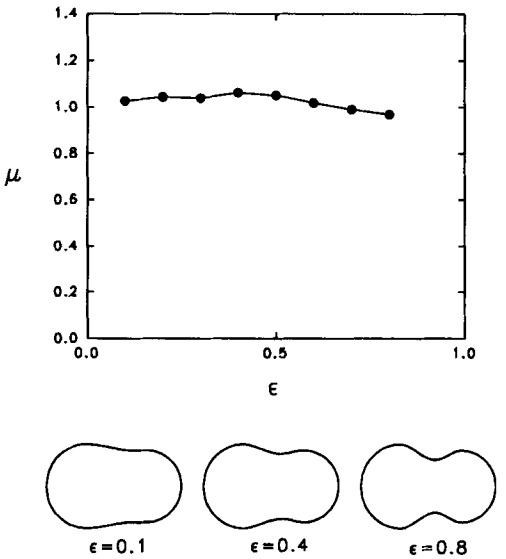

Figure 4. The variation of the chaos parameter μ with the neck parameter ϵ for two centre shell potential. Dots (•) denote the calculated values and the lines are to guide the eye. Lower part shows the evolution of the profile of the equipotential surface. The separation is held fixed at $R = 10$ fm for the cases above.

particle motion. In fission or collisions between two heavy nuclei, the nuclear shape evolves in a manner similar to that depicted in figure 3. Therefore, the one-body dissipation that comes into play in the dynamics of such processes has to be scaled by the chaos parameter for the appropriate shapes. In particular, we note that the system is fully chaotic at $R \approx 10$ fm which corresponds to the touching configuration of two nuclei of masses 100 and 160. Since in damped nuclear collisions, most of the damping occurs around this configuration, a scaling factor close to 1 is appropriate for the one-body dissipation in collisions. This, in turn, justifies the use of the original wall formula of Blocki *et al* in collision dynamics which, in fact, has been quite successful in reproducing [24] a large volume of experimental data.

On the other hand, energy dissipation takes place in fission when the nucleus evolves from a compact shape to the scission configuration. It has been observed from phenomenological studies [30, 31] of fission dynamics that a coordinate dependence of friction is essential for simultaneous reproduction of both the fission probability and the pre-scission neutron multiplicity. A reduced friction at compact shapes followed by a gradual increase upto the scission point was used successfully in the above studies. This shape dependence is exactly similar to our findings as discussed earlier. Therefore, the shape dependence of nuclear friction arising out of the chaoticity of the single particle states offers a possible explanation of the phenomenological [30, 31] shape dependence.

After studying the separation R dependence of the single particle chaos, it is of interest to study how it depends on other shape parameters. In particular, we would be interested to find the dependence of the chaos parameter on the neck parameter ϵ since the negative curvature in the potential profile in the neck region provides a defocussing effect on the single particle motion. With this view, the chaos parameter is obtained for a number of

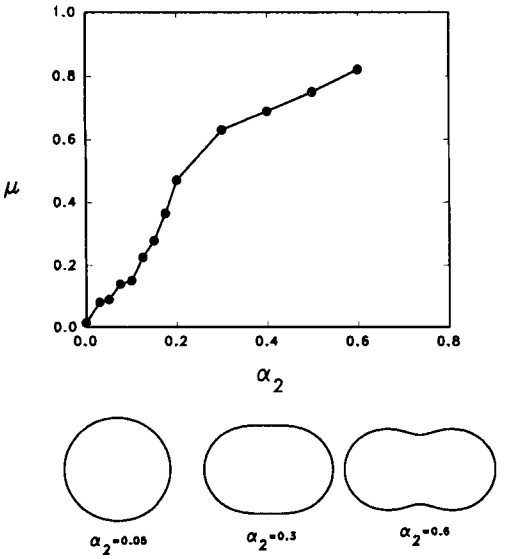

Figure 5. The variation of the chaos parameter μ with deformation α_2 for quadrupole cavities. Dots (•) denote the calculated values and the lines are to guide the eye. Lower part shows the evolution of the cavity shape with increasing deformation.

values of the neck parameter (for a given separation) and the results are given in figure 4. It shows that the single particle motion is fully chaotic over a broad range of the neck potential. This implies that the choice of the neck parameter is not very critical so far as its role in inducing chaos is concerned.

4.2 *Chaos parameter for a deformed cavity*

We shall now consider single particle motion in an axially symmetric cavity with quadrupole deformation. The main motivation for considering this system is twofold. Firstly, we would like to study systems for which larger number of levels can be numerically obtained and secondly, the relevence of quadrupole deformed shapes for the study of giant resonances in nuclei [29]. Specifically, we shall consider a cavity whose surface is deformed according to the Legendre polynomial of second order,

$$R(\theta) = \frac{R_0}{\lambda}[1 + \alpha_2 P_2(\cos\theta)] \tag{4.3}$$

where R_0 is the radius of a sphere having the same volume as the deformed cavity, λ is a volume preserving factor and α_2 is the strength of the quadrupole deformation. The single particle eigenenergies were obtained in ref. [18] by numerically solving the Schrodinger equation. The nearest neighbour level-spacing distributions were obtained from these eigenenergy sequences. The spacing distributions were subsequently fitted by the Brody distribution to yield the chaos parameter μ for cavities of various deformations.

We shall presently extend the above study to a wider range of deformation. A better description of the variation of the chaos parameter with deformation will be obtained here

compared to the two-centre potential system. This is essentially because the chaos parameter is obtained here at smaller intervals of the deformation and larger number of levels (≈ 1000) are employed compared to the two-centre potential calculation. The variation of the chaos parameter with deformation is shown in figure 5. The nature of the variation is similar to the two-centre potential system, namely a regular to chaos transition is observed as the system is progressively deformed. It is of interest to note here that in typical giant quadrupole oscillations of nuclei, the amplitude of the deformation lies within 0.1 and one finds in figure 5 that the chaos parameter is small ($\mu \leq 0.3$) in this range. The one-body mechanism for the damping of giant resonances is, therefore, a very weak mechanism [29].

5. Summary and discussion

In the preceding sections, we have considered the wall formula for one body dissipation by relaxing the 'full randomization' assumption in order to make it applicable to systems in which the mixing in phase space is only partial. In particular, we investigated the effect of irregularity in the shape of the one body potential on the dissipation mechanism. Closely following the theoretical framework of one body dissipation developed earlier by Koonin and Randrup [22], we arrived at a dissipation rate which is a scaled version of the wall formula. We could show that the scaling factor is determined by a measure of chaos in the single particle motion.

Subsequently we calculated this measure for a two-centre potential system and a quadrupole deformed cavity. In both the cases, an order to chaos transition in the single particle motion was observed with increasing deformation of the system. This also corresponds to a gradual increase of the one-body dissipation as the system is progressivly deformed. Since the one-body dissipation is expected to be the dominant mechanism in slow collective processes such as low energy nucleus-nucleus collisions and fission, it is of interest to note that a shape dependent friction is also found to be essential for simultaneous reproduction of both the fission probabilty and the pre-scission neutron multiplicity in fission dynamics calculations [30, 31]. It was found in these works that a phenomenological friction which is very weak for compact configurations and increases after the neck is formed in a fissioning nucleus can describe a large number of experimental data. Interestingly, this corresponds exactly to our own conclusions regarding the shape dependence of nuclear friction. It is thus quite likely that the original wall formula determined from a series of simplifying assumptions may require some readjustments to account for the relaxtion of these assumptions towards more realistic situations. The present article describes some of the efforts in this direction and it is expected that the effects of shape-dependence would serve as an essential input in evolving a better model of one-body dissipation.

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