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Radiative decays of heavy baryons with heavy quark symmetry

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Abstract. The decay widths for the radiative decays of heavy baryons are calculated in the heavy quark effective theory. Introducing the interpolating fields for heavy baryons we obtain the transition matrix elements and the corresponding decay widths. Considering the $SU(6)$ flavor-spin wave functions for heavy baryons, the coupling constants are calculated in the nonrelativistic quark model. Since the masses of the heavy baryons are not available, we have taken the predicted bag model masses. We find our results are quite different from that of the heavy quark bag model calculations.

Keywords. Heavy quark effective theory; interpolating fields.

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1. Introduction

In this brief report we intend to study the radiative decays of heavy baryons in the heavy quark effective theory (HQET) [1–4]. Earlier Cheng *et al* [5] had studied these decays using the basic assumptions of heavy quark symmetry alongwith chiral symmetry. Here we have presented a direct and elegant calculation for the radiative decay widths of the heavy baryons. Although we have adopted the procedure as done in [5], our calculations are more reliable than theirs since we have related the coupling constants to the proton magnetic moment which is measurable to a high accuracy whereas they have evaluated them in terms of the quark magnetic moments. Since the experimental masses of the heavy baryons are still unknown, we have taken the predicted bag model masses [6] for them and obtained the decay widths for their radiative transitions. However, a definite prediction can be made only after the experimental masses are known.

2. Theory

A heavy baryon contains a single heavy quark and two light quarks, which are often referred as diquark. Since each light quark is represented as a triplet in $SU(3)$ flavor space, the diquarks form two different $SU(3)$ multiplets. The diquark in the flavor symmetric sextet has spin 1 and when it combines with the spin of heavy quark gives the spin-1/2 (B_6) and spin-3/2 (B_6^*) baryons. In terms of 3×3 matrix the spin-1/2 baryons are represented as [7]

$$B_6 = \begin{bmatrix} \Sigma_Q^{+1} & \frac{1}{\sqrt{2}}\Sigma_Q^0 & \frac{1}{\sqrt{2}}\Xi_Q'^{1/2} \\ \frac{1}{\sqrt{2}}\Sigma_Q^0 & \Sigma_Q^{-1} & \frac{1}{\sqrt{2}}\Xi_Q'^{-1/2} \\ \frac{1}{\sqrt{2}}\Xi_Q'^{1/2} & \frac{1}{\sqrt{2}}\Xi_Q'^{-1/2} & \Omega_Q \end{bmatrix}. \tag{1}$$

The matrix for B_6^* is similar to B_6. The superscript indicates the value of the isospin quantum number I_3. The diquark in the antisymmetric antitriplet state has spin 0. Thus the combination gives the spin-1/2 baryons ($B_{\bar{3}}$) which are given in terms of an antisymmetric matrix as [7]

$$B_{\bar{3}} = \begin{bmatrix} 0 & \Lambda_Q & \Xi_Q^{1/2} \\ -\Lambda_Q & 0 & \Xi_Q^{-1/2} \\ -\Xi_Q^{1/2} & -\Xi_Q^{-1/2} & 0 \end{bmatrix}. \tag{2}$$

In terms of the interpolating fields [2] these heavy baryons are described as

$$B_{\bar{3}}(v,s) = \bar{u}(v,s)\phi_v h_v, \tag{3}$$

and

$$B_6(v,s,k) = \bar{B}_\mu \phi_v^\mu h_v; \quad k = 1,2, \tag{4}$$

where $k = 1$ for spin-1/2 baryons and $k = 2$ for spin-3/2 baryons. In the above equations ϕ_v and h_v represent the diquark and the heavy quark field respectively. Since the ground state baryons have even parity, the diquarks must also have even parity. Therefore the diquark in $B_{\bar{3}}$ has spin parity 0^+ and represented by the Lorentz scalar ϕ whereas the diquark in B_6 multiplet has spin parity 1^+ and represented by the axial vector field ϕ_μ. The wave functions $\bar{B}_\mu$ as given by Georgi [2] are

$$\bar{B}_\mu(v,s,k=1) = \frac{1}{\sqrt{3}}\bar{u}(v,s)\gamma_5(\gamma_\mu + v_\mu) \tag{5}$$

and

$$\bar{B}_\mu(v,s,k=2) = \bar{u}_\mu(v,s), \tag{6}$$

$u_\mu(v,s)$ and $u(v,s)$ being the Rarita–Schwinger vector spinor field and the usual Dirac spinor respectively, satisfying the relations

$$\not{v}B_\mu(v,s) = B_\mu(v,s) \quad \text{and} \quad v^\mu B_\mu(v,s) = 0. \tag{7}$$

The spin sum of the spinors are taken as

$$\sum_s u_\mu(v,s)\bar{u}_\nu(v,s) = \frac{\not{v}+1}{2}[-g_{\mu\nu} + \tfrac{1}{3}\gamma_\mu\gamma_\nu + \tfrac{1}{3}(\gamma_\mu v_\nu - \gamma_\nu v_\mu) + \tfrac{2}{3}v_\mu v_\nu], \tag{8}$$

and

$$\sum_s u(v,s)\bar{u}(v,s) = \frac{\not{v}+1}{2}. \tag{9}$$

In the HQET the heavy baryons are taken as bound states of a heavy quark and the diquark fields. Hence in the radiative M1 transitions of the heavy baryons the photon is emitted either from the heavy quark or from the light diquark system. For the transition processes $B_6(B_6^*) \to B_{\bar{3}}\gamma$, the diquark system undergoes a spinflip transition and the heavy quark remains as a spectator whereas for $B_6^* \to B_6\gamma$ the photon may be emitted by the heavy quark and the diquark system remains as spectator.

The transition amplitude describing the process $B_i \to B_f\gamma$ is given by

$$\mathcal{M}(B_i \to B_f\gamma) = \varepsilon^\mu \langle B_f | J_\mu^{\text{e.m.}} | B_i \rangle, \tag{10}$$

where ε^μ denotes the polarization vector of the emitted photon. $J_\mu^{\text{e.m.}}$ is the electromagnetic current which can be split into heavy and light quark components, i.e. $J_\mu^{\text{e.m.}} = J_\mu^{\text{Q}} + J_\mu^{\text{q}}$ with the heavy quark current as $J_\mu^{\text{Q}} = e_{\text{Q}}\bar{\text{Q}}\gamma^\mu \text{Q}, e_{\text{Q}}$ is the charge of the heavy quark field Q.

To calculate the transition amplitude for $B_6 \to B_{\bar{3}}\gamma$ process

$$\mathcal{M}(B_6(v,s) \to B_{\bar{3}}(v',s')\gamma) = \varepsilon^\mu \langle B_{\bar{3}}(v',s') | J_\mu^{\text{Q}} + J_\mu^{\text{q}} | B_6(v,s) \rangle, \tag{11}$$

it is essential to evaluate the r.h.s. of eq. (11) for both heavy and light quark sector. Using the interpolating fields for heavy baryons from eqs (3) and (4), we obtain the matrix element for the heavy quark current part as

$$\langle B_{\bar{3}}(v',s') | J_\mu^{\text{Q}} | B_6(v,s) \rangle = e_{\text{Q}} \bar{u}_{\bar{3}}(v',s')\gamma_\mu B^\nu(v,s) \langle 0 | \phi_{v'} \phi_{v\nu}^\dagger | 0 \rangle. \tag{12}$$

In the above equation we have used $\langle 0 | h_{v'} \bar{h}_v | 0 \rangle = ((\not{v} + 1)/2)$, the heavy quark propagator, as in the heavy quark limit the velocity of heavy quark before and after interaction is approximately equal i.e. $v \sim v'$. It has been emphasized earlier that the diquark field $\phi_{v'}$ and $\phi_{v\nu}^\dagger$ are scalar and axial vectors respectively, so $\langle 0 | \phi_{v'} \phi_{v\nu}^\dagger | 0 \rangle$ is an axial vector. Since it is impossible to construct an axial vector out of v and v' only, therefore $\langle 0 | \phi_{v'} \phi_{v\nu}^\dagger | 0 \rangle = 0$, hence the heavy quark current sector does not contribute to the transition amplitude. Now for the light quark sector we have

$$\langle B_{\bar{3}}(v',s') | J_\mu^{\text{q}} | B_6(v,s) \rangle = \langle 0 | \bar{u}_{\bar{3}}(v',s') M_{\mu\nu} B^\nu(v,s) | 0 \rangle, \tag{13}$$

where $M_{\mu\nu} = \langle 0 | \phi_v J_\mu^{\text{q}} \phi_{v\nu}^\dagger | 0 \rangle$. Lorentz invariance implies that

$$M_{\mu\nu} = i\eta \varepsilon_{\mu\nu\alpha\beta} k^\alpha v^\beta, \tag{14}$$

where η is the coupling parameter. With (5), (13) and (14) we obtain the transition amplitude for $B_6 \to B_{\bar{3}}\gamma$ process from (11) as

$$\mathcal{M}(B_6(v,s) \to B_{\bar{3}}(v',s')\gamma) = \frac{i}{\sqrt{3}} \eta \bar{u}_{\bar{3}}(v',s') \sigma_{\mu\nu} k^\mu \varepsilon^\nu u_6(v,s) \tag{15}$$

and the corresponding decay width to be

$$\Gamma(B_6 \to B_{\bar{3}}\gamma) = \frac{1}{\pi} k^3 \left(\frac{\eta}{\sqrt{3}} \right)^2. \tag{16}$$

For $B_6^* \to B_{\bar{3}}\gamma$ processes there is also no contribution to the transition amplitude from the heavy quark current sector as the diquark field is same for B_6 and B_6^* baryons, implying $\langle 0|\phi_{v'}\phi^{\dagger}_{vv}|0\rangle = 0$. For the light quark sector we have

$$\langle B_{\bar{3}}(v',s')|J^{q}_{\mu}|B^{*}_{6}(v,s)\rangle = \bar{u}_{\bar{3}}(v',s')M_{\mu\nu}u^{\nu}(v,s). \tag{17}$$

Substituting the value of $M_{\mu\nu}$ from (14) with η replaced by η_1 we obtain the transition amplitude as

$$\mathcal{M}(B^{*}_{6}(v,s)\to B_{\bar{3}}(v',s')\gamma) = -i\eta_1\varepsilon_{\mu\nu\alpha\beta}\bar{u}_{\bar{3}}(v',s')\varepsilon^{\mu}k^{\nu}\gamma^{\alpha}u^{\beta}(v,s) \tag{18}$$

and the corresponding decay width is

$$\Gamma(B^{*}_{6}\to B_{\bar{3}}\gamma) = \frac{k^3}{12\pi}\eta_1^2\left(3+\frac{m_{\rm f}^2}{m_{\rm i}^2}\right), \tag{19}$$

where $m_{\rm i}$ and $m_{\rm f}$ are the masses of the initial and final baryons in the decay process.

For $B^{*}_{6}\to B_{6}\gamma$ process, the contribution from the heavy quark current sector is given as

$$\langle B_{6}(v',s')|J^{\rm Q}_{\mu}|B^{*}_{6}(v,s)\rangle = e_{\rm Q}\bar{B}^{\alpha}(v',s')\gamma_{\mu}u^{\beta}(v,s)\zeta_{\alpha\beta}(v\cdot v'), \tag{20}$$

where $\zeta_{\alpha\beta}(v\cdot v') = \langle 0|\phi_{v'\alpha}\phi^{\dagger}_{v\beta}|0\rangle$ and in the limit $v\sim v'$, $\zeta_{\alpha\beta}(v\cdot v') = -g_{\alpha\beta}\xi(v\cdot v')$, where $\xi(v\cdot v')$ is the Isgur–Wise function. Equation (20) implies that the heavy quark current does not induce a magnetic coupling and therefore it does not contribute to the radiative M1 transitions. For the light quark current we have obtained the matrix element as

$$\langle B_{6}(v',s')|J^{q}_{\mu}|B^{*}_{6}(v,s)\rangle = \bar{B}^{\alpha}(v',s')M_{\mu\alpha\beta}u^{\beta}(v,s), \tag{21}$$

where $M_{\mu\alpha\beta} = \langle 0|\phi_{v'\alpha}J^{q}_{\mu}\phi^{\dagger}_{v\beta}|0\rangle$. Taking the general expression for $M_{\mu\alpha\beta}$ from ref. [5] as

$$M_{\mu\alpha\beta} = f_1 g_{\alpha\beta}(v+v')_{\mu} + f_3(g_{\mu\alpha}k_{\beta} - g_{\mu\beta}k_{\alpha}) + f_5 v_{\alpha}v'_{\beta}(v+v')_{\mu}, \tag{22}$$

we obtain

$$\langle B_{6}(v',s')|J^{q}_{\mu}|B^{*}_{6}(v,s)\rangle = \frac{f_3}{\sqrt{3}}\bar{u}(v',s')\gamma_5(\gamma_{\mu}k_{\beta} - g_{\mu\beta}\not{k})u^{\beta}(v,s). \tag{23}$$

Hence the transition amplitude for the process $B^{*}_{6}\to B_{6}\gamma$ is given by

$$\mathcal{M}(B^{*}_{6}(v,s)\to B_{6}(v',s')\gamma) = \frac{if_3}{\sqrt{3}}\bar{u}(v',s')\gamma_5\gamma^{\nu}F_{\nu\beta}u^{\beta}(v,s), \tag{24}$$

where $F_{\nu\beta}$ is the electromagnetic field strength tensor i.e. $F_{\nu\beta} = i(k_{\nu}\varepsilon_{\beta} - k_{\beta}\varepsilon_{\nu})$. Using eq. (8) we obtain the decay width to be

$$\Gamma(B^{*}_{6}\to B_{6}\gamma) = \frac{k^3}{12\pi}\left(\frac{f_3}{\sqrt{3}}\right)^2\left(3+\frac{m_{\rm f}^2}{m_{\rm i}^2}\right), \tag{25}$$

with $m_{\rm i}$ and $m_{\rm f}$ are the masses of the initial and final baryons in the decay process. Thus the general expression for the decay widths $B_6\to B_{\bar{3}}\gamma$, $B^{*}_{6}\to B_{\bar{3}}\gamma$ and $B^{*}_{6}\to B_{\bar{6}}\gamma$ are given in eqs (16), (19) and (25) respectively. The coupling constants η, η_1 and f_3 present in the expressions are calculated considering the nonrelativistic quark model. In this model the M1 transitions are described by the spin operator of the quarks given as $M = \Sigma_{\rm q}\mu_{\rm q}e_{\rm q}\sigma_{\rm q}$, where $e_{\rm q}$ and $\sigma_{\rm q}$ are the charge and spin operator of the quarks. $\mu_{\rm q}$

denotes the quark scale magnetic moment which is identical with the proton magnetic moment for u and d quarks, while for s quark it is taken as $\mu_s = (3/5)\mu_p$ [8]. For the heavy quarks we have taken $\mu_Q = (1/2m_Q)$. Considering the $SU(6)$ flavor-spin wave functions for the heavy baryons from ref. [5] we obtain the transition matrix elements of the spin operator for various possible M1 transitions. Since we have used the flavor-spin wave functions for the baryons to calculate the above matrix elements, the results obtained are equal to the couplings in (15), (18) and (24) which are given as

$$\frac{\eta}{\sqrt{3}}(\Sigma_Q^0 - \Lambda_Q) = -\frac{1}{\sqrt{3}}(\mu_u e_u - \mu_d e_d),$$

$$\frac{\eta}{\sqrt{3}}(\Xi_Q'^{1/2} - \Xi_Q^{1/2}) = -\frac{1}{\sqrt{3}}(\mu_u e_u - \mu_s e_s),$$

$$\frac{\eta}{\sqrt{3}}(\Xi_Q'^{-1/2} - \Xi_Q^{-1/2}) = -\frac{1}{\sqrt{3}}(\mu_d e_d - \mu_s e_s),$$

$$\eta_1(\Sigma_Q^{0*} - \Lambda_Q) = \frac{2}{\sqrt{6}}(\mu_u e_u - \mu_d e_d),$$

$$\eta_1(\Xi_Q'^{*1/2} - \Xi_Q^{1/2}) = \frac{2}{\sqrt{6}}(\mu_u e_u - \mu_s e_s),$$

$$\eta_1(\Xi_Q'^{*-1/2} - \Xi_Q^{-1/2}) = \frac{2}{\sqrt{6}}(\mu_d e_d - \mu_s e_s),$$

$$\frac{f_3}{\sqrt{3}}(\Sigma_Q^{*+1} - \Sigma_Q^{+1}) = \frac{2\sqrt{2}}{3}(\mu_u e_u - \mu_Q e_Q),$$

$$\frac{f_3}{\sqrt{3}}(\Sigma_Q^{*0} - \Sigma_Q^0) = \frac{\sqrt{2}}{3}(\mu_u e_u + \mu_d e_d - 2\mu_Q e_Q),$$

$$\frac{f_3}{\sqrt{3}}(\Xi_Q'^{*1/2} - \Xi_Q'^{1/2}) = \frac{\sqrt{2}}{3}(\mu_u e_u + \mu_s e_s - 2\mu_Q e_Q),$$

$$\frac{f_3}{\sqrt{3}}(\Xi_Q'^{*-1/2} - \Xi_Q'^{-1/2}) = \frac{\sqrt{2}}{3}(\mu_d e_d + \mu_s e_s - 2\mu_Q e_Q)$$

and

$$\frac{f_3}{\sqrt{3}}(\Omega_Q^* - \Omega_Q) = \frac{2\sqrt{2}}{3}(\mu_s e_s - \mu_Q e_Q). \tag{26}$$

3. Results and discussion

Here we have estimated the radiative decay widths for different radiative processes. The masses of the heavy quarks are taken as $m_c = 1{\cdot}6\,\mathrm{GeV}$ and $m_b = 5\,\mathrm{GeV}$. The masses of the Σ_c, Λ_c, Ξ_c and Λ_b baryons are taken from ref. [9] and the other heavy baryon masses from ref. [6]. The proton magnetic moment is taken as $\mu_p = 2{\cdot}793$ nm. The centre of mass momentum of the photon k is obtained from the rest frame of the decaying baryon. Substituting the value of the coupling constants from eq. (26) in the expressions for the partial decay widths (16), (19) and (25) we tabulate the results in table 1 and compare them with the results of Izzat *et al* [6]. We observe that our results are quite different from them. Since there is no model dependent parameter in our calculation we expect our results to be more appreciative than that of ref. [6].

In the present investigation we have estimated the radiative decay widths of heavy baryons in heavy quark effective theory. Using the interpolating fields for the heavy

Table 1. The radiative decay widths for the heavy baryons in the HQET as well as in bag model.

$B_i \to B_f \gamma$	k in MeV	$\Gamma(B_i \to B_f \gamma)$ in keV (Present work)	$\Gamma(B_i \to B_f \gamma)$ in keV (from heavy quark bag model [6])
$\Sigma_c^+ \to \Lambda_c^+ \gamma$	162·90	93·17	17
$\Sigma_b \to \Lambda_b \gamma$	137·33	55·82	53
$\Xi_c'^+ \to \Xi_c^+ \gamma$	34·66	0·674	4
$\Xi_c'^0 \to \Xi_c^0 \gamma$	29·52	0·009	0·07
$\Xi_b'^0 \to \Xi_b^0 \gamma$	108·97	20·95	20
$\Xi_b'^- \to \Xi_b^- \gamma$	108·97	0·496	0·3
$\Sigma_c^{*+} \to \Lambda_c^+ \gamma$	194·47	104·676	177
$\Sigma_b^* \to \Lambda_b \gamma$	176·25	77·48	107
$\Xi_c'^{*+} \to \Xi_c^+ \gamma$	131·4	23·87	66
$\Xi_c'^{*0} \to \Xi_c^0 \gamma$	126·46	0·504	1
$\Xi_b'^{*0} \to \Xi_b^0 \gamma$	148·11	34·63	50
$\Xi_b'^{*-} \to \Xi_b^- \gamma$	148·11	0·82	0·7
$\Sigma_c^{*++} \to \Sigma_c^{++} \gamma$	36·63	0·252	9
$\Sigma_c^{*+} \to \Sigma_c^+ \gamma$	35·94	$0{\cdot}598 \times 10^{-3}$	0·08
$\Sigma_b^{*+} \to \Sigma_b^+ \gamma$	39·86	0·574	0·8
$\Sigma_b^{*0} \to \Sigma_b^0 \gamma$	39·86	0·043	0·06
$\Xi_c'^{*+} \to \Xi_c'^+ \gamma$	98·08	0·154	0·3
$\Xi_c'^{*0} \to \Xi_c'^0 \gamma$	98·08	0·93	4
$\Xi_b'^{*0} \to \Xi_b'^0 \gamma$	39·87	0·079	0·09
$\Xi_b'^{*-} \to \Xi_b'^- \gamma$	39·87	0·072	0·1
$\Omega_c^* \to \Omega_c \gamma$	98·15	2·056	3
$\Omega_b^* \to \Omega_b \gamma$	39·87	0·038	0·06

baryons we calculate the decay widths for various possible radiative transitions. The decay widths so calculated contain unknown couplings, which are calculated in the nonrelativistic quark model using the $SU(6)$ flavor-spin wave functions of the baryons. The masses of the heavy baryons are taken from Izatt *et al* [6] as their experimental masses are not available. However there is an uncertainty in the predicted widths which may be removed after experimental values for the baryon masses are available. Although we have followed the procedure of Cheng *et al*, the main difference between our calculation and that of theirs is as follows. We have related the effective coupling constant of the $B_i B_f \gamma$ vertex in the HQET to the proton magnetic moment which is directly measurable to a high accuracy, whereas they have related the couplings to the quark magnetic moments which are not directly observed. We therefore feel that our calculation is more reliable in the spirit of effective field theory where one tries to obtain the branching ratio predictions in terms of directly observable quantities.

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