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Nonadiabatic gravitational collapse in multidimensional spacetime

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Abstract. We extend to higher dimensions an earlier work of Santos regarding junction conditions for a spherical fluid distribution with heat flux and an electromagnetic field. It is observed that the pressure at the surface of distribution does not vanish when the heat flow is present.

Keywords. Gravitational collapse; higher dimensional spacetime; heat flow.

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1. Introduction

Finding a theory that unifies gravity with other forces in nature remains an elusive goal in quantum field theory. Most recent efforts in this search have been directed at theories in which the dimensions of space-time is greater than the $(3+1)$ of the world that we observe. Further, the advances concerning supergravity in 11 dimensions and superstrings in 10 dimensions indicate that the multidimensionality of space is apparently a fairly adequate reflection of the dynamics of interaction over the distances $r \ll 10^{-16}$ cm where unification of all types of forces is possible [1].

Several attempts have been made to formulate and solve equations of gravitational collapse of a sphere containing heat flow and outgoing radiation flux. Misner and Sharp [2] considered the gravitational collapse problem with energy transport by heat flow. The matching of an interior metric representing an outgoing heat flux is not possible with the exterior Schwarzschild space time for obvious reasons. The matching conditions and other allied matters for non adiabatic flow have been extensively discussed in the literature by de Oliveira and Santos [3] where it is found that when one takes a Reissner–Nordstrom–Vaidya exterior for matching, one gets the interesting result that the pressure of the fluid in the interior does not vanish at the boundary in this case – rather it is related to the heat flow. It may now appear worthwhile to study the above scenario when one introduces space-like extra dimensions particularly in view of the renewed interest in higher-dimensional theories as well as the urgency of taking into account radiation from localized bodies (particularly starlike objects in astrophysics).

We have already studied in the literature the higher dimensional generalization of Schwarzschild and Vaidya type radiating solutions, which are characterized by spherical symmetry, [4–6] and one can use them for the exterior space-time.

In the present paper, junction conditions are studied at the boundary of a gravitating fluid sphere under very general conditions in higher dimensional space time. It is further assumed that the fluid has some excess charge giving rise to an electric field in the radial direction. We obtain expressions of the extrinsic curvature and the mass function in $(n+2)$ dimensions and to our knowledge such study has not been done so far.

As mentioned earlier, the pressure at the boundary does not vanish when energy momentum tensor contains heat flow. Section 2 addresses primarily the query whether one arrives at the same result when the topology of space time is R^{2+n} and we have spherical symmetry in n-dimensions including the extra ones. Here we get the answer that the dimensionality of space and also the presence of an electric field, does not change the four-dimensional result referred to earlier. It is to be noted that the same boundary conditions can be obtained if one use Synge–O' Brien [7] or Lichnerowicz [8] junction conditions.

2. Interior and exterior higher dimensional space-time and junction conditions

We follow the method of Israel [9] in matching the interior with the exterior at the boundary. We consider an $(n+2)$-dimensional spherically symmetric distribution with its motion described by a time-like $(n+1)$ space Z. The time-like $(n+1)$-space z divides the interior from the exterior and the dividing hypersurface distinguishes between the two $(n+2)$-dimensional manifolds V^- and V^+; both of which contain Z as a part of their boundaries.

The intrinsic metric to z is

$$ds^2 = g_{ij}\mathrm{d}\lambda^i \mathrm{d}\lambda^j = -\mathrm{d}\tau^2 + R^2(\tau)\mathrm{d}X_n^2, \tag{1}$$

where $\mathrm{d}X_n^2 = \mathrm{d}\theta_1^2 + \mathrm{d}\theta_2^2 \sin^2\theta_1 + \cdots + \sin^2\theta_1 \sin^2\theta_2 \cdots \sin^2\theta_{n-1}\mathrm{d}\theta_n^2$ and the co-ordinates λ^i being $\tau, \theta_1, \ldots, \theta_n$.

The interior metric in $(n+2)$-dimensional space time is taken as

$$ds_-^2 = -A^2\mathrm{d}t^2 + B^2(\mathrm{d}r^2 + r^2\mathrm{d}X_n^2), \tag{2}$$

where A, B and R are functions of the radial co-ordinate 'r' and time 't' and it contains a charged dissipative fluid characterized by radiation and heat flow in the radial direction.

For the exterior spacetime V^+ described by higher-dimensional Reissner–Nordstrom–Vaidya metric [5, 6] which represents radiating Vaidya metric expressed in the null form and including the charge parameter as the source of the electromagnetic field is given by

$$ds_+^2 = -\left[1 - \frac{2m(v)}{(n-1)\bar{r}^{n-1}} + \frac{Q^2}{(n-1)\bar{r}^{2n-2}}\right]\mathrm{d}v^2 - 2\mathrm{d}v\mathrm{d}\bar{r} + \bar{r}^2\mathrm{d}X_n^2, \tag{3}$$

where v is the retarded time co-ordinate.

In order to have a unique intrinsic geometry at the boundary hypersurface Z, both $g^-_{\alpha\beta}$ and $g^+_{\alpha\beta}$ must include the same intrinsic metric on Z, so that

$$(\mathrm{d}s_-)^2_Z = (\mathrm{d}s^2_+)_Z = \mathrm{d}S^2_Z. \tag{4}$$

The above condition guarantees the continuity of the metric components across the boundary Z and equations of Z in $V^\pm$ are given by

$$x^\alpha_\pm = x^\alpha_\pm(\lambda^i). \tag{5}$$

The second continuity condition imposed on z takes the form

$$[K_{ij}] = K^+_{ij} - K^-_{ij} = 0. \tag{6}$$

The extrinsic curvature components as given by Eisenhart are expressed as

$$K^\pm_{ij} = -n^\pm_\alpha \frac{\partial^2 x^\alpha_\pm}{\partial\lambda^i\partial\lambda^j} - n^\pm_\alpha \Gamma^\alpha_{ab} \frac{\partial x^a_\pm \partial x^b_\pm}{\partial\lambda^i\partial\lambda^j}. \tag{7}$$

The equation of the surface which is the boundary Z will have the equation in the interior-co-ordinates

$$f(r,t) = r - r_Z = 0, \tag{8}$$

where r_Z is a constant, being the radial co-ordinates at the boundary in the co-moving system. Since $\partial f/\partial x^\alpha_-$ is a vector orthogonal to Z, the unit normal vector to this surface in the co-ordinates x^α_- will be

$$n^-_\alpha = (0, B(r_Z,t), 0, 0\ldots). \tag{9}$$

The unit tangent vector to Z becomes

$$1^-_\alpha = (A^2 t, 0, 0\ldots). \tag{10}$$

Using equations (2), (7) and (9), we can calculate the following components of the extrinsic curvature

$$K^-_{\tau\tau} = \left(-\frac{\partial A}{\partial r}\middle| AB\right)_Z, \tag{11}$$

$$K^-_{\theta_1\theta_1} = \frac{1}{\sin^2\theta_1} K^-_{\theta_2\theta_2} \equiv \left(r\frac{\partial(Br)}{\partial r}\right)_Z, \tag{12}$$

$$K^-_{ij} = 0, \quad i \neq j, \tag{13}$$

where τ stands for the proper time measured along the stream lines. We consider the junction condition (4) and comparing (1) with (2) with $\mathrm{d}r = 0$, we obtain

$$A(r_z,t)\dot{t} = 1, \tag{14}$$

$$r_z B(r_z,t) = R_z(\tau), \tag{15}$$

where overhead dot represents differentiation with respect to τ. As mentioned earlier the interior space-time V^- is due to a spherically symmetric distribution of a charged fluid with radial heat flow. Hence in our problem. we have two energy momentum tensors, one is matter energy momentum tensor and the other is electromagnetic energy tensor.

The matter energy momentum tensor $T_{\alpha\beta}$ is given by

$$T_{\alpha\beta} = (\rho + p)v_\alpha v_\beta + p g_{\alpha\beta} + q_\alpha v_\beta + q_\beta v_\alpha, \tag{16}$$

where ρ is the energy density of the fluid, p the isotropic fluid pressure, v_α is the $(n+2)$-velocity and g_α is the radial heat flux such that

$$v^\alpha = \frac{1}{A}\delta^\alpha_0, \tag{17}$$

$$q^\alpha = (0, q, 0, 0 \ldots). \tag{18}$$

The electromagnetic energy tensor is given by

$$E_{\alpha\beta} = \frac{1}{4\pi}\left[F^\nu_\alpha F_{\beta\nu} - \frac{1}{4}g_{\alpha\beta}F^{\nu\delta}F_{\nu\delta}\right], \tag{19}$$

where $F_{\alpha\beta}$ is the electromagnetic field tensor. Maxwell's equations are

$$F_{\alpha\beta} = \varphi_{\beta,\alpha} - \varphi_{\alpha,\beta}, \tag{20}$$

$$F^{\alpha\beta}_{;\beta} = 4\pi J^\alpha, \tag{21}$$

where φ_α is the $(n+2)$ potential and J_α is the $(n+2)$ current. As we have used co-moving co-ordinate system, there will be no magnetic field present in this local co-ordinate system and thus the $(n+2)$ potential and $(n+2)$ current can be given by

$$\varphi_\alpha = [\varphi(r,t), 0, 0 \ldots], \tag{22}$$

$$J^\alpha = \sigma v^\alpha, \tag{23}$$

where σ is the charge density.

Einsteins's field equation is

$$G^-_{\alpha\beta} = (T_{\alpha\beta} + E_{\alpha\beta}). \tag{24}$$

Using eq. (22), we get the non-vanishing component of (20) as

$$F_{01} = -F_{10} = -\frac{\partial\varphi}{\partial r}. \tag{25}$$

Putting (23), (25) in Maxwell's equation, straight-forward calculation yields

$$\frac{\partial^2\varphi}{\partial r^2} + \left(\frac{1}{A}\frac{\partial A}{\partial r} + \frac{(n+1)}{B}\frac{\partial B}{\partial r} + \frac{n}{r}\right)\frac{\partial\varphi}{\partial r} = 4\pi A B^2 \sigma \tag{26}$$

and

$$\frac{\partial}{\partial t}\left(\frac{1}{A^2B^2}\frac{\partial\varphi}{\partial r}\right) + \left(\frac{1}{A}\frac{\partial A}{\partial t} + \frac{(n+1)}{B}\frac{\partial B}{\partial t}\right)\frac{1}{A^2B^2}\frac{\partial\varphi}{\partial r} = 0. \tag{27}$$

From the above equations (26) and (27), we get after integration

$$\frac{\partial\varphi}{\partial r} = \frac{A}{B}\frac{l(r)}{r^n}, \tag{28}$$

where

$$l(r) = 4\pi\int_0^r \sigma B^{(n+1)} r^n \mathrm{d}r. \tag{29}$$

Outside the spherical surface Z, observe that σ vanishes and thus $l(r)$ given by (29) becomes a constant Q. At large distance from the source we have from (28), $\partial\varphi/\partial r = Q/r^n$. Hence we conclude that $l(r)$ is the total charge distributed up to the radius r.

The non vanishing components of Einstein's field equation (24) with (17), (18) and (28) and metric (2) are given by

$$G^-_{00} = -\frac{n(n+1)}{2}\frac{\dot{B}^2}{B^2} + \frac{A^2}{B^2}\left(n\frac{B''}{B} + \frac{n(n-3)}{2}\frac{B'^2}{B^2} + \frac{n^2 B'}{Br}\right)$$
$$= -\left(\rho A^2 + \frac{A^2}{B^4}\frac{l^2(r)}{r^{2n}}\right), \tag{30}$$

$$G^-_{11} = n\frac{A'B'}{AB} + n\frac{A'}{Ar} + \frac{n(n-1)}{2}\frac{B'^2}{B^2} + \frac{n(n-1)B'}{Br} + \frac{B^2}{A^2}$$
$$\times\left(-n\frac{\ddot{B}}{B} - n\frac{n-1}{2}\frac{\dot{B}^2}{B^2} + \frac{n\dot{A}\dot{B}}{AB}\right)$$
$$= \left(pB^2 - \frac{l^2}{B^2 r^{2n}}\right), \tag{31}$$

$$G^2_2 = G^3_3 \cdots = G^n_n = \frac{1}{B^2}\left(\frac{A''}{A} + (n-1)\frac{B''}{B} + (n-1)\frac{A'}{Ar} + \frac{(n-1)(n-4)}{2}\frac{B'^2}{B^2}\right.$$
$$\left. + (n-1)^2\frac{B'}{Br} + (n-2)\frac{A'B'}{AB}\right) - \frac{1}{A^2}\left(n\frac{\ddot{B}}{B} + \frac{n(n-1)}{2}\frac{\dot{B}^2}{B^2} - \frac{n\dot{A}\dot{B}}{AB}\right)$$
$$= \left(p - \frac{1}{B^4}\frac{l^2}{r^{2n}}\right), \tag{32}$$

$$G^-_{01} = -\frac{n\dot{B}'}{B} + \frac{n\dot{B}B'}{B^2} + \frac{nA'\dot{B}}{AB} = -qB^2A. \tag{33}$$

When $n = 2$, the above field equations reduce to four dimensional case. The isotropy of the pressure gives the relation

$$\left(\frac{A'}{A} + (n-1)\frac{B'}{B}\right)' = \left(\frac{A'}{A} + \frac{B'}{B}\right)^2 - \frac{1}{r}\left(\frac{A'}{A} + (n-1)\frac{B'}{B}\right)$$
$$+ 2\left(\frac{A'}{A}\right)^2 - (n-2)\left(\frac{B'}{B}\right)^2. \tag{34}$$

As mentioned earlier the exterior space-time V^+ described by higher dimensional Reissner–Nordstron–Vaidya metric represents spherically symmetric metric with electric field and out going radial flux of unpolarized radiation given in equation (3). The equation of Z in the exterior Vaidya co-ordinate is

$$f(\bar{r}, v) = \bar{r} - r_z(v) = 0 \tag{35}$$

and the unit normal vector on z in the exterior space-time is

$$n^+_\alpha = \left[1 - \frac{2m}{(n-1)\bar{r}^{n-1}} + \frac{2\mathrm{d}\bar{r}_2}{\mathrm{d}v} + \frac{Q^2}{(n-1)\bar{r}^{2n-2}}\right]\left(-\frac{\mathrm{d}\bar{r}_Z}{\mathrm{d}v}, 1, 0, 0\ldots\right). \tag{36}$$

The junction condition (4) yields

$$\bar{r}_Z(v) = R(\tau) \tag{37}$$

and

$$\left(\frac{d\tau}{dv}\right)^2_z = \left[1 - \frac{2m}{(n-1)\bar{r}^{n-1}} + 2\frac{d\bar{r}_z}{dv} + \frac{Q^2}{(n-1)\bar{r}^{2n-2}}\right]. \tag{38}$$

Using (38) we can write unit-normal vector as

$$n^+_\alpha = (-\dot{\bar{r}}, \dot{v}, 0, 0, \ldots)_Z. \tag{39}$$

The unit tangent vector to z is

$$1^+_\alpha = \left(\dot{\bar{r}} - \frac{1}{\dot{v}}, -\dot{v}, 0, 0, \ldots\right). \tag{40}$$

The extrinsic curvature to z is calculated with the help of equations (7), (3), (38) and (39) and we get

$$K^+_{\tau\tau} = \left[\frac{\ddot{v}}{\dot{v}} - \dot{v}\left(\frac{m}{\bar{r}^n} - \frac{Q^2}{\bar{r}^{2n-1}}\right)\right]_z, \tag{41}$$

$$K^+_{\theta\theta} = \frac{1}{\sin^2\theta} K^+_{\varphi\varphi} = \left[\dot{v}\bar{r}\left(1 - \frac{2m}{(n-1)\bar{r}^{n-1}} + \frac{Q^2}{(n-1)\bar{r}^{2n-2}}\right) + \dot{\bar{r}}\bar{r}\right], \tag{42}$$

$$K^+_{ij} = 0 \text{ for } i \neq j. \tag{43}$$

The Einstein tensor for the metric (3) is

$$G^+_{\alpha\beta} = (G^+_{\alpha\beta})_{\text{rad.}} + (G^+_{\alpha\beta})_{\text{el.}}, \tag{44}$$

$$(G^+_{\alpha\beta})_{\text{rad.}} = -\frac{n}{\bar{r}^n}\frac{dm}{dv}\delta^0_\alpha\delta^0_\beta, \tag{45}$$

$$(G^+_{\alpha\beta})_{\text{el.}} = \frac{Q^2}{r^{2n}}\left[1 - \frac{2m}{(n-1)\bar{r}^{n-1}} + \frac{Q^2}{(n-1)\bar{r}^{2n-2}}\right]\delta^0_\alpha\delta^0_\beta + \frac{Q^2}{\bar{r}^{2n}}\delta^0_\alpha\delta^1_\beta + \frac{Q^2}{\bar{r}^n}(\delta^2_\alpha\delta^2_\beta + \sin^2\theta_1\delta^3_\alpha\delta^3_\beta + \cdots + \sin^2\theta_1\cdots\sin^2\theta_n\delta^n_\alpha\delta^n_\beta). \tag{46}$$

Now from the second continuity condition at the boundary $K^+_{ij} = K^-_{ij}$ and comparing $K_{\theta\theta}$ at the boundary z from (42) and (12), we get

$$\left[r\frac{\partial(Br)}{\partial r}\right]_Z = \left[\dot{v}\bar{r}\left(1 - \frac{2m}{(n-1)\bar{r}^{n-1}} + \frac{Q^2}{(n-1)\bar{r}^{2n-2}}\right) + \dot{\bar{r}}\bar{r}\right]_z. \tag{47}$$

With the help of (14), (15), (37) and (38) we get an expression for higher dimensional mass function as

$$m_z = (n-1)(B\bar{r})^{n-1}\left[\frac{\bar{r}^2\dot{B}^2}{2A^2} - \bar{r}\frac{B'}{B} - \frac{\bar{r}^2B'^2}{B^2} + \frac{Q^2}{2(n-1)(\bar{r}B)^{n-1}}\right]_Z. \tag{48}$$

Again comparing (41), (11) and (47), (48) yield the important relations

$$[-(\partial A/\partial r)/AB]_z = \left[\frac{\ddot{v}}{v} - \dot{v}\left(\frac{m}{\bar{r}^n} - \frac{Q^2}{\bar{r}^{2n-1}}\right)\right]_Z, \tag{49}$$

$$p_z = (qB)_z. \tag{50}$$

Relation (50) clearly indicates that in the absence of dissipation such as heat flow, the boundary condition reduces to simply $p_z = 0$, but in this case the radiation cannot exist and the exterior space-time V^+ is the higher dimensional Reissner–Nordstron space-time.

In other words, the present analysis extends to higher dimensions an important observation of Santos in 4D case that for a non-adiabatic heat flow pressure does not vanish at the boundary.

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