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A model of the universe with decaying vacuum energy

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Abstract. The consequences of taking the total active gravitational mass of the universe phasewise constant together with a decaying vacuum energy in the background of Robertson–Walker space-time are investigated. The model so determined admits a contracted Ricci-collineation along the fluid flow vector v^i. It is geometrically closed but ever-expanding and does not possess the initial singularity, horizon, entropy, monopole or cosmological constant problems of the standard big bang cosmology. Estimates of the present matter; radiation and vacuum energy densities, the age of the universe and the present values of the deceleration parameter and the scale factor are also obtained.

Keywords. RW space-time; cosmological constant-time variation; vacuum energy; active gravitational mass.

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1. Introduction

In recent times the cosmological constant Λ has interested theoreticians and observers for various reasons [1]. The non-trivial role of the vacuum in the early universe generates a Λ-term in the Einstein field equations that leads to the inflationary phase [2]. The inflationary cosmology postulates that during an early exponential phase, the vacuum energy was a large cosmological constant. Therefore, in view of the smallness of the cosmological constant observed at present, it is natural to assume that the cosmological constant Λ, representing the energy density of vacuum, is a variable dynamic degree of freedom which being initially very large relaxes to its small present value in an expanding universe. The idea of a dynamically decaying cosmological constant with cosmic expansion has been considered by several authors in the past few years [3–8]. Linde [9] has proposed that Λ is a function of temperature and is related to the process of broken symmetries. Gasperini [6] in this regard, argues that the cosmological constant Λ can also be interpreted as a measure of the temperature of cosmic vacuum which should decrease, like the radiation temperature, with cosmic expansion.

In the present paper, a time-varying cosmological constant representing the energy density of vacuum is considered within the framework of general relativity. Motivation is given for taking the total active gravitational mass of an isotropic and homogeneous universe phasewise constant and a model of the universe is investigated. It is found that the vacuum decays differently in the different phases of evolution depending upon the equation of state of the matter content. The resulting model is geometrically closed but ever-expanding and is free from the entropy, horizon, initial singularity, cosmological constant and monopole problems of the standard big bang cosmology.

2. The field equations

The universe is assumed to be filled with distribution of matter represented by the energy-momentum tensor of a perfect fluid

$$T_{ij} = (\rho + p)v_i v_j + p g_{ij}, \quad \text{(in the units with } c = 1), \tag{2.1}$$

where ρ is the energy density of the cosmic matter and p is its pressure. The geometry of an isotropic and homogeneous universe is described by the Robertson–Walker metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + R^2(t)\left\{\frac{\mathrm{d}r^2}{(1-kr^2)} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right\} \tag{2.2}$$

characterized by its scale factor $R(t)$ and the curvature parameter $k(=\pm 1, 0)$. The gravitational effects of matter and vacuum in the universe together control the nature of expansion and provide a tool to determine $R(t)$ and k through the Einstein field equations

$$R_{ij} - \tfrac{1}{2}R^h_h g_{ij} = -8\pi G\left(T_{ij} - \frac{\Lambda(t)}{8\pi G}g_{ij}\right). \tag{2.3}$$

In view of the vanishing divergence of the Einstein tensor, Λ would become constant in the absence of matter (i.e. for $T_{ij} = 0$) implying that the presence of matter is essential for a time-varying Λ. Thus the incorporation of a time-dependent cosmological constant $\Lambda(t)$ in the Einstein field equations amounts to assuming that in the presence of the usual energy momentum tensor of the matter content T_{ij} of the universe, there is an additional piece $-(\Lambda/8\pi G)g_{ij}$ which may be regarded as the energy momentum tensor of vacuum

$$T^{(\mathrm{vac})}_{ij} = -\frac{\Lambda}{8\pi G}g_{ij}, \tag{2.4}$$

representing a perfect fluid with its energy density ρ_{v} and homogeneous, isotropic pressure p_{v} satisfying the equation of state

$$p_{\mathrm{v}} = -\rho_{\mathrm{v}} = -\frac{\Lambda}{8\pi G}. \tag{2.5}$$

Thus the total energy momentum tensor of the universe due to the matter and the vacuum can be written as

$$T^{(\mathrm{tot})}_{ij} \equiv T_{ij} + T^{(\mathrm{vac})}_{ij} = (\rho_{\mathrm{t}} + p_{\mathrm{t}})v_i v_j + p_{\mathrm{t}} g_{ij}, \tag{2.6}$$

where $\rho_{\mathrm{t}} = \rho + \rho_{\mathrm{v}}$ and $p_{\mathrm{t}} = p + p_{\mathrm{v}}$. Obviously it is the total energy momentum tensor, due to the matter and the vacuum, which is conserved and not the two separately. However, the representation of Λ term by (2.4) and (2.5) is only a gross way of describing the vacuum. One would really like to replace Λ by a term that gives a proper description of the physics of the vacuum as T_{ij} does for the matter. But the main thing which is lacking for a unified theory of gravitation and particle physics is a proper energy momentum tensor for the vacuum [10]. In the context of (2.1), (2.2) and (2.5), the field equation (2.3) yields

$$-\frac{\ddot{R}}{R}=\frac{4\pi G}{3}(\rho+3p-2\rho_v)\equiv\frac{4\pi G}{3}(\rho_t+3p_t) \tag{2.7}$$

and

$$\frac{\dot{R}^2}{R^2}+\frac{k}{R^2}=\frac{8\pi G}{3}(\rho+\rho_v)\equiv\frac{8\pi G}{3}\rho_t. \tag{2.8}$$

Equation (2.7) may be deemed as an analogue of the Newtonian force law and suggests that the force per unit mass at each space-time point is determined by the active gravitational mass density $(\rho+3p)$ causing an attractive force together with the vacuum energy acting as a repulsive force. It is this repulsive force which drives expansion, with the classical de-Sitter inflation, in the early vacuum dominated universe. Here we note that the gravitational pull is exerted not only by ρ, as in the Newtonian theory, but rather by $(\rho+3p)$ which exhibits a relativistic effect. It is this additional pressure and internal energy contribution to the gravitational force which is the major cause of the problem of gravitational collapse in general relativity.

In the standard model (with $\rho_v=0$), (2.7) and (2.8) obtain $\rho R^3=\text{constant}$ in the present pressureless phase of evolution which can be interpreted as the conservation of the active gravitational mass of the comoving sphere of radius R. As there is no justification of conferring a special status upon the present epoch, we speculate that this constancy feature of the active gravitational mass is met not only in the present phase of evolution but in the early phases too. We thus assume that in the presence of pressure, the active gravitational mass is constant, i.e.

$$(\rho+3p)R^3=\text{constant}. \tag{2.9}$$

It may be noted that $\rho R^3=\text{constant}$ is a consequence of the field equations in the absence of pressure, whereas $(\rho+3p)R^3=\text{constant}$ is the result of the restriction that we have imposed on the model.

Equation (2.9), taken together with the energy-momentum conservation equation

$$\dot{\rho}+3(\rho+p)\frac{\dot{R}}{R}=0, \tag{2.10}$$

leads to

$$p^3=K(\rho+3p)^2, \quad K=\text{constant}\geqslant 0, \tag{2.11}$$

which is the equation of state for the perfect fluid constituting the matter content of the universe. It is a physically reasonable equation of state since

$$\frac{\mathrm{d}p}{\mathrm{d}\rho}=\frac{2}{3}\cdot\frac{p}{(\rho+p)}, \tag{2.12}$$

indicating that $\mathrm{d}p/\mathrm{d}\rho<1/3$ for $\rho>p$. The equation of state (2.11) taken together with the field equations leads to big bang models which we have already discussed [11].

In the presence of ρ_v, where the active gravitational mass density is modified as $(\rho_t + 3p_t)$ or equivalently as $(\rho + 3p - 2\rho_v)$, (2.9) representing the conservation of the active gravitational mass is then modified as

$$(\rho_t + 3p_t)R^3 \equiv (\rho + 3p - 2\rho_v)R^3 = \text{constant} = A(\text{say}), \tag{2.13}$$

and the energy-momentum conservation equation (2.10) is replaced by

$$\dot{\rho}_t + 3(\rho_t + p_t)\frac{\dot{R}}{R} = 0. \tag{2.14}$$

It may be noted that the constant A appearing in (2.13) may assume different values (even zero and negative ones) in the different phases of evolution specially for a decaying vacuum energy which may generate relativistic or non-relativistic particles. This is because the total active gravitational mass A is the resultant of two components; (i) a gravitational attraction caused by the material part $(\rho + 3p)$ and (ii) a repulsion produced by the vacuum. The value of A at any epoch therefore depends upon the relative contribution of these two components and hence on the relative dominance of matter and vacuum at that epoch. In the early vacuum dominated phase, $A < 0$ as (2.13) indicates. As soon as the vacuum balances its material counterpart, A becomes zero. When the vacuum further reduces and gets dominated by matter, A becomes positive. Thus the total active gravitational mass is conserved phasewise only and not throughout the evolution. The non-conservation of the active mass across phases may be a result of two phase transitions occurring in the early universe which lead to the restoration of the originally broken symmetry between weak, strong and electromagnetic interactions and thus results in a number of striking and unusual effects which take place during the phase transitions. Almost all elementary particles become massless, weak and strong interactions become long-range like electromagnetic interactions and large inhomogeneities in the energy density is created forming black holes which, after the phase transition, are soon evaporated due to the Hawking effect [12]. These phase transitions also account for the changes in $\ddot{R}$ via the discontinuities in A as is clear from

$$\ddot{R} = -\frac{4\pi G A}{3R^2}, \tag{2.15}$$

which may be obtained from (2.7) and (2.13).

As the conservation laws are closely related with the symmetry properties of the physical systems, it is natural to look for the symmetry of the space-time linked with the conservation law (2.13). In this connection, we note that a space-time is said to admit Ricci-collineation along a field vector η^i [13] if $\mathscr{L}_\eta R_{ij} = 0$, where $\mathscr{L}_\eta$ denotes the Lie-derivative along η^i. If the Ricci tensor is Lie-transported along η^i, then there exists a conservation law generator of the form $[R^j_{\rm m}\eta^{\rm m}]_{;j} = 0$ [14] and if Einstein's field equation (2.3) is satisfied, this can be written as $[T^j_{\rm m}\eta^{\rm m} + (\rho_v - \frac{1}{2}T)\eta^j]_{;j} = 0$ which provides the symmetries of the stress energy tensor depending upon the specific character of the symmetry vector η^i. Also a space-time is said to admit a family of contracted Ricci-collineation if $g^{ij}\mathscr{L}_\eta R_{ij} = 0$ which leads to a conservation law generator of the form $[\sqrt{-g}\{T^j_{\rm m}\eta^{\rm m} + (\rho_v - \frac{1}{2}T)\eta^j\}]_{;j} = 0$ [15]. Now we recall the symmetries of the

Robertson–Walker metric belonging to the family of contracted Ricci-collineation discussed by Green *et al* [16] for the symmetry vector proportional to the fluid flow vector v^i. Out of the three possible choice for $\mathscr{L}R_{ij}$ studied by them, the one having $\mathscr{L}R_{ij} = \lambda(R_{ij} - \frac{1}{4}R^{\rm h}_{\rm h}g_{ij})$ leads to a symmetry vector $\eta^i = \psi v^i$ given by $\psi = \text{constant}/\ddot{R}R^2$. Thus, in view of (2.15) which is a consequence of the conservation law (2.13), it follows that the space time admits a contracted Ricci-collineation along the fluid flow vector v^i. The conservation law (2.13) also follows directly from the conservation law generator $[\sqrt{-g}\{T^j_{\rm m}\eta^{\rm m} + (\rho_{\rm v} - \frac{1}{2}T)\eta^j\}]_{;j} = 0$ for the symmetry vector $\eta^i = \text{constant} \times v^i$ and T_{ij} given by (2.1).

It may also be noted that the left hand side of (2.7) is the Gaussian curvature of the two dimensional surface specified by varying r and t, keeping θ and ϕ constant in (2.2) and may be considered as the curvature of the isotropic, homogeneous space-time (2.2). Thus (2.7) indicates that the curvature of the space-time is governed by the total active gravitational mass density of the universe which continuously rolls down in an expanding universe, as (2.13) indicates. This consequently transforms the space-time from a state of large curvature to a state of flatness as $t \to \infty$.

The integration of (2.15) gives

$$\dot{R}^2 = \frac{8\pi GA}{3R} + B, \tag{2.16}$$

where B is a constant of integration. It is important to note that (2.16), whose integral supplies the time-variation of the scale factor $R(t)$, is independent of k – at least in the absence of any further constraint imposed on the model. It may be noted that in the standard model, three competing terms drive the universal expansion: a matter term, a cosmological constant term and a curvature term, via (2.8), for an universe which is respectively open, flat and closed as $k = -1, 0$ and $+1$. In the present case, the curvature index $k = \pm 1, 0$ plays its role via the equation of state and the initial conditions.

Equations (2.8), (2.13) and (2.16) may be used to obtain

$$\rho_{\rm t} \equiv \rho + \rho_{\rm v} = \frac{A}{R^3} + \frac{3(B+k)}{8\pi GR^2} \tag{2.17}$$

and

$$p_{\rm t} \equiv p - \rho_{\rm v} = -\frac{(B+k)}{8\pi GR^2}. \tag{2.18}$$

One can observe that the different suitable choices of B and k in (2.18) may obtain the variations of $\rho_{\rm v}$, in the present pressureless phase of evolution, as obtained by various authors in their models [3,4]. If one chooses B and k in such a way that $B + k = \delta$, where δ is a positive constant of order 1, one would get $\rho_{\rm v} = \delta/8\pi GR^2$ as obtained by Chen and Wu [3] from a dimensional analysis made in the spirit of quantum cosmology. For $B = 2$ with $k = 1$, (2.18) gives $\rho_{\rm v} = 3/8\pi GR^2$ and (2.16)–(2.18) give $\rho = \rho_{\rm c}(\equiv 3H^2/8\pi G)$ as obtained by Ozer and Taha [4].

As we have only two independent equations (2.17) and (2.18) in three unknowns ρ, p and $\rho_{\rm v}$, we need an extra equation to solve the system uniquely. This is supplied by the equation of state of the matter content. We assume that the energy density of the cosmic matter is the sum of rest mass and radiation energy densities and its pressure p is due to the radiation only. That is,

$$\rho = \rho_m + \rho_r, \quad p = p_r = \tfrac{1}{3}\rho_r. \tag{2.19}$$

Thus the assumption (2.13), taken together with Einstein's field equations, determines the time variation of the scale factor $R(t)$ and leaves the vacuum energy density ρ_v to be treated on the same footing as the material energy densities ρ_m and ρ_r so that like ρ_m and ρ_r, ρ_v too varies differently in the different phases of evolution.

Equations (2.17) and (2.18), together with (2.16) imply that

$$\rho + p = \rho_c, \quad \text{provided we take } B = 2k. \tag{2.20}$$

However, $k = -1$ demands $\rho_v < 0$ for $p = 0$. Equation (2.20), taken together with (2.19), suggests that

$$\frac{\rho}{\rho_c} \simeq \begin{cases} \frac{3}{4}, & \text{when } \rho \simeq \rho_r \gg \rho_m \text{ (in the early radiation dominated universe)}, \\ 1, & \text{when } \rho \simeq \rho_m \gg \rho_r \text{ (in the present matter dominated universe)}. \end{cases} \tag{2.21}$$

This favours the presence of some sort of dark matter which has not been detected so far.

The total-energy conservation equation (2.14), which may alternatively be written as

$$\frac{d}{dt}(\rho R^3) + p\frac{dR^3}{dt} + R^3\frac{d\rho_v}{dt} = 0, \tag{2.22}$$

suggests that the change in the entropy of matter content of the universe, given by

$$TdS \equiv d(\rho R^3) + p dR^3 = -R^3 d\rho_v, \tag{2.23}$$

is always increasing for a decaying vacuum energy though the total entropy, of matter and vacuum taken together, remains conserved as is clear from (2.14). Thus there is a spontaneous creation of matter in the form of radiation out of the vacuum energy provided $\dot{\rho}_v < 0$, which is the case in our model as we shall see later on. This can also be seen by integrating (2.22) which, in the early pure radiation era ($\rho_m = 0$) with the equation of state given by (2.19), gives

$$\rho_r = \rho_{r0}\left(\frac{R_0}{R}\right)^4 - \frac{1}{R^4}\int_{R_0}^{R} R'^4 \frac{d\rho_v}{dR'} dR', \tag{2.24}$$

where $\rho_r = \rho_{r0}$ at $R = R_0$. The subscript zero stands for the value of the quantity at $t = 0$. When matter and radiation are both present ($\rho = \rho_m + \rho_r$), equation (2.22) reads

$$\frac{d}{dt}(\rho_m R^3) + \frac{d}{dt}(\rho_r R^3) + p\frac{dR^3}{dt} + R^3\frac{d\rho_v}{dt} = 0. \tag{2.25}$$

Vacuum coupling primarily to matter would imply the continuous creation of baryon-antibaryon pairs. Naively, one would expect this effect to be greatly suppressed relative to radiation production. Moreover, observations so restrict the size of such a matter creation term that one would get [5]

$$\frac{d}{dt}(\rho_m R^3) \simeq 0, \tag{2.26}$$

so that (2.24) may be considered approximately valid for the radiation component in both—the radiation dominated as well as matter dominated eras, which shows that for $\dot{\rho}_v < 0$ with $\dot{R} > 0$, the vacuum generates a positive contribution to ρ_r. Even when $\rho_{r0} = 0$, (2.24) is valid and the vacuum energy then represents the curvature of the empty space-time which soon makes the universe non-empty by generating radiation. Thus the radiation will be created from a decaying vacuum energy all the time transforming the space-time from a state of large curvature to a state of flatness as $t \to \infty$.

We thus see that the energy momentum tensor of vacuum represented by (2.4), which is a negative field, acts just like a creation field proposed by Hoyle and Narlikar [17] operating through the interaction of a zero rest mass scalar filed C of negative energy, with matter. Because of its negative energy, the C-field has a repulsive effect tending to expand the space-time as does the positive cosmological constant Λ. In fact continuous creation can be accounted for with the equations of conventional general relativity plus an equation of state that includes negative pressure and a positive matter density [18].

The model is now completely specified in dynamical structure. The provisions for an initial condition and an equation of state, which is supplied by (2.19), determine its physical content. In the following sections, we investigate its cosmological consequences, starting with the initial phase of pure radiation.

3. The very early universe

We choose the initial time $t = 0$, when $\rho_{r0} = \dot{R} = 0$ and hence the whole vacuum energy is then locked up in potential form in the curvature of empty space-time. With this, (2.16), (2.17) and (2.18) suggest that $A = -3kR_0/4\pi G$, $B = 2k$ and $\rho_{v0} = 3k/8\pi G R_0^2$. As ρ_v is always positive in this model, we have $k > 0$ so that

$$k = 1, \quad A = -\frac{3R_0}{4\pi G} \text{ and } B = 2. \tag{3.1}$$

It is noteworthy that the initial conditions necessarily require the curvature index k to be positive. Further, the curvature can be looked upon as being generated by the Λ term which, as evolution proceeds, itself generates the radiation as evidenced by (2.24).

By the use of (3.1), (2.15) reduces to

$$\ddot{R} = \frac{R_0}{R^2}, \tag{3.2}$$

which indicates that $\ddot{R} > 0$ implying that the expansion is being driven by some general (non-exponential) type of inflation throughout the era. With (3.1), the solution of (2.16) obtains

$$R = R_0 \operatorname{cosec}^2 \psi, \quad t = \frac{R_0}{\sqrt{2}}\left(\ln \cot\frac{\psi}{2} + \operatorname{cosec}\psi \cot\psi\right). \tag{3.3}$$

The parameter ψ appearing in (3.3) can be eliminated, giving

$$t = \frac{R_0}{\sqrt{2}} \ln\left(\frac{\sqrt{R} + \sqrt{R - R_0}}{\sqrt{R_0}}\right) + \sqrt{\frac{R(R - R_0)}{2}}. \tag{3.4}$$

With (3.1), equations (2.17) and (2.18) obtain

$$\rho_r = \frac{\alpha}{R^2}\left[1 - \frac{R_0}{R}\right], \tag{3.5}$$

$$\rho_v = \frac{\alpha}{R^2}\left[1 - \frac{R_0}{3R}\right], \tag{3.6}$$

where $\alpha = 9/16\pi G$. The differentiation of (3.5) and (3.6) with respect to the cosmic time t yields

$$\dot{\rho}_r = -\frac{2\alpha\dot{R}}{R^4}\left[R - \frac{3}{2}R_0\right], \tag{3.7}$$

$$\dot{\rho}_v = -\frac{2\alpha\dot{R}}{R^4}[R - \tfrac{1}{2}R_0], \tag{3.8}$$

which indicate that for $\dot{R} \geqslant 0$, $\dot{\rho}_r \lesseqgtr 0$ according as $R \gtreqless 1{\cdot}5R_0$ while $\dot{\rho}_v \leqslant 0$ throughout the era implying that the vacuum is continuously decaying in this era and transforming into radiation whose energy density ρ_r, which is zero initially, reaches its maximum at $R = 1{\cdot}5R_0$ or equivalently at $t = 1{\cdot}24R_0$. Thereafter, it begins to decrease which may be a result of fast expansion of the universe. As soon as the created radiation reaches thermal equilibrium, it may be characterized by its temperature T given by

$$\rho_r = \frac{\pi^2}{30}N(T)T^4, \tag{3.9}$$

in suitable units. Here the effective number of spin degrees of freedom $N(T)$ at temperature T is given by

$$N(T) = N_b(T) + \tfrac{7}{8}N_f(T), \tag{3.10}$$

where $N_b(T)$ and $N_f(T)$ correspond to bosons and fermions respectively. We assume $N(T)$ to be constant throughout the pure radiation era. From (3.5) and (3.9), we have

$$T = \left[\frac{30\alpha}{\pi^2 N}\right]^{1/4}\left[\frac{1}{R^2}\left(1 - \frac{R_0}{R}\right)\right]^{1/4}. \tag{3.11}$$

Thus the temperature T is zero initially. It remains finite for all finite values of t and reaches a maximum at $t = 1{\cdot}24R_0$, given by

$$T_{\max} = \left[\frac{40\alpha}{9\pi^2 N R_0^2}\right]^{1/4}. \tag{3.12}$$

The model although starts from a cold state but need not, however, be much different from a hot initiation, for a small R_0, since the temperature increases very rapidly within a time scale of order R_0.

In order to have an estimate of R_0, we relate it with the Planck mass M_{pl}, by assuming that

$$T_{\max} = M_{pl} = G^{-1/2}. \tag{3.13}$$

This amounts to

$$R_0 \simeq 0{\cdot}28 M_{\rm pl}^{-1} N^{-1/2} \simeq 2{\cdot}3 \times N^{-1/2} \times 10^{-20} ({\rm GeV})^{-1}. \tag{3.14}$$

Thus the maximum temperature, at $R = 1{\cdot}5R_0$, is achieved at a very early time, at $t \simeq 10^{-44}$ sec.

For $R \gg 1{\cdot}5R_0$, (3.4) obtains

$$R \simeq \sqrt{(2)}t. \tag{3.15a}$$

Hence for $R \gg 1{\cdot}5R_0$, (3.5), (3.6) and (3.11) reduce to

$$\rho_{\rm r} \simeq \rho_{\rm v} \simeq \frac{\alpha}{(\sqrt{2}t)^2}, \tag{3.15b}$$

and

$$T \simeq \left[\frac{30\alpha}{\pi^2 N}\right]^{1/4} \frac{1}{(\sqrt{2}t)^{1/2}}. \tag{3.15c}$$

The corresponding expressions for these variables in the standard model are given by

$$R_{\rm SM} \sim t^{1/2}, \tag{3.16a}$$

$$\rho_{\rm rSM} = \frac{\alpha}{(\sqrt{6}t)^2} \tag{3.16b}$$

and

$$T_{\rm SM} = \left[\frac{30\alpha}{\pi^2 N}\right]^{1/4} \frac{1}{(\sqrt{6}t)^{1/2}}. \tag{3.16c}$$

Thus for $R \gg 1{\cdot}5R_0$, the values of the radiation energy density and temperature attained at time t in the standard model are attained at time $\sqrt{3}t$ in the present model. We thus see that although the present model is clearly different from the standard model in several respects such as cold initiation, non-adiabaticity and regularity at $t = 0$, it possesses for $T \geqslant T_{\rm max}$ almost the same thermal history as the standard model. However, the time dependence of R in the present model is altogether different from that of the standard model which may be effective in solving the horizon problem. As the universe is geometrically closed ($k = 1$) in the present model, it is possible to determine the time $t = t_{\rm cau}$ when the whole universe becomes causally connected. This is given by [4, 19]

$$\int_0^{t_{\rm cau}} \frac{{\rm d}t}{R(t)} = \int_0^1 \frac{{\rm d}r}{\sqrt{(1 - r^2)}} = \frac{\pi}{2} \tag{3.17}$$

which, by the use of (3.3), obtains

$$t_{\rm cau} \simeq 2{\cdot}4 R_0 \simeq 0{\cdot}68 M_{\rm pl}^{-1} N^{-1/2} \tag{3.18}$$

indicating that the global causal connection in the present model has been established at a very early time – of the order of Planck time. Thus the present model does not have a horizon problem. When the global causality is established, the maximum temperature is surpassed and the whole universe attains the temperature $T_{\rm cau}$, given by

$$T_{\rm cau} = 0{\cdot}859 T_{\rm max}, \tag{3.19}$$

which indicates that the whole universe becomes causally connected soon after attaining the Planck temperature.

4. Period of rest mass generation

Equations (3.5) and (3.6) indicate that as R becomes larger than R_0, ρ_v gradually tends to ρ_r. When $\rho_v = \rho_r$ at $R = R_1$ (say), which may be taken as the first phase transition in the model, $A = \ddot{R} = 0$, as (2.13) and (2.15) indicate, and the universe starts to expand linearly. In this phase, the radiation energy is converted into rest mass by some mechanism until the rest mass density ρ_m attains a maximum, say at $R = R_2$ which may be taken as the second phase transition leading to $\ddot{R} < 0$. Assuming that the early part of the evolution is very small compared to the matter dominated universe, one observes that R_2 may not be much larger than R_{eq} (where $\rho_m = \rho_r$) and hence one may take $R_2 \simeq R_{eq}$ with no consequence. The possibility of two phase transitions occurring in the present model is consistent with unified gauge field theories.

As $A = 0$ in this period, (2.13) suggests that

$$\rho + 3p - 2\rho_v = 0. \tag{4.1}$$

Now we consider the viscous effects of radiation in this phase, where the motion is one of pure expansion, by replacing the hydrostatic pressure p by p' given by

$$p' = p - \xi\theta, \tag{4.2}$$

where ξ is the coefficient of bulk viscosity and $\theta = 3\dot{R}/R$ is the volume expansion rate. The customary expression for ξ for a radiative fluid is given by [20]

$$\xi = \beta\rho_r, \tag{4.3}$$

where β is a constant of order unity. For $R \gg R_0$, (3.5) and (3.6) suggest that $\rho_v \simeq \rho_r \simeq \alpha/R^2$. Though this holds in the pure radiation era for $R \leqslant R_1$, it may be taken approximately valid in the rest mass generation period too as the latter period may not be very large. Thus (4.1), together with (2.19), (4.2) and (4.3), obtains

$$\rho_m R^3 \simeq 9\alpha\beta\dot{R}. \tag{4.4}$$

Thus $\rho_m R^3 \simeq$ constant, as $\ddot{R} = 0$ in this period. This indicates that the universe might have turned matter dominated by some dissipative mechanism in the rest mass generation period itself.

5. The matter and radiation era

In this period of evolution $R \geqslant R_2$ and (2.26) holds which implies that the total rest mass energy $E_m = \rho_m R^3$, remains approximately constant:

$$\rho_m R^3 = E_m^p, \tag{5.1}$$

where 'p' stands for the present value of the quantity.

Observations [8, 21] indicate that the present cosmic energy density ρ is very close to its corresponding critical value ρ_c. In our model, (2.21) indicates that $\rho \simeq \rho_c$ for $B = 2$ with $k = 1$ in the present matter dominated phase. We therefore, maintain $B = 2$ in this phase too in order to be in agreement with observations.

Now (2.13) together with (5.1) obtains

$$A = E_{\rm m}^p + 2(\rho_{\rm r}^p - \rho_{\rm v}^p)R_p^3. \tag{5.2}$$

The vacuum energy density $\rho_{\rm v}^p$, at the present epoch, is below the radiation energy density $\rho_{\rm r}^p$ and at most it is of the order of $\rho_{\rm r}^p$ [5]. Also from (2.18) together with $B = 2$ and $k = 1$, we conclude that $\rho_{\rm v}^p > \frac{1}{3}\rho_{\rm r}^p$. These constraints form limits for the present value of $\rho_{\rm v}$ as $\frac{1}{3}\rho_{\rm r}^p < \rho_{\rm v}^p \lesssim \rho_{\rm r}^p$ implying

$$\rho_{\rm v}^p = \left(1 - \frac{w}{2}\right)\rho_{\rm r}^p, \quad 0 \lesssim w < 4/3, \tag{5.3}$$

w being a dimensionless pure number. With (5.3), (5.2) gives the value of A in this era as

$$A = E_{\rm m}^p + wE_{\rm r}^p. \tag{5.4}$$

Equations (2.17) and (2.18), in this phase, thus yield

$$\rho_{\rm r} = \frac{\alpha}{R^2}\left[1 + \gamma\frac{R_{\rm p}}{R}\right] \tag{5.5}$$

and

$$\rho_{\rm v} = \frac{\alpha}{R^2}\left[1 + \frac{\gamma}{3}\frac{R_{\rm p}}{R}\right], \tag{5.6}$$

where $\gamma = (4\pi wG/3)\,\rho_{\rm r}^p R_{\rm p}^2$ is a positive dimensionless number. Equation (5.5) indicates that the total radiation content of the universe $E_{\rm r}$, which is given by

$$E_{\rm r} \equiv \rho_{\rm r} R^3 = \alpha\gamma R_{\rm p} + \alpha R, \tag{5.7}$$

is a function of R and increases as R increases. This is altogether different from the case in Ozer-Taha model [4] which has a decreasing $E_{\rm r}$ in a certain interval of time, in spite of radiation always being created.

Equations (5.1) and (5.5) imply that

$$\rho_{\rm m} \gtreqless \rho_{\rm r} \quad \text{according as} \quad R \lesseqgtr \frac{1}{\alpha}\left(E_{\rm m}^p - \frac{3w}{4}E_{\rm r}^p\right). \tag{5.8}$$

Thus $\rho_{\rm m} = \rho_{\rm r}$ is obtained at (i) $R = R_{\rm eq}(\simeq R_2)$ and (ii) $R = 1/\alpha(E_{\rm m}^p - (3w/4)E_{\rm r}^p) = R_{\rm eq}^{\rm II}$ (say).

Thus a second era of radiation dominance starts at $R_{\rm eq}^{\rm II}$ which, for the estimates of $R_{\rm p}, \rho_{\rm r}^p$ and $\rho_{\rm m}^p$ as obtained later on, is found to be $R_{\rm eq}^{\rm II} \simeq 9{\cdot}6 \times 10^{47}(\text{GeV})^{-1}$.

The time-dependence of the scale factor $R(t)$ in this era may be obtained from (2.16) as

$$R = \frac{3A}{8\alpha}(\cosh 2\psi - 1), \quad t = \frac{3A}{8\sqrt{2}\alpha}(\sinh 2\psi - 2\psi) + C, \tag{5.9a}$$

where A is given by (5.4) and C, the constant of integration, is some function of R_2. Equation (5.9a) indicates that $R \to \infty$ and $t \to \infty$ although $k = 1$.

Equation (5.9a) may alternatively be written as

$$t = [8^{-1}R(4R + 3\alpha^{-1}A)]^{1/2} + 16^{-1}A\,3\sqrt{2}\alpha^{-1} \times \ln\left[\frac{\sqrt{4R + 3\alpha^{-1}A} - 2\sqrt{R}}{\sqrt{4R + 3\alpha^{-1}A} + 2\sqrt{R}}\right] + C. \tag{5.9b}$$

The radiation energy density ρ_r, at present, is given by

$$\rho_r^p = \frac{\pi^2 N(T_p) T_p^4}{30}. \tag{5.10}$$

For the variation of ρ_r given by (5.5), one can observe, along the lines of Ozer–Taha [4], that $N(T_p) = 43/11$ in the present model. Hence (5.10), with the experimental value of $T_p = 2{\cdot}7°K$, yields

$$\rho_r^p \simeq 3{\cdot}8 \times 10^{-51} (\text{GeV})^4. \tag{5.11}$$

Equation (2.20) may now be used to obtain the present total (non-vacuum) energy density $\rho_p (= \rho_m^p + \rho_r^p)$ as

$$\rho_p = \frac{2\alpha}{3} H_p^2 - \tfrac{1}{3}\rho_r^p. \tag{5.12}$$

Taking $H_p = 75\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$ as favoured by the recent observations, (hence $\rho_c^p = 4{\cdot}4 \times 10^{-47} (\text{GeV})^4$), we get

$$\rho_p \simeq 4{\cdot}4 \times 10^{-47} (\text{GeV})^4. \tag{5.13}$$

Thus the present universe is matter dominated with $\rho_p \simeq \rho_c^p \simeq \rho_m^p$.

In order to obtain an estimate of the present values of the observables R_p and ρ_v^p, one may use (5.11) in (5.5) and (5.3) giving

$$R_p = \left[\frac{\alpha}{(1 - \frac{3}{4}w)\rho_r^p}\right]^{1/2} = (1 - \tfrac{3}{4}w)^{-1/2} \times 8{\cdot}4 \times 10^{43} (\text{GeV})^{-1}, \tag{5.14}$$

$$\rho_v^p = \left(1 - \frac{w}{2}\right) \times 3{\cdot}8 \times 10^{-51} (\text{GeV})^4. \tag{5.15}$$

Another important quantity is the deceleration parameter q defined by $q = -R\ddot{R}/\dot{R}^2$ which, from (2.15) and (2.16) obtains

$$q = \left[2 + \frac{8\alpha R}{3A}\right]^{-1}, \tag{5.16}$$

so that the present value of q is given by

$$q_p \simeq \tfrac{1}{2}\left[1 - 0{\cdot}00012\left(1 - w\frac{\rho_r^p}{\rho_m^p}\right)\right] \simeq \tfrac{1}{2}, \tag{5.17}$$

independently of w. Although w, which is not exactly known, occurs in the expressions for R_p and ρ_v^p, it is possible to find the estimates of these quantities as the maximum uncertainty in the value of w is $\frac{4}{3}$ which does not much effect the values of these observables given by (5.14) and (5.15).

We thus have

$$R_p \gtrsim 8{\cdot}4 \times 10^{43} (\text{GeV})^{-1}, \quad \rho_v^p \lesssim 3{\cdot}8 \times 10^{-51} (\text{GeV})^4. \tag{5.18}$$

The present value of the cosmological constant is then obtained as

$$\Lambda_p \lesssim 6{\cdot}4 \times 10^{-88} (\text{GeV})^2, \tag{5.19}$$

which is well within the upper limit of the observed value of Λ_p found as $10^{-82}(\mathrm{GeV})^2$ [22].

Assuming that the early evolution (pure radiation era followed by the rest mass generation regime) lasts for a very small time and the maximum part of the evolution is matter dominated, we can drop the constant of integration C in (5.9) giving the approximate age of the universe as

$$t_p \simeq \tfrac{2}{3} \cdot H_p^{-1} \simeq 8{\cdot}9 \times 10^9\,\text{years}, \tag{5.20}$$

which is approximately the same as in the standard model. Thus the model has the same age problem as does the standard model. That is, the age of the universe is smaller than $12–18 \times 10^9$ yrs, the age of the globular clusters. However, the comparison of the age of the universe from observations with that from models is some what inconclusive [23]. Moreover, there is still an uncertainty in the value of H_p which varies by a factor of $2{:}50 \leqslant H_p \leqslant 100\,\mathrm{kms}^{-1}\,\mathrm{Mpc}^{-1}$ rendering the age of the universe in our model as $6{\cdot}7–13{\cdot}3 \times 10^9$ yrs.

Finally we consider the monopole problem of the standard cosmology. Although our discussion so far has not been field theoretic as the present model is presented in purely classical terms, the model does solve the monopole problem associated with grand unified theories. The present monopole number density is found to be given by [24]

$$n_{\mathrm{M}}(t_p) \simeq \mathrm{d}_{\mathrm{H}}^{-3}(t_c)\left[\frac{R(t_c)}{R_p}\right]^3, \tag{5.21}$$

where t_c is the time of monopole production corresponding to the grand unifications – phase transition-temperature $T_c \simeq 10^{15}$ GeV and $\mathrm{d}_{\mathrm{H}}(t_c)$ is the horizon distance at time $t = t_c$ defined by [19]

$$\mathrm{d}_{\mathrm{H}}(t_c) = R(t_c)\int_0^{r_{\mathrm{H}}(t_c)} \frac{\mathrm{d}r}{\sqrt{1-kr^2}} = R(t_c)\int_0^{t_c}\frac{\mathrm{d}t'}{R(t')}, \tag{5.22}$$

which with (3.3), obtains

$$\mathrm{d}_{\mathrm{H}}(t_c) = \sqrt{2}\,R(t_c)\ln\left[\frac{\sqrt{R(t_c)} + \sqrt{R(t_c) - R_0}}{\sqrt{R_0}}\right]. \tag{5.23}$$

Noting that $R(t_c) \gg R_0$, we have

$$\mathrm{d}_{\mathrm{H}}(t_c) \simeq \sqrt{2}\,R(t_c)\ln\left[2\sqrt{\frac{R(t_c)}{R_0}}\right], \tag{5.24}$$

which, by the use of (3.11) and (3.12), obtains

$$\mathrm{d}_{\mathrm{H}}(t_c) \simeq \sqrt{2}\,R(t_c)\ln\left[108^{1/4}\left(\frac{T_{\max}}{T_c}\right)\right], \tag{5.25}$$

where we have taken $N(T_c) = N(T_{\max})$. Now (5.21) yields

$$n_{\mathrm{M}}(t_p) \simeq \left[\sqrt{2}\,R_p \ln\left\{108^{1/4}\left(\frac{T_{\max}}{T_c}\right)\right\}\right]^{-3}. \tag{5.26}$$

For R_p given by (5.18) and $T_{max} = M_{p1} = 1{\cdot}22 \times 10^{19}$ GeV, (5.26) gives $n_M(t_p) \lesssim 5 \times 10^{-136}(\text{GeV})^3$. Thus for monopoles of mass 10^{16} GeV, the present monopole energy density $\lesssim 5 \times 10^{-120}(\text{GeV})^4$ indicating that the present cosmic energy density is clearly far from being dominated by monopoles.

6. Conclusions

Assuming the total active gravitational mass of the universe phasewise constant, we have investigated a cosmological model admitting a contracted Ricci-collineation along the fluid flow vector. The resulting model is geometrically closed ($k = 1$) but ever expanding and is free from the main problems of big bang cosmology, viz. the initial singularity, horizon, entropy, monopole and cosmological constant problems. The physical picture of the model is described in three phases:

Phase 1: The universe evolves from rest with $S = T = \rho = 0$ and $R = R_0$. Radiation is created by the unfolding of space time curvature, as is clear from (2.24), heating up the universe to a maximum temperature $T_{max} = M_{pl}$ at $t = 1{\cdot}24R_0$ with $\rho_r = \rho_{r\,max}$ too. Shortly after attaining T_{max}, the universe is globally casually connected at $t = 2{\cdot}4R_0$ with $T_{cau} \simeq 0{\cdot}86T_{max}$. For $t \gg R_0$, $T(t) \sim t^{-1/2}$ and $\rho_r \sim t^{-2}$ as in the standard model while $R(t) \sim t$ which is different from $R(t) \sim t^{1/2}$ of the standard model. This phase is vacuum dominated, however, for $R \gg R_0, \rho_v \to \rho_r$.

Phase 2: As soon as ρ_v equals to ρ_r at $R = R_1$, $\ddot{R} = 0$ and the universe starts to expand linearly. This is interpreted as the first phase transition in the model. Rest mass emerges from the created radiation and reaches a maximum at $R = R_2 \simeq R_{eq}$, which has been taken as the second phase transition in the model resulting in the reversal of sign of $\ddot{R}$. Though the mechanism of generation of rest mass is not known, it is possible to infer from the dissipative effects of radiation that the whole rest mass content of the universe would have been generated in this period itself and the universe turned matter dominated.

Phase 3: This is the present phase of matter dominance with $\ddot{R} < 0$. For the present radiation temperature $T_p = 2{\cdot}7°$K and the present value of the Hubble parameter $H_p = 75\,\text{kms}^{-1}\text{Mpc}^{-1}$, the predicted values of some observables are found. These include $\rho_r^p = 3{\cdot}8 \times 10^{-51}(\text{GeV})^4$, $\rho_m^p \simeq 4{\cdot}4 \times 10^{-47}(\text{GeV})^4$, $R_p \gtrsim 8{\cdot}4 \times 10^{43}(\text{GeV})^{-1}$, $q_p \simeq \frac{1}{2}$, the age of the universe $t_p \simeq 8{\cdot}9 \times 10^9$ years, $\Lambda_p \lesssim 6{\cdot}4 \times 10^{-88}(\text{GeV})^2$ and the present monopole energy density $\lesssim 5 \times 10^{-120}(\text{GeV})^4$. The value of Λ_p is well within the experimental limit which is remarkable since the initial value of Λ is very large.

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